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Interaction of Two Electrons by Interchanging the Phonon Occured in the Beam-Plasma System

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Abstract

Nonlinear interaction between beam electrons in the presence of ion wave by exchanging the phonon is analysed with renormalization theory. We obtain the electric field of envelope solitary wave. Attractive phenomena between the solitary waves around the ion wave are found. Many phase gaps of the soliton's carrier appear when ion wave intensity is strongly enough. Thus the possibility of stochastic accelleration by the phase gap is also discussed.

Keywords:
nonlinear plasma, renormalization, solitary wave, ion wave, attractive force

1. The nonlinear theory

We consider here the nonlinear interaction between beam electrons and envelope solitary wave under presence of the ion waves by using *the renormalization technique* which was developed by Al'tshul, Karpmann, Dupree, Weinstock, Kono, Ichikawa [1]. Then we used a Gausian-type Green's function that was proposed initially by Horton [2]. In the linear system of interaction, retarded Green function has a form $-\mathrm{i}/(\omega - \boldsymbol{k}\cdot\boldsymbol{v} + \mathrm{i}\nu)$. The $(\omega, \boldsymbol{k})$ indicate the frequency and wave number and the ν is a collision frequency [3]. The collision frequecy, has been authorized as a *self energy term* at the nonlinear system, whose real part is the collision frequency and the imaginary part shows resonance broadening or frequency shift.

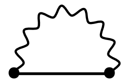
As an envelope solitary wave (soliton) is third order nonlinear quantity, the retarded Green function, $G_\alpha(\boldsymbol{k}, \boldsymbol{v}, \omega; k', \boldsymbol{v}', \omega')$, includes a second order self-energy term, $\sum_\alpha(\boldsymbol{k}, \boldsymbol{v}, \omega; \boldsymbol{k}', \boldsymbol{v}', \omega')$, then the electric field is composed of the Green function times elctron distribution, $f_\alpha(\boldsymbol{k}', \boldsymbol{v}', \omega')$, and a factor, $-\mathrm{i}ke_\alpha/(\varepsilon_0 \boldsymbol{k}^2)$:

$$\boldsymbol{E}(\boldsymbol{k}, \omega) = -\sum_\alpha \frac{e_\alpha}{\varepsilon_0}\cdot\frac{\mathrm{i}\boldsymbol{k}}{\boldsymbol{k}^2}\int \mathrm{d}\boldsymbol{v}\int \mathrm{d}\boldsymbol{v}'\int\frac{\mathrm{d}\omega'}{2\pi}\int\frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^3} \cdot G_\alpha(\boldsymbol{k}, \boldsymbol{v}, \omega; \boldsymbol{k}', \boldsymbol{v}', \omega')\cdot f_\alpha^{(0)}(\boldsymbol{k}', \boldsymbol{v}', \omega') \tag{1}$$

where the Green function is written by :

$$G_\alpha(\boldsymbol{k}, \boldsymbol{v}, \omega; \boldsymbol{k}', \boldsymbol{v}', \omega') = \frac{-\mathrm{i}}{\omega - \boldsymbol{k}\cdot\boldsymbol{v} + \mathrm{i}\sum_\alpha(\boldsymbol{k}, \boldsymbol{v}, \omega; \boldsymbol{k}', \boldsymbol{v}', \omega')}\delta_{k, k'}\,\delta_{\omega, \omega'}\,\delta(\boldsymbol{v} - \boldsymbol{v}'), \tag{2}$$

$$\sum_\alpha(\boldsymbol{k}, \boldsymbol{v}, \omega; \boldsymbol{k}', \boldsymbol{v}', \omega') =$$



$$= \left(\frac{e_\alpha}{m_\alpha}\right)^2 \sum_{k_1}\int\frac{\mathrm{d}\omega_1}{2\pi}E(\boldsymbol{k}_1, \omega_1)\frac{\partial}{\partial \boldsymbol{v}} \left\{G_\alpha(\boldsymbol{k} - \boldsymbol{k}_1, \boldsymbol{v}, \omega - \omega_1; \boldsymbol{k}', \boldsymbol{v}', \omega')E(-\boldsymbol{k}_1, -\omega_1)\cdot\frac{\partial}{\partial \boldsymbol{v}}\right\}. \tag{3}$$

The complex function (3) depends on the intensity of the electric field, $|E(\boldsymbol{k}_1, \omega_1)|^2$. If the wave, $(\boldsymbol{k}_1, \omega_1)$, corresponds to ion wave, then two electron interaction, that one beam electron emits a phonon and another electron absorb it, can be taken into the wave-particle dynamics.

