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# Theoretical Investigation of Langmuir Wave Instability in Irradiated Dusty Plasmas

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**Abstract**

Instability of high frequency wave, such as, Langmuir wave (LW) is studied using fluid model in collisional and streaming dusty plasma with dust charge fluctuation. In irradiated dusty plasma, dust charge fluctuation is considered to be due to the combined effect of LW perturbation and photoelectron emission. The piled up electrons on a dust grain due to LW perturbation can be swept out by photoelectric effect. The LW can be unstable due to dust charge fluctuation effect when the streaming velocity of electrons exceeds the phase velocity of the wave, provided that the electron neutral collisional damping is negligible. Growth rate of the instability of LW is discussed.

**Keywords:**

dusty plasma, Langmuir wave, instability, dust charge fluctuation, irradiation, photoelectric effect, streaming, collision

## 1. Introduction

Dust grains with in the range of 10 nm to 100 μm can be immerged both in natural and laboratory plasmas. Due to the interaction of electrons, ions, and background radiation, dust grains can be charged with the order of $Q_d \sim 10^3e - 10^4e$. The dust grains can be charged by absorbing the electrons and ions flowing onto their surface. Due to the high mobility of the electrons, the grain charge is usually negative. On the other hand, in a radiation back ground, dust grains which emit photoelectrons, may become positively charged. Negative and positive dust grains can coexist in some situations of dusty plasma [1,2]. In some special circumstances of dusty plasma, dust grains can be charged significantly due to thermonic emission, impact ionization, *etc*.

Presence of the dynamics of dust grains in plasma can modify the existing plasma modes or may introduce new eigenmodes [3]. Theoretical prediction of the modified low frequency mode by dust dynamics–dust ion acoustic mode–and new dust modes of below the ion cyclotron frequency–dust acoustic mode–have been confirmed in a laboratory [4-7]. In a dusty plasma, collective perturbation of plasma parameters can exhibit self consistent dust charge fluctuations in response to oscillations in the plasma currents flowing into them. The self consistent dust charge fluctuation oscillation can be sustained by the low frequency oscillation of the order of ion oscillation and usually leads the damping of the mode [8,9]. Usually, high frequency electron plasma wave (such as Langmuir wave) perturbations can not influence the self consistent dust charge fluctuation oscillation. However, dust charge fluctuation is a new dynamical variable in dusty plasma. Properties of this dynamical variable with different charging mechanisms have been studied [10,11]. Studies on dust charge fluctuation in a dusty plasma have also been carried out in laboratory experiment [12].

In this paper, a different mechanism of sustaining high frequency dust charge fluctuation oscillation by the combined effect of LW and photoelectron emission is proposed. The pilled up perturbed electron on the dust grain by Langmuir oscillation can be removed by photoelectric emission. It is found that the LW can be unstable due to dust charge fluctuation effect in irradiated streaming dusty plasma, provided electron neutral collisional damping is negligible. In a collisionless plasma, when streaming velocity of electrons exceeds the wave phase velocity then the growth rate of LW due to dust charge fluctuation effect depends on equilibrium dust grain density, radius of dust grain, thermal velocity of electrons, and electron streaming velocity. The manuscript is organized as follows: In Sec. **2**, dispersion relation of LW is derived using fluid model. Finally, in Sec. **3**, discussion and conclusion of our results are given.

## 2. Dispersion relation

We have considered a homogeneous and uniform dusty

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plasma consisting of electrons, ions, negatively/positively charged dust grains and neutrals. The neutral gas is taken to be at rest and the electrons are streaming with a velocity $v_{e0}$. The dust charge $Q_d = \pm Z_d e$, where $Z_d$ is the number of charge on a dust grain and $e$ is the electron charge. Let us consider a high-frequency electrostatic wave propagating in dusty plasma. Due to the presence of this mode ($\omega$, $\boldsymbol{K}$), the dust will acquire a perturbed charge, $Q_{d1}$, where $\omega$ and $\boldsymbol{K}$ are the frequency and propagation vector of the mode, respectively. The high frequency oscillation like Langmuir oscillation is so fast that the massive ions and dusts do not have time to respond to the oscillating field and may be considered as fixed. Therefore, we have ignored the ions and dusts dynamics and also have ignored the ions contribution in the fluctuation of dust grain charge.

Momentum and continuity equations for electrons in the dusty plasma are taken as follows, respectively:

$$mn_e\left[\frac{\partial \boldsymbol{v}_e}{\partial t} + (\boldsymbol{v}_e.\nabla)v_e\right] = -en_e\boldsymbol{E} - \nabla p_e - v_e n_e m_e \boldsymbol{v}_e, \quad (1)$$

$$\frac{\partial n_e}{\partial t} + \nabla.(n_e\boldsymbol{v}_e) = 0, \quad (2)$$

where the suffix $e$ denotes the electron quantities and $m$, $n$, $\boldsymbol{v}$, and $p$ are the mass, density, velocity, and pressure of electron, respectively. The quantity $\boldsymbol{E}$ is the electric field and $v_e$ is the collisional frequency of electrons with neutral atoms/molecules.

