J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 573–576

Magnetic Field Perturbations Correlated with Large Amplitude Lower-Hybrid Waves in a High-Voltage Linear Plasma Discharge

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(Received: 9 December 2003 / Accepted: 2 February 2004)

Abstract

Theoretical description of experiments on excitation of magnetic field perturbations correlated with large amplitude lower-hybrid (LH) wave bursts in a high-voltage linear plasma discharge is presented. It is shown that the magnetic field perturbations, which are excited in the experiments mostly in the paramagnetic sense, are associated with the development of the magneto-modulational processes. The theoretical results are compared with the experimental data using the following parameters: the values of the magnitude of the magnetic field perturbations; the direction of the vector of the magnetic field perturbation; the correlation between the magnetic field perturbations and the electron density perturbations. It is shown that for LH waves propagating in one plane theoretical predictions are in a good agreement with the experimental results.

Keywords:

high-current linear plasma discharge, lower-hybrid wave, modulational interaction, magnetic field perturbation, magneto-modulational process

1. Introduction

The experiments in a high-current linear plasma discharge carried out at the Nihon University [1] have shown the possibility of the excitation of magnetic field perturbations related to intense lower-hybrid turbulence. The value of the magnitude of the perturbations is $|\delta \boldsymbol{B}| \approx 10$ G and $|\delta \boldsymbol{B}|/|\boldsymbol{B}_0| \approx 0.01$, where $\boldsymbol{B}_0$ is the unperturbed magnetic field. The characteristic features of the observations are: 1) the magnetic field perturbations are excited in the experiments mostly in the paramagnetic sense; 2) there is a correlation between the magnetic field perturbations and the electron density perturbations; 3) the power spectrum of the associated electric field fluctuations shows widely broadened profile and has multiple peaks around the lower-hybrid (LH) frequency.

All the results show that the excitation of the magnetic field perturbations can be associated with the development of the magneto-modulational processes [2]. We check this assumption.

2. Equation for magneto-modulational perturbations

The theoretical investigation of the magneto-modulational processes with the participation of LH waves has been performed for tokamak plasmas [3,4]. The conditions of the laboratory experiments [1] differ strongly from those of tokamak plasmas. In particular, in the case of the laboratory experiments [1] the electron plasma frequency ω_{pe} is far higher than the electron gyrofrequency ω_{Be}. The equation for the magneto-modulational processes given in Refs. [3,4] assumes that the inequality $\omega_{pe} < \omega_{Be}$ is fulfilled. Here we present the main steps of the derivation of the equation for the magneto-modulational perturbations caused by LH waves in general case.

Although LH waves are "almost" electrostatic, we are interested in the modulational excitation of the magnetic field perturbations. This means that we have to take into account low-frequency (with the frequencies much less than those of LH waves) transversal fields which can appear in process of the modulational interactions. We start from Maxwell's equations, which in Fourier representation take the form (see, *e.g.*, [2])

$$\left[\frac{\omega^2}{c^2}\varepsilon_{ij} + k_i k_j - \delta_{ij}\boldsymbol{k}^2\right] E_{k,j} = -\frac{4\pi i\omega}{c^2}\,(j^{(2)}_{k,i} + j^{(3)}_{k,i} + \cdots), \qquad (1)$$

where $j^{(2)}$, $j^{(3)}$, ... are the quadratic, cubic, etc. (in LH electric

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field $\boldsymbol{E}$) currents, respectively, $\varepsilon_{ij}(\omega, \boldsymbol{k})$ is the dielectric permittivity tensor, the subscript $k = \{\omega, \boldsymbol{k}\}$ is a four-vector characterizing the Fourier component, ω is the frequency, $\boldsymbol{k}$ is the wave vector, δ_{ij} is the Kronecker symbol, and c is the velocity of light.

To denote the low-frequency fields we use the symbol tilde. The direction of the unperturbed magnetic field $\boldsymbol{B}_0$ is chosen along the axis Oz. Assuming that the low-frequency fields obey the inequality $\tilde{\omega} \ll \tilde{k}_z v_{Te}$, taking into account only the quadratic current $j^{(2)}$, and using the following relationship for the components of ε_{ij} under the condition $\omega \ll k_z v_{Te}$ [5]

$$\varepsilon_{xx} = \varepsilon_{yy} = 1,\ \varepsilon_{zz} \approx 1 + \frac{\omega_{pe}^2}{k_z^2 v_{Te}^2},$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_{xz} = \varepsilon_{zx} = \varepsilon_{yz} = \varepsilon_{zy} = 0, \tag{2}$$

we find the relationship

$$E_{\tilde{k}} = \frac{4\pi \mathrm{i}}{\tilde{\omega}} \left[\frac{\tilde{\omega}^2 \boldsymbol{j}_{\tilde{k},\perp}^{(2)}}{\tilde{\boldsymbol{k}}^2 c^2} - \frac{\tilde{\boldsymbol{k}}(\tilde{\boldsymbol{k}} \cdot \boldsymbol{j}_{\tilde{k},\perp}^{(2)})}{\tilde{k}_z^2 \varepsilon_{zz}} + \frac{\tilde{\omega}^2 \tilde{\boldsymbol{k}}_\perp (\tilde{\boldsymbol{k}}_\perp \cdot \boldsymbol{j}_{\tilde{k},\perp}^{(2)})}{\tilde{k}_z^2 \tilde{\boldsymbol{k}}^2 c^2} \right]. \tag{3}$$