Figure 1 shows Feynman-diagrams. If we use diagram of the left side, we need the form of the ion wave, for example, as a cosine function and if we take the right side, conditions are satisfied consistently.

Poles of ω for the Green's function, eq. (2), are complex functions, thus the system has an nonlinear character such as

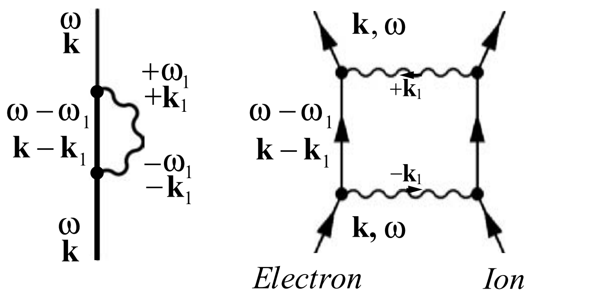


Fig. 1 Feynman diagram of interaction between the electron and the ion wave is shown.

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growing, saturation, and damping. On the one hand, eq. (2) can be separated into a principal part representing the non-resonant reaction and a delta function. The δ-function means a resonant interaction and is the most important term. However we seek such the Green's function that at time $t = t'(=0)$, the function equal to $\delta(\boldsymbol{v}-\boldsymbol{v}')$ and at time $t\,(t>0)$, the width of the function becomes broaden. Then we use a Gausian-type Green's function that was proposed initially by Horton [2] and define a diffusion tensor D_{ij}, then the Green function is described as:

$$G_\alpha(\boldsymbol{k},\boldsymbol{v},t;\boldsymbol{k}',\boldsymbol{v}',t'=0) = -\,\mathrm{i}\left\{\frac{4\pi}{\boldsymbol{k}^2}\,\boldsymbol{k}_i\boldsymbol{k}_j\boldsymbol{D}_{ij}(\boldsymbol{k},\boldsymbol{v})\,t\right\}^{-\frac{3}{2}} \cdot\exp\left[-\frac{\boldsymbol{k}_i(\boldsymbol{v}-\boldsymbol{v}')_i\,\boldsymbol{k}_j(\boldsymbol{v}-\boldsymbol{v}')_j}{4\boldsymbol{k}_i\boldsymbol{k}_j\boldsymbol{D}_{ij}(\boldsymbol{k},\boldsymbol{v})\,t} - \frac{\boldsymbol{k}_i\boldsymbol{k}_j\boldsymbol{D}_{ij}(\boldsymbol{k},\boldsymbol{v})}{12}\,t^3 - \frac{\mathrm{i}}{2}\,\boldsymbol{k}\cdot(\boldsymbol{v}+\boldsymbol{v}')\,t\right]\delta_{\boldsymbol{k},\boldsymbol{k}'} \tag{4}$$

Fourier transformation for the above Geen's function, when we assume the D_{ij} to be constant with time in zero order, becomes:

$$G(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega') = -\,2\mathrm{i}\left\{4\pi\left(\frac{\boldsymbol{k}_i\boldsymbol{k}_j}{\boldsymbol{k}^2}\right)\cdot D_{ij}(\boldsymbol{k},\boldsymbol{v})\right\}^{-\frac{3}{2}} \cdot\sum_{m=0}^{\infty}\frac{(-b)^m}{m!}\left\{\frac{a}{\mathrm{i}(-\omega+c)}\right\}^{\frac{3m}{2}-\frac{1}{4}} \cdot K_{3m-\frac{1}{2}}\left\{2\sqrt{\mathrm{i}a(-\omega+c)}\right\}\cdot\delta_{\boldsymbol{k},\boldsymbol{k}'}\,\delta_{\omega,\omega'} \tag{5}$$

$$a = \frac{\boldsymbol{k}_i(\boldsymbol{v}-\boldsymbol{v}')_i\,\boldsymbol{k}_j(\boldsymbol{v}-\boldsymbol{v}')_j}{4\boldsymbol{k}_i\boldsymbol{k}_j\cdot D_{ij}(\boldsymbol{k},\boldsymbol{v})},\quad b = \frac{\boldsymbol{k}_i\boldsymbol{k}_j\cdot D_{ij}(\boldsymbol{k},\boldsymbol{v})}{12},\quad c = \boldsymbol{k}\cdot\boldsymbol{v} - \mathrm{i}\Sigma_\alpha,$$