The momentum and continuity equations, *i.e.*, eqs. (1) and (2) will be coupled to the following set of equations, namely: Poisson's equation

$$-\nabla^2\phi = 4\pi e\left[\frac{Q_d n_d}{e} - n_e\right]. \quad (3)$$

Quasi-neutrality equation

$$n_e = n_i + \frac{Q_d n_d}{e}, \quad (4)$$

and basic dust charging equation

$$\frac{dQ_d}{dt} = I_e + I_{pe} + I_i, \quad (5)$$

where $I_e$, $I_i$, and $I_{pe}$ are, respectively, the electron current, ion current collected by the dust grains and photoelectric current.

If the electron streaming velocity becomes much larger than the thermal velocity of electrons, then the approximated expression of electron current to positively charged dust grain is given by:

$$I_e = -\pi r_d^2(en_e v_e)\left[1 - \frac{2e(\phi_p - \phi_d)}{m_e v_e^2}\right]. \quad (6)$$

The photoelectric current is given by

$$I_{pe} = \pi r_d^2 e n_e\sqrt{\frac{2}{m_e}(h\nu_p - e\phi_0)}, \quad (7)$$

where $h$, $\nu_p$ and $\phi_0$ are the Planck's constant, irradiation frequency, and work function, respectively. The quantities, $r_d$, $\phi_p$, and $\phi_d$ are the dust radius, bulk plasma potential, and dust grain surface potential, respectively. Expression of ion current to dust grain is not discussed, because the contribution of ion current to dust charge fluctuation is not considered in this study.

Linearizing eqs. (1)–(7) by the usual technique and considering the perturbed quantities vary as *exp* [i($K.r - \omega t$)], then we get:

$$\bar{\omega} n_{e1} - K n_{e0} v_{e1} = 0, \quad (8)$$

$$v_{e1}(\bar{\omega} + \mathrm{i}\nu_e) - K v_{the}^2\frac{n_{e1}}{n_{e0}} + \frac{e}{m_e}K\phi_1 = 0, \quad (9)$$

$$\phi_1 = \frac{4\pi e}{K^2}\left[-n_{e1} + \frac{n_{d0}}{e}Q_{d1}\right], \quad (10)$$

$$\frac{dQ_{d1}}{dt} = I_{e1} + I_{pe1}, \quad (11)$$

and

$$I_{e1} = -e\pi r_d^2(n_{e0}v_{e1} + n_{e1}v_{e0})\left[1 + \frac{2e\phi_{d0}}{m_e v_{e0}^2}\right], \quad (12)$$

$$I_{pe1} = \pi r_d^2 e n_{e1}\sqrt{\frac{2}{m_e}(h\nu_p - e\phi_0)}, \quad (13)$$

where Doppler shifted frequency, $\bar{\omega} = \omega - Kv_{e0}$ and electron thermal velocity, $v_{the} = \sqrt{k_B T_e/m_e}$. At equilibrium, $\nabla n_{e0} = \phi_{p0} = 0$ and $I_{e0} + I_{p0} + I_{i0} \approx 0$. Noting that $I_{i1} = 0$.

From eqs. (8) and (9), we get the perturbed electron density as

$$n_{e1} = -\frac{e n_{e0} K^2 \phi_1}{m_e[\bar{\omega}(\bar{\omega} + \mathrm{i}\nu_e) - K^2 v_{the}^2]}. \quad (14)$$

Using eqs. (12), (13) and (8), we get the dust charge fluctuation from Eq.(11) as:

$$Q_{d1} = \frac{\mathrm{i}|I_{pe0}|}{\omega}\frac{n_{e1}}{n_{e0}} - \frac{\mathrm{i}|I_{e0}|}{Kv_{e0}}\frac{n_{e1}}{n_{e0}}, \quad (15)$$

Using the value of $Q_{d1}$ from eq. (15), we get the linearized Poission's equation of eq.(10) as:

$$\phi_1 = \frac{4\pi e}{K^2}\left[-1 + \frac{\mathrm{i}\beta_{pe}}{\omega} - \frac{\mathrm{i}\beta}{Kv_{e0}}\right]n_{e1}, \quad (16)$$

where $\beta = (|I_{e0}|/e)(n_{d0}/n_{e0}) = [1 + (2e\phi_{d0})/(m_e v_{e0}^2)]\pi r_d^2 n_{d0} v_{e0}$, $\beta_{pe} = (|I_{pe0}|/e)(n_{d0}/n_{e0}) = \pi r_d^2 n_{d0} v_{pe}$, and the velocity of photo electron is $v_{pe} = \sqrt{2(h\nu_p - e\phi_0)/m_e}$. The i$\beta$ and i$\beta_{pe}$ terms are arising through coupling to dust charge fluctuations. The coupling parameter $\beta$ is like an effective collision frequency of the streaming electrons with the dust grains. On the other hand, $\beta_{pe}$ is the effective detachment frequency of photo electrons from the dust grains.

