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Three-Dimensional Electrostatic Particle Simulation of Parallel-Flow-Shear Driven Low-Frequency Plasma Instabilities

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Abstract

Three-dimensional electrostatic particle simulations are performed in order to investigate the effects of ion and electron flows parallel to magnetic-field lines and their velocity shears on low-frequency plasma instabilities in detail. It is clarified that not only the electron flow but also the ion flow can excite the instabilities in the same way. In addition, the parallel ion flow velocity shear can excite the ion-acoustic instability, even when the ion flow velocity is so small that the instability can not take place.

Keywords:

three-dimensional electrostatic particle simulation, parallel flow velocity shear, ion-cyclotron instability, ion-acoustic instability, electron velocity distribution function

1. Introduction

It is well known that sheared plasma flows perpendicular to magnetic-field lines are recognized as an important element in the transition from L to H mode confinement in fusion-oriented plasmas [1], which are also related to the suppression of low-frequency instabilities. On the other hand, magnetic field-aligned (parallel) plasma flow velocity shear has been regarded as playing an important role in the generation of low-frequency plasma instabilities, which was first investigated theoretically and experimentally as a destabilization origin of the Kelvin-Helmholtz (D'Angelo mode) instability over three decades ago [2,3]. Furthermore, in recent theoretical works using a kinetic treatment, the parallel flow velocity shear is found to cause not only the D'Angelo mode instability but also electrostatic ion-cyclotron and ion-acoustic instabilities [4]. In order to experimentally clarify the effects of the velocity shear on the instabilities in the real situation, we have carried out laboratory experiments using newly-developed plasma sources [5], which can externally control a radial profile of the field-aligned flow velocity, that is to say the parallel flow velocity shear, in the absence of perpendicular velocity shear. According to the experimental results, it is demonstrated that the drift-wave and D'Angelo mode instabilities are excited and suppressed by the parallel velocity shear, where the destabilizing and stabilizing mechanisms are well explained by the kinetic theory [6]. In the experimental investigation, however, it is difficult to change the shape and the location of the velocity shear, and the plasma parameters such as the ratio of the ion to electron temperature, which is very effective in the growth rate of the shear driven instabilities. In this sense, a particle simulation is very useful method to clarify the effects of the velocity shear, because the simulation can easily set these parameters. Concerning the particle simulation on this subject, the relation between the parallel flow velocity shear and the low-frequency plasma instabilities is investigated by Gavrishchaka *et al.* [7]. They adopt a two-and-half dimensional (2-1/2D) electrostatic particle simulation code and nonlinear properties of multiple ion cyclotron harmonic waves are discussed for the purpose of understanding results of astronomical observations. From the viewpoint of investigating the general properties of the velocity shear driven instabilities, however, the simulation should be performed in the three dimensional (3D) system because in most cases waves propagate obliquely or perpendicularly to the direction of the flow velocity gradient under the influence of the velocity shear. Therefore, in order to understand the experimental results and clarify the essential mechanism of the shear driven instabilities, we have attempted to investigate the time evolutions of various spatial Fourier modes of the ion density by using the three dimensional electrostatic particle simulation and to compare the simulation results with the kinetic theory.

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2. Simulation model

We employ a three dimensional electrostatic particle simulation with a periodic boundary model, where an external uniform magnetic field directs to the positive z direction. Our simulation code applies the particle-in-cell (PIC) method [8], which follows particle motions in the self-consistent electric field produced by charged particles. We calculate full ion and electron dynamics. Electrons and ions are uniformly loaded in the system with $L_x \times L_y \times L_z$ at $t = 0$. The system sizes L_x, L_y and L_z are $128\lambda_{De}$, $128\lambda_{De}$ and $512\lambda_{De}$, respectively, and $128 \times 128 \times 512$ spatial grid system is used. Here, λ_{De} is the Debye length. The number of electrons and ions per unit cell is 64. The ion to electron mass ratio m_i/m_e is fixed at 400. The ratio of the electron cyclotron to electron plasma frequency is $\omega_{ce}/\omega_{pe} = 5$, and the ratio of the ion-cyclotron frequency to electron plasma frequency is $\omega_{ci}/\omega_{pe} = 0.0125$. The ion to electron temperature ratio is $T_i/T_e = 0.5$. The time step width Δt is $0.1\omega_{pe}^{-1}$. The parallel ion flow velocity shear is introduced by means of changing the ion flow velocity v_{di} spatially in the x direction, the shape of which is given by

