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Statistical Characteristics from Gyro-fluid Transport Simulation

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Abstract

Statistical characteristics are analyzed from gyrofluid simulation for sheared slab electron temperature gradient turbulence. It is found that the autocorrelation time is enhanced by zonal flow formation. For turbulent plasma, relatively high correlation dimension (~ 8) is obtained. On the other hand, the formation of the zonal flows is found to cause the reduction of the correlation dimension less than 3. Then, it is concluded that the variation of turbulent structure is well reflected in the correlation dimension. For turbulent plasma the deviation of probability density function from Gaussian profile at high amplitude region is supposed to be the origin of intermittent large heat flux.

Keywords:

plasma turbulence, ETG (electron temperature gradient), zonal flow, statistical analysis, fractal, correlation dimension, probability density function, gyro-fluid simulation

1. Introduction

For magnetic confinement fusion research, it is important to clarify and control the turbulent transport mechanism that governs the performance of confinement plasma. In recent years, it has been clarified that magnetic confinement plasma autonomously generates a variety of structure such as internal transport barrier through dynamics of the fluctuations with various temporal and spatial scales [1-2]. Then, it is necessary to find out the rules to identify what kind of turbulent structure is generated there.

For understanding the transport mechanism in toroidal plasmas, the statistical characteristics of the plasma turbulence have been investigated by experimental observations. The fractal dimension of density and temperature fluctuations was found to characterize the attractor of the turbulence [3-4]. It was shown that the fractal dimension is affected by the difference of the plasma heating method [3]. The relation between the tail component of probability density function (PDF) and the intermittency of turbulent structure have been discussed in recent years. function (PDF) is affected by the turbulent structure. However, from a viewpoint of difficulty of preparing detail experimental condition, the knowledge of such statistical characteristics enough to identify turbulent structure has not been understood well.

Then, it is instructive to simulate microturbulence for finding out the rules to identify turbulent structure by inductive method, in particular, from a viewpoint of feasibility of preparing detail condition. In this research, the relations between statistical characteristics of turbulent structure and parameters such as temperature gradient, magnetic shear, zonal flow intensity, which characterize plasma performance, have been systematically investigated by using gyrofluid simulation of sheared slab ETG modes [5].

2. Simulation model and analysis method

2.1 Model equations

Gyrofluid models for ITG (ion temperature gradient) modes have been developed in recent years [6]. For ETG turbulence, gyro-fluid models including the Landau damping effect are described from the symmetry to those of ITG turbulence, although the treatment for electron adiabatic response to zonal flows is different from ion one [6]. Thus, the gyrofluid equations for sheared slab configuration in the collisionless limit are described as

$$(1-\hat{\nabla}_\perp^2)\frac{\partial\hat{\phi}}{\partial\hat{t}} = [1+(1+\eta_e)\hat{\nabla}_\perp^2]\frac{\partial\hat{\phi}}{\partial\hat{y}} + [\hat{\phi},\hat{\nabla}_\perp^2\hat{\phi}] + \hat{\nabla}_\parallel\hat{v}_\parallel - \mu_\perp\hat{\nabla}_\perp^4\hat{\phi}, \tag{1}$$

$$\frac{\partial\hat{v}_\parallel}{\partial\hat{t}} = \hat{\nabla}_\parallel\hat{\phi} - \hat{\nabla}_\parallel\hat{p} - [\hat{\phi},\hat{v}_\parallel] + \eta_\perp\hat{\nabla}_\perp^2\hat{v}_\parallel, \tag{2}$$

$$\frac{\partial\hat{p}}{\partial\hat{t}} = -(1+\eta_e)\frac{\partial\hat{\phi}}{\partial\hat{y}} - [\hat{\phi},\hat{p}] - \Gamma\hat{\nabla}_\parallel\hat{v}_\parallel - (\Gamma-1)\sqrt{8/\pi}\,|k_\parallel|\,(\hat{p}+\hat{\phi}) + \chi_\perp\hat{\nabla}_\perp^2\hat{p}, \tag{3}$$

where $\Gamma = 5/3$, and coordinates are normalized to characteristic scales as $\hat{x} = x/\rho_e$, $\hat{y} = y/\rho_e$, $\hat{z} = L_n$ and $\hat{t} = v_{te}t/L_n$. Here, the perturbed quantities, the electrostatic potential, the electron parallel velocity and the electron pressure, are normalized as $\hat{\phi} = (L_n/\rho_e)(e\tilde{\phi}/T_e)$, $\hat{v}_\parallel = (L_n/\rho_e)(v_{e\parallel}v_{te})$ and $\hat{p} = (L_n/\rho_e)(\tilde{p}/p_0)$, with $L_n = (d\ln n/dx)^{-1}$, where ρ_e is the electron Larmor radius,

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v_{te} is the electron thermal velocity, T_e is the electron temperature, $v_{e\parallel}$ is the electron parallel velocity and n is the electron density. In this model, the adiabatic ion response is assumed, so that $\eta_e = \mathrm{dln}T_e/\mathrm{dln}n$. The magnetic shear parameter $\hat{s}$ is defined as $\hat{s} = L_n/L_s$, where $L_s = B_0/(\mathrm{d}B_y/\mathrm{d}x)$ and B_0 is the longitudinal magnetic field.