Here v_{Te} is the electron thermal velocity, the subscript $\perp$ denotes the vector component perpendicular to the unperturbed magnetic field. Thus if $\boldsymbol{j}_{\tilde{k},\perp}^{(2)} \neq 0$ then the presence of LH waves results in the excitation of magnetic field perturbations

$$\delta \boldsymbol{B}_{\tilde{k}} = \frac{c}{\tilde{\omega}} (\tilde{\boldsymbol{k}} \times \boldsymbol{E}_{\tilde{k}}). \tag{4}$$

To determine $\boldsymbol{j}_{\tilde{k},\perp}^{(2)} \equiv -e\int \boldsymbol{v}_\perp f_{\tilde{k}} \mathrm{d}\boldsymbol{p}/(2\pi)^3$ the kinetic equation for the electron distribution function $f(t, \boldsymbol{r}, \boldsymbol{p})$ is used (as it can be shown by direct calculations, the ion contribution to the magneto-modulational processes with participation of LH waves is negligibly small in comparison with the electron one). Here, $-e$ is the electron charge, $\boldsymbol{p}(\boldsymbol{v})$ is the electron momentum (velocity). The distribution function f is normalized as follows $n_e = \int f(t, \boldsymbol{r}, \boldsymbol{p}) \mathrm{d}\boldsymbol{p}/(2\pi^3)$, where n_e is the electron density. We assume that LH wave frequency ω_0 is much less than ω_{Be}.

We solve the kinetic equation using the theory of perturbations in powers of LH wave electric field and following the standard procedure of separation of high-frequency electric fields (which are associated with LH field) and low-frequency those [2]. We find

$$j_{\tilde{k},x}^{(2)} \approx \frac{\mathrm{i}}{4\pi} \frac{c\omega_{pe}^2}{|\boldsymbol{B}_0|\omega_0^2} \int (k_{1,z} E_{k_1,y}^+ E_{k_2,z}^- + k_{1,z} E_{k_2,y}^- E_{k_1,z}^+ + k_{2,z} E_{k_1,y}^+ E_{k_2,z}^- + k_{2,z} E_{k_2,y}^- E_{k_1,z}^+) \delta(\tilde{k} - k_1 - k_2) \mathrm{d}k_1 \mathrm{d}k_2, \tag{5}$$

$$j_{\tilde{k},y}^{(2)} \approx -\frac{\mathrm{i}}{4\pi} \frac{c\omega_{pe}^2}{|\boldsymbol{B}_0|\omega_0^2} \int (k_{1,z} E_{k_1,x}^+ E_{k_2,z}^- + k_{1,z} E_{k_2,x}^- E_{k_1,z}^+ + k_{2,z} E_{k_1,x}^+ E_{k_2,z}^- + k_{2,z} E_{k_2,x}^- E_{k_1,z}^+) \delta(\tilde{k} - k_1 - k_2) \mathrm{d}k_1 \mathrm{d}k_2, \tag{6}$$

where the superscripts “+” and “–” denote the positive- and negative-frequency parts of the electric field, respectively.

Using (3), (5) and (6), we obtain from (4) the equation determining the amplitude of the quasistationary magnetic field perturbations $\delta \boldsymbol{B}$ excited by intense LH waves of a given frequency ω_0

$$\Delta \delta \boldsymbol{B} = \frac{1}{|\boldsymbol{B}_0|} \frac{\omega_{pe}^2}{\omega_0^2} \nabla \times \nabla \times (\boldsymbol{E}_\perp (\boldsymbol{b} \cdot \boldsymbol{E}^*) + \boldsymbol{E}_\perp^* (\boldsymbol{b} \cdot \boldsymbol{E})), \tag{7}$$

where Δ is the Laplace operator; $\boldsymbol{b} = \boldsymbol{B}_0/|\boldsymbol{B}_0|$ is a unit vector along the unperturbed magnetic field; $\boldsymbol{E}$ is the complex amplitude of LH field, the field itself is $\mathrm{Re}(\boldsymbol{E}\exp\{-\mathrm{i}\omega_0 t\})$; the asterisk stands for the complex conjugate. Equation (7) is valid for the case $\omega_0 \ll \omega_{Be}$. This means that for LH waves, when $\omega_0 \sim \omega_{LH} \equiv \omega_{pi}/\sqrt{1 + \omega_{pe}^2/\omega_{Be}^2}$ (ω_{pi} is the ion plasma frequency), Eq. (7) is valid for arbitrary relationship between ω_{pe} and ω_{Be}.