$$v_{di}(x) = 0.3v_{te} + 0.2v_{te} \exp\left(\frac{x - 64\lambda_{De}}{24\lambda_{De}}\right), \qquad (1)$$

where $v_{te}(= (T_e/m_e)^{1/2})$ is electron thermal speed, T_e and m_e are electron temperature (energy unit) and electron mass, respectively. Here, the initial electron velocity distribution is a stationary Maxwellian and the initial ion velocity distribution is a shifted Maxwellian with the flow velocity v_{di}. Figure 1 shows profiles of the ion flow velocity $v_{di}(x)$ in the cases of uniform flow (solid line) and non-uniform flow (dashed line) described by Eq. (1). In the non-uniform flow case, the spatially averaged flow velocity is $0.37v_{te}$, and the maximum shear strength is $\mathrm{d}v_{di}/\mathrm{d}x = \pm\ 0.5\omega_{ci}$ around $x = 48\lambda_{De}$ and $80\lambda_{De}$. In the case of the uniform flow, the parallel flow is assigned to not only ions but also electrons as described in Sec. **3**. **1**.

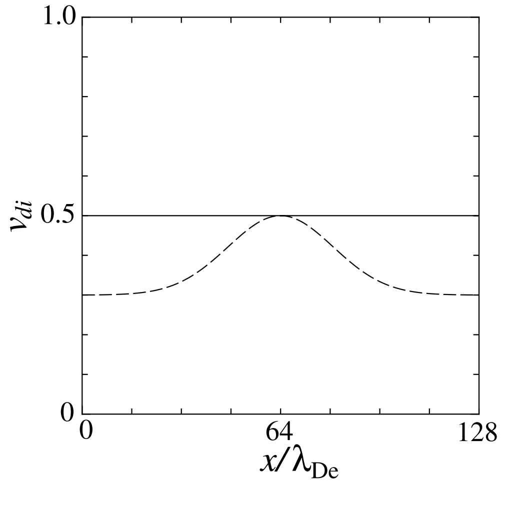

Fig. 1 Profiles of ion flow velocity v_{di} in the cases of uniform flow (solid line) and non-uniform flow (dashed line) in the x direction.

3. Simulation results and discussion

3.1 Effects of ion and electron flows on the instabilities

Before investigating the effects of the ion flow velocity shear on the instabilities, we compare the effects of uniform ion flow with those of uniform electron flow which is known to give rise to the conventional current driven instabilities. First, the parallel flow velocity is assigned to the electrons uniformly in the x direction and given by $v_{de} = 0.8v_{te}$ so as to verify the simulation results of Ishiguro *et al.* [9], where the parallel electron flow is adopted in the 2-1/2D simulation model and fluctuation properties of the ion-cyclotron instability such as the time evolutions of spatial Fourier modes are investigated in detail. Second, the parallel flow velocity is assigned to the ions uniformly and given by $v_{di} = 0.8v_{te}$ in the absence of the electron flow, where the ion flow velocity is the same as the electron flow velocity in the first case. The differences in the fluctuation properties between the two cases are discussed.

Figure 2 shows (a) time evolutions of the real (solid line) and the imaginary (dashed line) parts of the spatial Fourier mode of the ion density fluctuation $\tilde{n}_i/\bar{n}_i$($\bar{n}_i$: time averaged ion density) and (b) a frequency spectrum of the fluctuations with $k_x\rho_i = 0$, $k_y\rho_i = 0.412$, $k_z\rho_i = 0.103$ for the case with the uniform electron flow, where $\rho_i = (T_i/m_i\omega_{ci}^2)^{1/2}$ is the ion gyroradius, k_x, k_y and k_z are wave numbers in x, y and z directions, respectively. The amplitude of the ion density fluctuation is found to increase from 0.001 to 0.005, indicating a destabilization of this mode. The obtained growth rate is $\gamma/\omega_{ci} = 0.11$ and the fluctuation frequency is $0.015\omega_{pe}$