where the function $K_{3m-\frac{1}{2}}(x)$ is the modified Bessel function and $m=0$ term is very important and $K_{-\frac{1}{2}}(x) = K_{\frac{1}{2}}(x) = \sqrt{\pi/(2x)}\cdot\exp(-x)$. The terms $m\neq 0$ are proportional to $\boldsymbol{k}^{2m}$ then they vanish when we consider the limit $\boldsymbol{k}\to 0$. We assume that the medium is isotropic, then we can describe as $\boldsymbol{k}_i\boldsymbol{k}_j\boldsymbol{D}_{ij}/\boldsymbol{k}^2 \simeq \boldsymbol{D}_{ij}$, therefore

$$G_\alpha^{(m=0)}(\boldsymbol{k},\boldsymbol{v},\omega) = -\frac{\mathrm{i}}{4\pi D_{ij}}\cdot\frac{1}{|\boldsymbol{v}-\boldsymbol{v}'|} \cdot\exp\left(-\frac{|\boldsymbol{v}-\boldsymbol{v}'|}{\sqrt{D_{ij}}}\cdot\sqrt{\mathrm{i}(-\omega+c)}\right). \tag{5'}$$

For the $D_{ij}(\boldsymbol{k},\boldsymbol{v})$ and $f_\alpha(\boldsymbol{k},\boldsymbol{v},\omega)$, we get the following expression:

$$D_{ij}(\boldsymbol{k},\boldsymbol{v},\omega) = \left(\frac{e_\alpha}{m_\alpha}\right)^2\sum_{\boldsymbol{k}'}\sum_{\boldsymbol{k}_1}\int\mathrm{d}\boldsymbol{v}'\int\frac{\mathrm{d}\omega'}{2\pi}\int\frac{\mathrm{d}\omega_1}{2\pi} \cdot\frac{\boldsymbol{k}_{1i}\boldsymbol{k}_{1j}}{\boldsymbol{k}_1^2}\left|\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)\right|^2 G_\alpha(\boldsymbol{k}-\boldsymbol{k}_1,\boldsymbol{v},\omega-\omega_1;\boldsymbol{k}',\boldsymbol{v}',\omega') \tag{6}$$

$$f_\alpha(\boldsymbol{k},\boldsymbol{v},\omega) = \int\mathrm{d}\boldsymbol{v}' G_\alpha(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega')\cdot f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',\omega') \tag{7}$$

The above eqs. (1), (3), (5) or (5'), (6) and (7) construct a set of fundamental equation.

To get the $D_{ij}(\boldsymbol{k},\boldsymbol{v},t)$ or $\sum_\alpha(\boldsymbol{k},\boldsymbol{v},t)$, we take sum $\sum_{\boldsymbol{k}'}\int\mathrm{d}\omega'/(2\pi)\int\mathrm{d}\boldsymbol{v}'$ for the eqs. (3) and (6). As for the integration with ω_1, we carry out the $\left|\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)\right|^2$ outside the integration, since we suppose that $(\boldsymbol{k}_1,\omega_1)$ corresponds to ion wave and is independent of the solitary wave, $(\boldsymbol{k},\omega)$. We exchange the $\int\mathrm{d}\boldsymbol{v}'$ with $|\boldsymbol{v}-\boldsymbol{v}'|^3$, where the $|\boldsymbol{v}-\boldsymbol{v}'|$ is correlational length in velocity space, and finally we have done inverse transformation.

For the calculation $\sum_\alpha(\boldsymbol{k},\boldsymbol{v},\omega)$, we multiply the operator, eq. (3), to eq. (5') and after the summation, we divide it with eq. (5') again.

$$D_{ij}(\boldsymbol{k},\boldsymbol{v},\omega) \simeq \frac{1}{2\pi^2}\left(\frac{e_\alpha}{m_\alpha}\right)^2\sum_{\boldsymbol{k}_1}\frac{\boldsymbol{k}_{1i}\boldsymbol{k}_{1j}}{\boldsymbol{k}_1^2}\cdot\left|\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)\right|^2 \cdot\left[\frac{-1}{2\sqrt{D_{ij}}}\cdot\frac{1}{|\boldsymbol{v}-\boldsymbol{v}'|}\sqrt{\mathrm{i}(-\omega+\omega_1)+\mathrm{i}(\boldsymbol{k}-\boldsymbol{k}_1)\cdot\boldsymbol{v}+\Sigma_\alpha} + \frac{1}{2}\cdot\frac{1}{|\boldsymbol{v}-\boldsymbol{v}'|^2}\right] \cdot\exp\left\{-\frac{|\boldsymbol{v}-\boldsymbol{v}'|}{\sqrt{D_{ij}}}\sqrt{\mathrm{i}(-\omega+\omega_1)+\mathrm{i}(\boldsymbol{k}-\boldsymbol{k}_1)\cdot\boldsymbol{v}+\Sigma_\alpha}\right\} \tag{8}$$