The dynamics of fluid are computed in a three dimensional rectangular box with the Cartesian coordinates (x, y, z). A periodic boundary condition is adopted in y and z directions, Numerical parameters are selected as $L_x = 100\rho_e$, $L_y = 10\pi\rho_e$, $L_z = 2\pi\rho_e L_n$. For numerically stable and efficient calculation, the viscosity terms are introduced in Eqs. (1) ~ (3) as a damping effect for the instabilities with wave length smaller than electron Larmor radius, which does not change the linear growth rate of ETG turbulence. Then, the perpendicular viscosity $\mu_\perp$, $\eta_\perp$ and the perpendicular diffusivity $\chi_\perp$ are assumed to be 0.5. The detail of computation is described in Ref. 5.

2.2 Autocorrelation time

Autocorrelation function of time series data, $x(t_i)[i = 1,2,3,\ldots,N]$, is calculated as

$$\langle (x\,(t) - \langle x\rangle)(x\,(t+\tau) - \langle x\rangle)\rangle, \tag{4}$$

where $\langle\ldots\rangle$ means time average and τ is delay time. The autocorrelation time τ_{ac} is defined as the delay time when the autocorrelation function has zero at the first time, so that $\langle (x\,(t) - \langle x\rangle)(x\,(t+\tau_{ac}) - \langle x\rangle)\rangle = 0$.

2.3 Correlation dimension

To evaluate the fractal dimension D of $x(t_i)$, it is necessary to reconstruct a trajectory in multi-dimensional space by embedding theorem. For one-variable time series data, it is appropriate to use a time-delay coordinate,

$$\{x_i\,(t_i), x(t_i+\tau),\ldots,x\,(t_i+(m-1)\tau)\}, \tag{5}$$

in a viewpoint of noise reduction [7], where m is embedding dimension and τ is embedding log (time delay). In order to obtain precise D, m should be larger than the twice of resultant fractal dimension D, *i.e.* $m \geq 2D + 1$ [8]. The linear correlation between embedding coordinates would be too strong for too small τ. On the other hand, the deterministic relation between coordinates would disappear for too large τ. Then, the autocorrelation time τ_{ac} would be appropriate for the embedding log τ.

To evaluate the fractal dimension, the correlation dimension D_2 is useful from the point of view of computational feasibility. The correlation dimension can be calculated with Grassberger-Procaccia algorithm [9] by taking correlation integral of time series data as

$$C(r) = \lim_{N\to\infty}\frac{1}{N^2}\sum_{i,j=1}^{N} H(r - |x_i - x_j|) \tag{6}$$

where H is the Heaviside function defined by $H(r) = 1$ for $r \geq 0$ and $H(r) = 0$ for $r < 0$. For each m, $C(r)$ would have a scale like $C(r) \sim r^\nu$. The correlation dimension D_2 is obtained as the exponent ν saturated for enough embedding dimension m

3. Simulation results

We have simulated ETG turbulence in three cases. For the case (A) where magnetic shear is relatively weak ($\hat{s}$ = 0.1), zonal flow can be obviously excited for large η_e. Then, electron heat conductivity is restricted. Kelvin-Helmholtz (KH) instability is also generated as a result of zonal flow formation [5]. On the other hand, for small η_e, several modes compete and the formation of zonal flow is restricted. For the case (B), the magnetic shear is the same to (A). However, zonal flow component (0/0 mode) is artificially off to distinguish the pure effect of zonal flow component. For the case (C), magnetic shear is relatively strong ($\hat{s}$ = 0.2). In each cases, electron temperature gradient is changed within η_e = 3 ~ 6.

A typical behavior of time series data of poloidal electric field E_y for the strong ETG turbulence is shown in Fig.1. There is no periodic behavior and large spiky bursts are intermittently observed. On the other hand, for the coherence with zonal flow, the time series data has large periodic oscillation with small spiky fluctuation.

3.1 Autocorrelation time

Figure 2 shows the autocorrelation time τ_{ac} obtained from E_y after the saturation of linear growing modes. For weak shear case (A), the steep jump by one order is observed from η_e = 3 to 4. This is because the effect of zonal flow formation, that is, the 0/0 mode becomes dominant for $\eta_e \leq 4$. On the other hand, τ_{ac} is kept small regardless of η_e for (B) and (C). Then, it is found that the autocorrelation time is enhanced by zonal flow formation because long time scale characterizing KH mode appears through the dispersion relation.