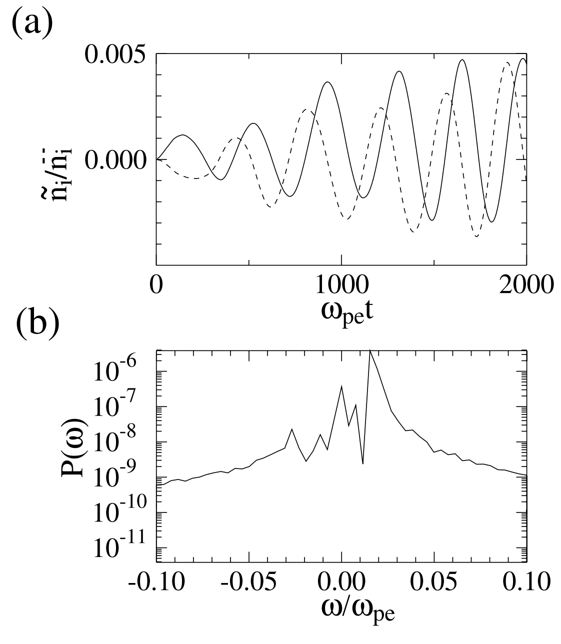

Fig. 2 (a) Time evolutions of the real (solid line) and the imaginary (dashed line) parts of the spatial Fourier mode of the ion density fluctuation $\tilde{n}_i/\bar{n}_i$, and (b) a frequency spectrum of the fluctuations with $k_x\rho_i = 0$, $k_y\rho_i = 0.412$, $k_z\rho_i = 0.103$ for the case with uniform electron flow $v_{de} = 0.8v_{te}$.

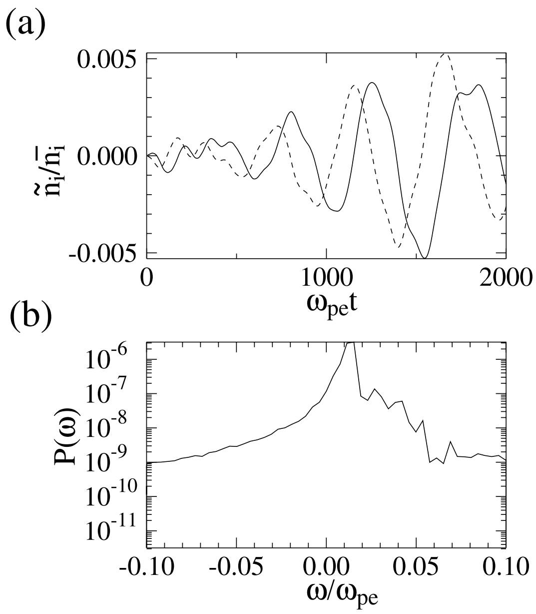

Fig. 3 (a) Time evolutions of the real (solid line) and the imaginary (dashed line) parts of the spatial Fourier mode of the ion density fluctuation $\tilde{n}_i/\bar{n}_i$, and (b) a frequency spectrum of the fluctuations with $k_x\rho_i = 0$, $k_y\rho_i = 0.412$, $k_z\rho_i = 0.103$ for the case with uniform ion flow $v_{di} = 0.8v_{te}$

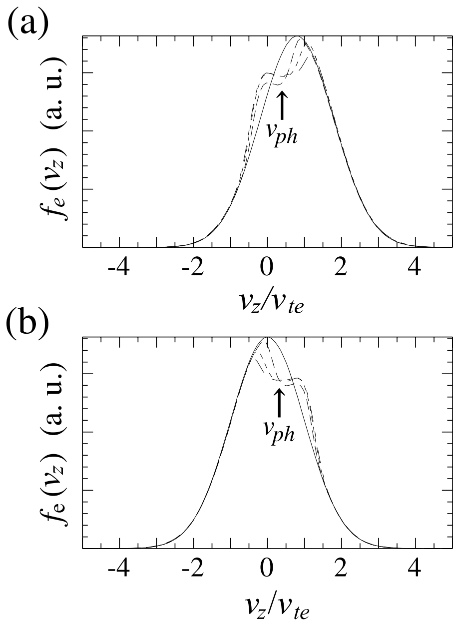

Fig. 4 Electron velocity distribution functions $f_e(v_z)$ at $\omega_{pe}t = 0$ (solid line), 1000 (dashed line), 2000 (short-dashed line) and 3000 (dot-dashed line) (a) for the case with uniform electron flow, and (b) for the case with uniform ion flow.

which is close to the ion-cyclotron frequency. The parallel component of the phase velocity of the fluctuation is calculated to be $v_{ph} = 0.41v_{te}$. As understood from the fluctuation frequency, the observed instability turns out to be the electrostatic ion-cyclotron instability. This result is consistent with that of Ishiguro *et al.* [9].