$$D_{ij}(\boldsymbol{k},\boldsymbol{v},t) \simeq \frac{1}{2\pi^2}\left(\frac{e_\alpha}{m_\alpha}\right)^2\sum_{\boldsymbol{k}_1}\frac{\boldsymbol{k}_{1i}\boldsymbol{k}_{1j}}{\boldsymbol{k}_1^2}\cdot\left|\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)\right|^2\cdot \left[\frac{1}{2\sqrt{D_{ij}}}|\boldsymbol{v}-\boldsymbol{v}'|\frac{1}{\sqrt{\pi t^3}} - \frac{1}{8D_{ij}^{\frac{3}{2}}}|\boldsymbol{v}-\boldsymbol{v}'|^3\frac{1}{\sqrt{\pi t^5}}\right] \cdot\exp\left\{-\frac{|\boldsymbol{v}-\boldsymbol{v}'|^2}{4D_{ij}t}\right\}\exp\{-\mathrm{i}\omega_1 t - \mathrm{i}(\boldsymbol{k}-\boldsymbol{k}_1)\cdot\boldsymbol{v}_0 t - \Sigma_\alpha t\} \tag{9}$$

For right hand side of the D_{ij} in eq. (9), we use the value of zero-order. Thus we obtain a relation between the $D_{ij}(\boldsymbol{k},\boldsymbol{v},t)$, and the $\Sigma_\alpha(\boldsymbol{k},\boldsymbol{v},t)$:

$$\Sigma_\alpha(\boldsymbol{k},\boldsymbol{v},t) = 5\cdot\frac{D_{ij}(\boldsymbol{k},\boldsymbol{v},t)}{|\boldsymbol{v}-\boldsymbol{v}'|^2} \tag{10}$$

The eq. (10) is the anomalous collision frequency by Tsytovich in qusilinear theory [4], if we exchange our correlational length in velocity space, $|\boldsymbol{v}-\boldsymbol{v}'|$, with his $\boldsymbol{v}$, except our factor 5.

Final results of the electric field of solitary wave, yet leaving the $\boldsymbol{k}$-space integration are given as follows :

$$\boldsymbol{E}(\boldsymbol{k},x) = -\sum_\alpha\frac{e_\alpha}{\varepsilon_0}\cdot\frac{\mathrm{i}\boldsymbol{k}}{\boldsymbol{k}^2}\int\frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^3}f_1^{(0)}(\boldsymbol{k}')\frac{(-\mathrm{i}n_b)}{\pi^{\frac{1}{2}}} \cdot\frac{|\boldsymbol{v}-\boldsymbol{v}'|}{2\boldsymbol{v}_0}\cdot x^{\frac{1}{2}}\cdot\exp\left\{-\frac{(\mathrm{Im}\Sigma_\alpha)^2}{\boldsymbol{k}^2|\boldsymbol{v}-\boldsymbol{v}'|^2}\cdot\frac{1}{x}\right\} \cdot\left[\exp\left\{-\frac{2(\boldsymbol{k}\cdot\boldsymbol{v}_0)\,\mathrm{Im}\Sigma_\alpha}{\boldsymbol{k}^2|\boldsymbol{v}-\boldsymbol{v}'|^2}\cdot\frac{1}{x}\right\} - 1\right] \cdot\exp\left\{-\frac{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}_0 x}{|\Sigma_\alpha|} - \frac{\mathrm{i}\mathrm{Im}\Sigma_\alpha\cdot x}{|\Sigma_\alpha|}\right\}\cdot\exp\left(-\frac{\mathrm{Re}\Sigma_\alpha\cdot x}{|\Sigma_\alpha|}\right) \tag{11}$$