The dispersion relation of the LW in a collisional and streaming dusty plasma with dust charge fluctuation is obtained from eq.(16) using the value of $n_{e1}$ from eq.(14):

$$\omega = \pm\sqrt{(\omega_{pe}^2 + K^2 v_{the}^2)} + Kv_{e0} - \frac{i}{2}\nu_e + \frac{i}{2}\left[\frac{|I_{pe0}|}{\omega(Kv_{e0}-\omega)} + \frac{|I_{e0}|}{Kv_{e0}(\omega - Kv_{e0})}\right]\frac{\omega_{pe}^2}{e}\frac{n_{d0}}{n_{e0}}, \quad (17)$$

where electron plasma frequency $\omega_{pe} = \sqrt{4\pi n_{e0}e^2/m_e}$. The first term in the right hand side of eq.(17) is the Langmuir mode. The second term is the term due to streaming of electrons and which shifts the real part of the Langmuir wave. Third term is the electron neutral collision term, which gives the damping of the mode. The fourth and fifth imaginary terms are due to the dust charge fluctuation.

In the case of $v_{e0} > \omega/K$ the dispersion relation, *i.e.*, eq. (17) reduces to

$$\omega = \pm\sqrt{(\omega_{pe}^2 + K^2 v_{the}^2)} + Kv_{e0} - \frac{i}{2}\nu_e + \frac{i}{2}\frac{|I_{pe0}|}{\omega K v_{e0}}\frac{\omega_{pe}^2}{e}\frac{n_{d0}}{n_{e0}}. \quad (18)$$

In the case of $K\lambda_{De} = 1$ and since, $v_{e0} > v_{the}$, the growth rate of Langmuir wave in a collisionless dusty plasma ($\nu_e = 0$) is given by

$$\gamma = \frac{\pi}{2}(r_d^2 n_{d0})\frac{v_{the}^2}{v_{e0}}, \quad (19)$$

where electron Debye length is $\lambda_{De} = v_{the}/\omega_{pe}$ and $v_{e0} \approx v_{pe}$ is considered. Therefore, in a collisionless dusty plasma where the electron streaming velocity exceeds the wave phase velocity then the amplitude of Langmuir wave grows according to eq. (19) due to dust charge fluctuation.

## 3. Discussion and conclusion

Instability of the high frequency wave, such as, Langmuir wave is studied using fluid model in a collisional and streaming dusty plasma with dust charge fluctuation. In irradiated dusty plasma, dust charge fluctuation is considered to be due to the combined effect of Langmuir wave perturbation and photoelectron emission. To discuss the properties of the dispersion relation of eq. (17) for Langmuir wave, at first we consider the case of without dust charge fluctuation. In this case, it is seen that the streaming velocity of electrons shifts the real part of the Langmuir wave frequency in the laboratory frame. The collisional effect of electrons with the neutrals gives the damping of the mode.

In the case of $v_{e0} > \omega/K$, it can be seen from the fourth term of eq. (17) or, eq. (18) that the dust charge fluctuation gives the instability of the Langmuir wave. Irradiated photon energy transfers to the electrons on a dust grain through the photoelectric effect. The photoelectric current is coupled with the Langmuir wave through dust charge fluctuation. Dust charge fluctuation makes the Langmuir wave to grow when the electron streaming velocity exceeds the wave phase velocity. The growth rate of Langmuir wave in the case of $K\lambda_{De} \approx 1$ is given by eq. (19). In this limit, the growth rate of LW due to dust charge fluctuation effect depends on equilibrium dust density, radius of dust, thermal velocity of electrons, and electron streaming velocity.

The approximated lighter particle current, such as, electron current to negatively charged dust grains, i.e., particle current to dust grain in retarding sheath potential can be obtained by changing the sign of dust potential ($\phi_d$) of eq.(6) [13]. In the case of $v_{e0} \approx v_p$, the obtained dispersion relation [cf. eq.(18)] is independent of $\phi_d$. In this limit, the dispersion relation of eq. (18) for Langmuir wave in the case of positively charged dust grains can be considered for the case of negatively charged dust grains. Present investigation clarifies many physical phenomenon related with high frequency mode both in laboratory and natural dusty plasmas, where the dust grains are positively/negatively charged.


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