We have also analyzed autocorrelation time for the radial electric field E_x, the electrostatic potential $\hat{\phi}$ and the heat

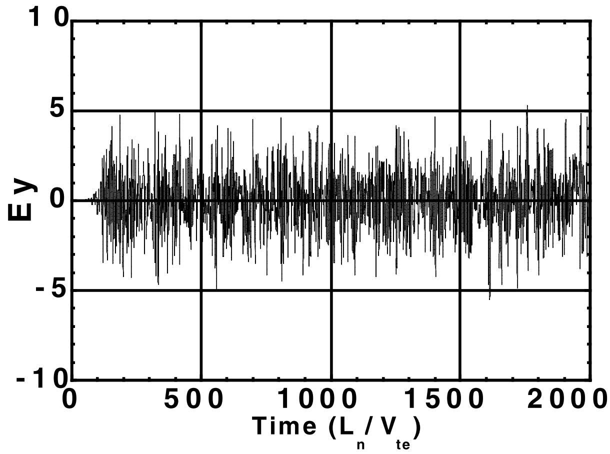

Fig. 1 Time series data of E_y for the case (A) $\hat{s}$ and η_e = 3.

flux Q (= $\hat{p}E_y$). The steep jump is also observed for all of these physical quantities. Then, hereafter, we mainly discuss the statistical characteristics for E_y.

3.2 Correlation dimension

Figure 3 shows the correlation dimension D_2 obtained from E_y of the case (A). As embedding dimension, D_2 linearly increases and saturates to the original correlation dimension of the dynamical system. For small temperature gradient ($\eta_e = 3$), relatively high dimension ($D_2 = 8 \sim 9$) is obtained because the strong turbulence with various temporal scales has large degree of complexity.

However, as increasing η_e, D_2 significantly decreases. In particular, D_2 becomes less than 3 for $\eta_e = 6$. At the same time, the time series data have dominant periodic oscillation with small fluctuations because the zonal flow excited by ETG turbulence drives KH instability. Then, D_2 is mainly determined by the coherent oscillation rather than the turbulent fluctuations.

Figure 4 shows D_2 as a function of η_e for the cases (A) and (B). By comparing D_2 for $\eta_e = 3$ and 6 of (B), it is found that the pure temperature gradient effect without zonal flow is small ($\Delta D_2 \sim 1$). The increase of electron temperature gradient without 0/0 mode is not accompanied with the variation of turbulent structure. Then, the degree of complexity is not so changed, although the electron heat conductivity is enhanced a lot.

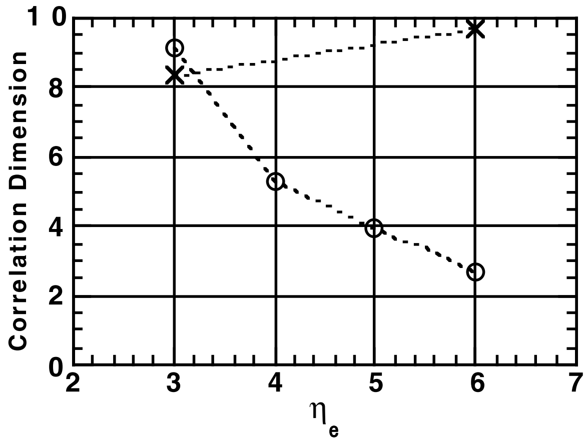

Fig. 4 Correlation Dimension of E_y. The open circles and crosses denote the case(A) $\hat{s} = 0.1$ with 0/0 mode and the case(B) $\hat{s} = 0.1$ without 0/0 mode, respectively.

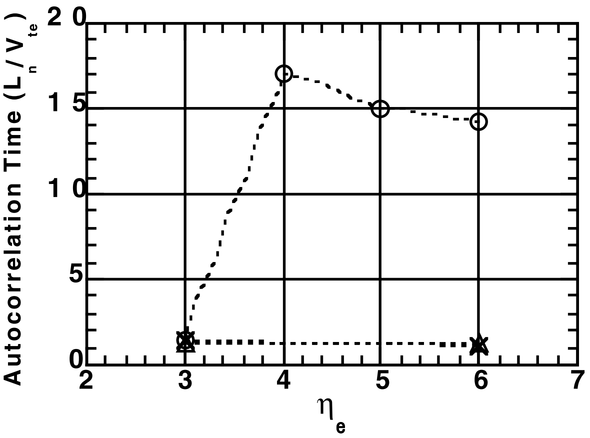

Fig. 2 Autocorrelation time of E_y. The open circles, crosses and triangles denote the cases (A) $\hat{s} = 0.1$ with 0/0 mode, (B) $\hat{s} = 0.1$ without 0/0 mode and (C) $\hat{s} = 0.2$, respectively.