$$\Sigma_\alpha(\boldsymbol{k},\boldsymbol{v}) = \frac{1}{2\pi^2}\left(\frac{e_\alpha}{m_\alpha}\right)^2 \sum_{k_1} \frac{\boldsymbol{k}_{1i}\boldsymbol{k}_{1j}}{\boldsymbol{k}_1^2} \left|\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)\right|^2 \cdot \frac{5}{\pi^{\frac{1}{2}}} \cdot \frac{\Sigma_\alpha}{|\boldsymbol{v}-\boldsymbol{v}'|^2} \cdot \left\{\frac{1}{x^{\frac{3}{2}}} - \frac{1}{x^{\frac{5}{2}}}\right\} \cdot \exp\left(-\frac{1}{x}\right) \cdot \exp\left\{-\mathrm{i}[\omega_1 + (\boldsymbol{k}-\boldsymbol{k}_1)\cdot\boldsymbol{v}_0] \cdot \frac{x}{|\Sigma_\alpha|} - \frac{\mathrm{i}\mathrm{Im}\Sigma_\alpha \cdot x}{|\Sigma_\alpha|}\right\} \cdot \exp\left(-\frac{\mathrm{Re}\Sigma_\alpha \cdot x}{|\Sigma_\alpha|}\right), \tag{12}$$

where the following notation and the values are used: the normalized time $x = |\Sigma_\alpha(0)| \cdot t$ ($|\Sigma_\alpha(0)|$: zero order self energy $\simeq 10^6(\mathrm{s}^{-1})$), electron mass m, beam velocity $\boldsymbol{v}_0 \simeq 2.6\times10^7$(m/s), beam density $n_b \simeq 10^{14}(m^{-3})$, beam forming factor in $\boldsymbol{k}'$-space $\int \mathrm{d}\boldsymbol{k}' f_1^{(0)}(\boldsymbol{k}')/(2\pi)^3 \simeq 1$, the carrier frequency $\omega \simeq 400$ (MHz), its wave number $\boldsymbol{k} \simeq 100$ (rad/m), ion wave frequency $\omega_1 \simeq 250$ (kHz), its wave number $\boldsymbol{k}_1 \simeq 100$ (rad/m), ion wave power in $\boldsymbol{k}_1$-space $|\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)|^2$, correlational length in velocity space $|\boldsymbol{v}-\boldsymbol{v}'| \simeq 10^5$ (m/s), respectively.

Carrier frequency of the electric field, eq. (11), is decided from resonance condition: $\omega - \boldsymbol{k}\cdot\boldsymbol{v}_0 + \mathrm{i}\sum_a = 0$, which is the pole of the Green's function, eq. (2). Then the carrier frequency ω is frequency modulated with the Im $\sum_\alpha$. Since the $\sum_\alpha$ is related to the D_{ij} of eq. (8), by the relation eq. (10), both quantities, Im $\sum_\alpha$ and D_{ij}, include the following function as in the eq. (8):

$$(\omega-\omega_1) - (\boldsymbol{k}-\boldsymbol{k}_1)\cdot\boldsymbol{v} + \mathrm{i}\Sigma_\alpha.$$

If this function is equal or nearly equal to zero, then nonlinear wave-particle interaction is extremely enhanced. Thus we assume that the resonance condition of 3-body interaction between the electron ($\boldsymbol{v}_0$), the solitary wave (ω), and the ion wave (ω_1) satisfy the relation :

$$(\omega-\omega_1) - (\boldsymbol{k}-\boldsymbol{k}_1)\cdot\boldsymbol{v}_0 + \mathrm{i}\Sigma_\alpha = 0, \tag{13}$$

then the oscillation, $\exp(-\mathrm{i}\omega t)$, becomes :

$$\exp(-\mathrm{i}\omega t) = \exp\{-\mathrm{i}[\omega_1 + (\boldsymbol{k}-\boldsymbol{k}_1)\cdot\boldsymbol{v}_0 - \mathrm{i}\Sigma_\alpha]t\}$$

which is appeared in the eq.(9) and the eq. (12).

2. Results

Figure 2 shows the soliton as the function of time and the fig. 3 shows the timing of ion wave (cosine function) and the Σ_α. Electron beam emits the soliton by Bremsstrahlung, when it collides to the negative phase of ion wave.

Figure 4 shows an electric field when the ion wave intensity is strongly enough. If we magnify the phase, we can see phase-gaps. One of the gap is shown in fig. 5, where the carrier frequency retraces back its progress at 2.771 μs-point.

3. Discussion

Electric field in the high-latitude ionosphere has become central to the auroral particle acceleration, where the keywards such as: streaming electron, ion cyclotron wave, density cavity, high frequency wave (AKR), are presented. All of these keywards appear in the beam-plasma discharge [5].

We present here the stochastic acceleration mechanism. Figure 6 shows the value of integration for the electric field. The value increases by the phase gaps which are caused by resonance as shown in eq. (13).