Figure 3 presents (a) time evolutions of the ion density fluctuation $\tilde{n}_i/\bar{n}_i$ and (b) a frequency spectrum of the fluctuations with $k_x\rho_i = 0$, $k_y\rho_i = 0.412$, $k_z\rho_i = 0.103$ for the case with the uniform ion flow. It is to be noted that the Doppler shift should be taken into account on the fluctuation frequency in the laboratory frame in the presence of the ion flow. The ion density fluctuation is observed to be enhanced with time. The obtained growth rate is $\gamma/\omega_{ci} = 0.11$, which is the same as that for the case with the electron flow. Furthermore, the observed fluctuation frequency in the laboratory frame $\omega \cong 0.011\omega_{pe}$ can be converted to the frequency in the ion frame $\omega_r \cong 0.017\omega_{pe}$ in consideration of the Doppler shift effect of the ion flow velocity, and as a result, the converted frequency in the ion frame almost agrees with the observed frequency for the case with the electron flow. The phase velocity in the laboratory frame is calculated to be $v_{ph} = 0.30v_{te}$.

In Fig. 4, the electron velocity distribution function parallel to the magnetic field $f_e(v_z)$ at $\omega_{pe}t = 0$ (solid line), 1000 (dashed line), 2000 (short-dashed line) and 3000 (dot-dashed line) are presented for the case with (a) the electron flow and (b) the ion flow. Arrows in Fig. 4 indicate the parallel component of the phase velocity of the fluctuation v_{ph}. In both the cases, the velocity space diffusions are given rise to, and as a result, the steepest region in the velocity distribution is changed. For the case with electron flow (Fig. 4(a)), the amount of electrons corresponding to $v_z \cong v_{ph}$ decreases while that at $v_z \cong 0$ increases. This means that the kinetic energy of electrons is transferred to the wave energy through the inverse Landau damping. The same situation also appears for the case with ion flow. The amount of electrons with the velocity corresponding to $v_z \cong v_{ph}$ in the laboratory frame decreases with time as is observed when the electron flow exists. On the other hand, the amount of electrons with the velocity corresponding to $v_z \cong v_{di}(= 0.8v_{te})$ increases, which is different from the case with the electron flow. However, when we think over the phenomena in the ion frame, namely take into account the Doppler shift effect of the ion flow velocity, $v_z \cong v_{di}(= 0.8v_{te})$ in the laboratory frame is consistent with $v_z = 0$ in the ion frame. Accordingly, it is confirmed that both the electron and the ion flows can excite instabilities in the same way in the ion frame.

3.2 Effects of ion flow velocity shear on the instabilities

Let us introduce ion flow velocity shear by means of changing the parallel ion flow velocity spatially in the x direction as shown in Fig. 1. In order to observe the effects of the velocity shear clearly, we make the uniform ion flow velocity small so as not to excite the instability in the absence of the velocity shear. Figure 5(a) shows time evolutions of spatial Fourier mode of the ion density fluctuation with $k_x\rho_i = 0$, $k_y\rho_i = 0.137$ and $k_z\rho_i = 0.069$, for the case that the ion flow is spatially uniform ($v_{di} = 0.5v_{te}$), that is, the shear strength $|\mathrm{d}v_{di}/\mathrm{d}x|/\omega_{ci} = 0$. The ion density fluctuation is observed not to grow temporally. This is because the ion flow velocity is too small to excite the instabilities. When the velocity shear

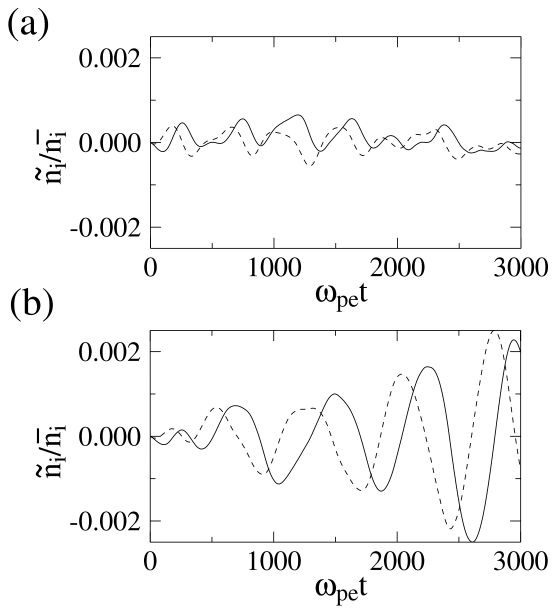

Fig. 5 Time evolutions of the real (solid line) and the imaginary (dashed line) parts of the spatial Fourier mode of the ion density fluctuation $\tilde{n}_i/\bar{n}_i$, with $k_x\rho_i = 0$, $k_y\rho_i = 0.137$ and $k_z\rho_i = 0.069$ (a) in the absence of velocity shear, and (b) in the presence of velocity shear for $|\mathrm{d}v_{di}/\mathrm{d}x|/\omega_{ci} = 0.5$.