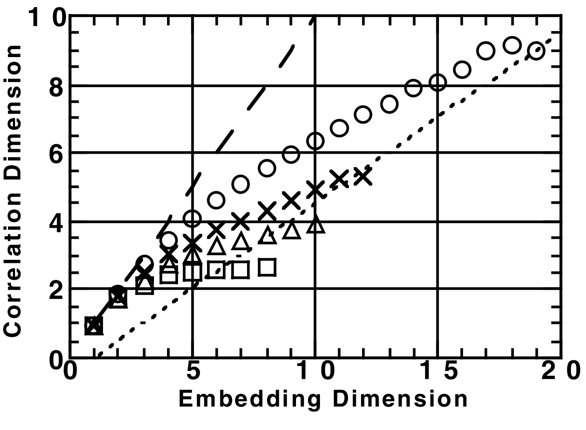

Fig. 3 Correlation dimension of E_y. The open circles, crosses, triangles and squares denote $\eta_e = 3$, 4, 5 and 6, respectively. The broken line and dotted line mean white noise case and the enough condition for embedding, respectively.

On the other hand, by comparing D_2 of (A) and (B) for $\eta_e = 6$, the pure effect of zonal flow formation is relatively large ($\Delta D_2 \sim 7$ for $\eta_e = 6$). Therefore, it is clarified that the variation of turbulent structure is well reflected in the correlation dimension.

3.3 PDF

Figure 5 shows PDF of E_y for $\eta_e = 3$ and 6 of the case (A). In general, for strong turbulence, PDF is close to Gaussian profile. Fig. 5(a) has good agreement with Gaussian at small amplitude region. However, the deviation from Gaussian is observed at large amplitude region. This tail component corresponds to the large spiky bursts observed in time series data of Fig.1.

In the PDF of heat flux for $\eta_e = 3$ of the case (A), the deviation from Gaussian is found to be localized at positive amplitude region. This asymmetry generates net heat flux. Then, it is supposed that E_y and the pressure perturbation $\hat{p}$ could synchronize at the intermittent burst,

On the other hand, Fig. 5(b) is also different from Gaussian because the periodic oscillation with small fluctuations is dominant in the presence of KH instability. Then, it is concluded that the variation of turbulent structure is also well reflected in PDF.

4. Summary and discussion

For sheared slab ETG turbulence, the relation between the statistical characteristics and parameters characterizing plasma performance have been systematically investigated by using gyrofluid simulation. It is found that the autocorrela-

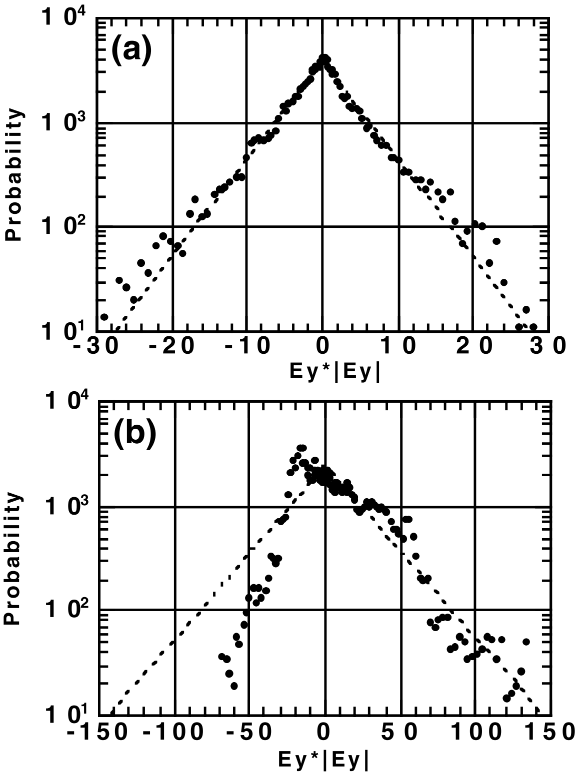

Fig. 5 Probability distribution function of E_y for (a) $\hat{s}$ = 0.1, η_e = 3 and (b) $\hat{s}$ = 0.1, η_e = 6. The dashed lines indicate Gaussian profile for reference.

tion time is enhanced by zonal flow formation because long time scale characterizing KH mode appears through the dispersion relation.

For turbulent plasma, relatively high correlation dimension (~ 8) is obtained. On the other hand, it is found that the formation of the zonal flows causes the reduction of the correlation dimension less than 3. Therefore, it is clarified that the variation of turbulent structure is well reflected in the correlation dimension.

The deviation of probability density function from Gaussian profile is observed for both turbulent plasma and coherent plasma with zonal flow. For turbulent plasma the deviation at high amplitude region is supposed to be the origin of intermittent large heat flux. For coherent plasma, PDF is very different from Gaussian due to the periodic oscillation excited by KH instability.

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