The envelope of the wave, *cross-type* character, in fig. 4 fits well in the results of fluid model [6], the character comes from a factor, $[\exp(\cdots)-1]$, in eq. (11). In model, interaction that one beam electron emits a phonon and another electron absorb it, is possible. Thus electrons will not work as Fermion but work as Bosson as in the superconductivity. The above factor behaves as the inverse of the Bose distribution [7].

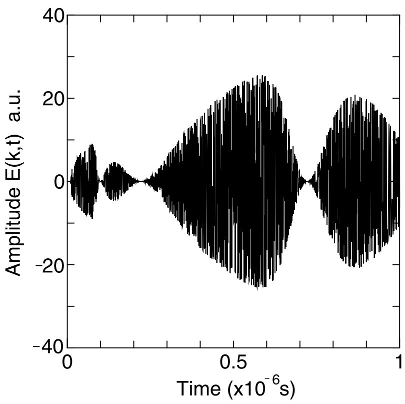

Fig. 2 A solution of electron wave is shown. Calculational condition is the following : ω_1 = 1 MHz, $|E(\omega_1, k_1)|$ = 4, $n_b = 10^{14}$ (m^{-3}), $|k| \simeq 100$ (rad/m), $|v - v'| = 10^5$ (m/s), phase of ω_1 = 80 (deg), zero-order of $\Sigma_\alpha(0) = 10^7$.

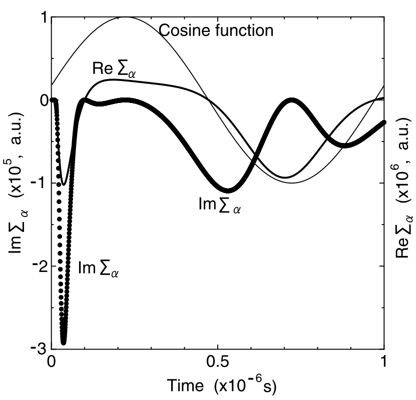

Fig. 3 Ion wave (cosine function), ReΣ_α, ImΣ_α are represented. They correspond to the above solution.

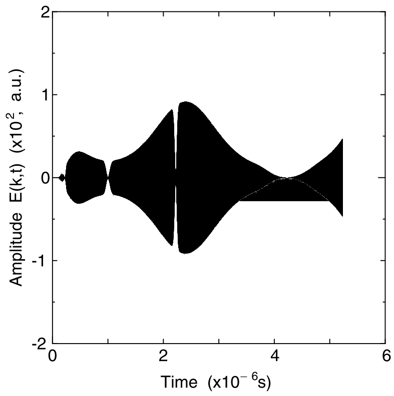

Fig. 4 A solution of electron wave is represented at the condition : ω_1 is 0.25 MHz, $|E(\omega_1, k_1)| = 5$, $n_b = 10^{14}$ (m^{-3}), $|k| \simeq 100$ (rad/m), $|v - v'| = 10^5$ (m/s), phase of ω_1= 20 (deg), zero-order of $\Sigma_\alpha(0) = 10^6$.

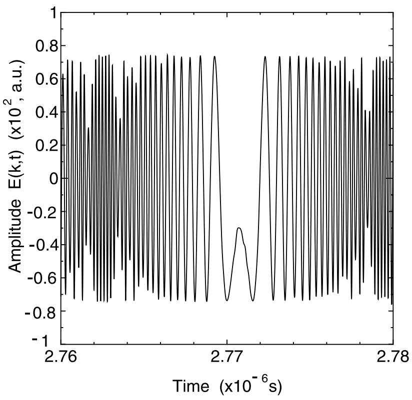

Fig. 5 Magnification of the carrier in the above figure is shown. The carrier is frequency modulated with ImΣ_α so that the phase gap by the resonance apears, which means the possibility of stochastic acceleration of particles by an integration of field.

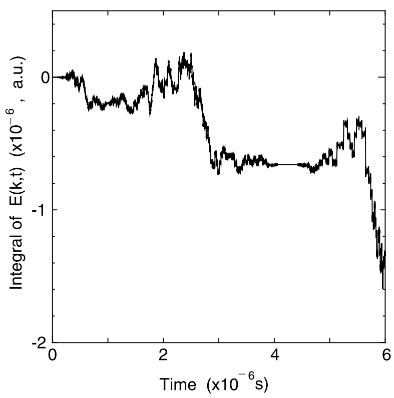

Fig. 6 Time integrated value of electric field at above solution is shown, where the ion wave intensity is in strongly high level.

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