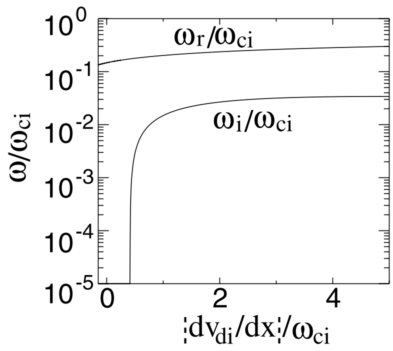

Fig. 6 Dependences of real frequency ω_r/ω_{ci} and growth rate ω_i/ω_{ci} of instability on shear strength $|\mathrm{d}v_{di}/\mathrm{d}x|/\omega_{ci}$ obtained from general kinetic dispersion relation.

which is shown as the dashed line in Fig. 1 exists ($|\mathrm{d}v_{di}/\mathrm{d}x|/\omega_{ci} = 0.5$), on the other hand, the fluctuation amplitude $\tilde{n}_i/\bar{n}_i$ gradually increases with time, as given in Fig. 5(b). According to the fluctuation frequency in the ion frame $\omega_r \cong 0.002\omega_{pe}$ which is nearly equal to k_zC_s (C_s: ion acoustic velocity), the observed instability turns out to be an obliquely propagating ion-acoustic instability. Since the spatially averaged ion flow velocity in the presence of the velocity shear is $0.37v_{te}$ and is smaller than that ($v_{di} = 0.5v_{te}$) in the absence of the velocity shear, the instability is found to be enhanced by not the ion flow but the ion flow velocity shear. This indicates that the parallel flow velocity shear plays an important role in destabilizing the ion-acoustic instability.

Here we will discuss the dependence of the shear strength on the growth rate of the instability by analyzing the general kinetic dispersion relation given by Ganguli *et al.* [10]

$$1 + \sum_{j=i,e}\sum_{n} \frac{\Gamma_n(b)}{k^2\lambda_{Dj}^{\ 2}} \times \left(1 - \frac{k_y}{k_z}\ \frac{\mathrm{d}v_{dj}/\mathrm{d}x}{\omega_{cj}}\right)$$
$$\left[1 + \frac{\omega - k_zv_{dj}}{\sqrt{2}\,k_zv_{tj}} Z\left(\frac{\omega - k_zv_{dj} + n\omega_{cj}}{\sqrt{2}\,k_zv_{tj}}\right)\right] = 0, \qquad (2)$$

where $\Gamma_n(b) \equiv I_n(b)\exp(-b)$, $b \equiv (k_y\rho_i)^2$, I_n are the modified Bessel functions, and Z is the plasma dispersion function.

In Fig. 6, the calculated growth rate ω_i/ω_{ci} is presented in terms of the ion-acoustic instability together with the real frequency ω_r/ω_{ci} in the ion frame. On the calculation, the flow velocity is $v_{di} = 0.5v_{te}$, and the other parameters are set to the same value as the parameters in the simulation. It is found that the growth rate is negative for $|\mathrm{d}v_{di}/\mathrm{d}x|/\omega_{ci} = 0$, however, changes into positive when $|\mathrm{d}v_{di}/\mathrm{d}x|/\omega_{ci}$ exceeds the threshold, steeply increasing with an increase in $|\mathrm{d}v_{di}/\mathrm{d}x|/\omega_{ci}$. This tendency is qualitatively in agreement with the simulation results. Since the absolute values of the calculated growth rate and threshold for the instability are not entirely consistent with the simulation results, the more detailed analysis is necessary for the quantitative discussion and is planned for future work.

4. Summary

We have performed the three-dimensional electrostatic particle simulations in order to investigate the effects of the parallel flow velocity shear on the low-frequency plasma instabilities in detail. When the electron flow velocity or the ion flow velocity is sufficiently large, the current-driven ion-cyclotron instabilities are observed and it is confirmed that the growth rates in both the cases are almost the same. In the case that the ion flow velocity is so small that the low-frequency instabilities cannot take place, on the other hand, the ion-acoustic instability is destabilized by introducing the parallel ion flow velocity shear. It is found that the relation between the shear strength and the growth rate of the instability obtained from the particle simulation results has the same tendency as the theoretically calculated growth rate.

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