3. Magnetic field perturbations

Here we compare the results obtained on the basis of (7) with the data [1]. In general, Eq. (7) describes the excitation of the magnetic field perturbations which have the components both parallel and perpendicular to the unperturbed magnetic field $\boldsymbol{B}_0$. Let us find the condition of the excitation of the magnetic field perturbations mostly in the paramagnetic sense, i.e., when the perturbations are “almost” parallel to $\boldsymbol{B}_0$. We denote

$$\boldsymbol{a}_\perp \equiv \frac{1}{|\boldsymbol{B}_0|} \frac{\omega_{pe}^2}{\omega_0^2} (\boldsymbol{E}_\perp (\boldsymbol{b} \cdot \boldsymbol{E}^*) + \boldsymbol{E}_\perp^* (\boldsymbol{b} \cdot \boldsymbol{E})). \tag{8}$$

Eq. (7) have solutions obeying the relationship

$$\nabla \delta B_z = \frac{\partial \boldsymbol{a}_\perp}{\partial z}. \tag{9}$$

This means that δB_z does not depend on z, while

$$\frac{\partial \delta B_z}{\partial x} = \frac{\partial a_x}{\partial z},\ \frac{\partial \delta B_z}{\partial y} = \frac{\partial a_y}{\partial z}. \tag{10}$$

Using the fact that $\partial \delta B_z/\partial z = 0$ and the relationships (10) we find from Eq. (7)

$$\Delta \delta \boldsymbol{B}_\perp = \mathbf{i} \frac{\partial}{\partial y} \left[\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \right] - \mathbf{j} \frac{\partial}{\partial x} \left[\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \right], \tag{11}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the directions of the axes Ox and Oy, respectively. Using Fourier representation we find

$$\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \propto \int \mathrm{d}\boldsymbol{k}_1 \mathrm{d}\boldsymbol{k}_2 (E_{k_1} E_{k_2}^* e^{\mathrm{i}(\boldsymbol{k}_1 - \boldsymbol{k}_2)\cdot \boldsymbol{r}}$$
$$+ E_{k_2} E_{k_1}^* e^{\mathrm{i}(\boldsymbol{k}_2 - \boldsymbol{k}_1)\cdot \boldsymbol{r}})[k_{2y} k_{1x} - k_{1y} k_{2x}], \quad (12)$$

where we take into account the longitudinal character of LH waves ($E_{k,i} = (k_i/|\boldsymbol{k}|)E_k$, $E_{k,i}^* = (k_i/|\boldsymbol{k}|)E_k^*$). If for two arbitrary LH waves with the wave vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ the relationship

$$k_{1y} / k_{1x} = k_{2y} / k_{2x} = \mathrm{const} \quad (13)$$

is fulfilled, then the excitation of the magnetic field perturbations in the direction perpendicular to the unperturbed magnetic field is negligible, and the magnetic field perturbations can be excited in the paramagnetic sense. This realationship means that LH wave spectrum consists of the waves propagating in one plane. In this case we obtain

$$\delta B_z \propto \frac{\omega_{pe}^2}{\omega_0^2} \frac{\boldsymbol{E}^2}{|\boldsymbol{B}_0|} \cos^2\theta_0, \quad (14)$$

where $\cos\theta_0 = k_z/|\boldsymbol{k}|$. The estimate (14) is valid for the spectrum which is rather broad and/or consists of several peaks. The width of LH wave spectrum in $\boldsymbol{k}$-space determines the characteristic wave vector of the low-frequency magnetic field perturbations. For the data of the experiments [1] (the characteristic frequency $f \approx 55$ MHz, $\omega_0 = 2\pi f \approx 3.46\cdot 10^8$ s^{-1}, $|\boldsymbol{E}| \sim 10$ kV/cm, $|\boldsymbol{B}_0| \approx 1.2$ kG, the electron density $n_e \sim 10^{12}$ cm^{-3}) under the assumption that $\cos\theta_0$ takes the typical (for LH waves) magnitude $\cos\theta_0 \sim \sqrt{m_e/m_i}$, we get from Eq. (14): $\delta B_z \sim 18$ G for hydrogen plasmas; $\delta B_z \sim 9$ G for deuterium plasmas. We see a good agreement of this estimate with the experimental data $\delta B_z \sim 10$ G.

4. Electron density perturbations

The electron density perturbations δn are related to the usual development of the modulational processes with the participation of LH waves. They appear in the places of LH field localization (see, e.g., [2]). Let us determine δn using the theory of perturbations. The dielectric permittivity for LH waves propagating under the angle θ_0 with respect to the external magnetic filed is

$$\varepsilon_k = (\varepsilon_0)_k - \frac{\delta n}{n_0} \equiv 1 - \frac{\omega_{pe}^2}{\omega^2}\cos^2\theta_0$$
$$- \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{\omega_{Be}^2} - \frac{\delta n}{n_0}, \quad (15)$$

where the electron (ω_{pe}) and ion (ω_{pi}) plasma frequencies correspond to the unperturbed electron and ion densities ($n_{e0} = n_{i0} = n_0$), respectively, $(\varepsilon_0)_k$ is the unperturbed dielectric function. Poisson's equation can be written in the form

$$(\varepsilon_0)_k E_k^+ = \frac{1}{n_0|\boldsymbol{k}|}(\boldsymbol{k}\cdot[\delta n \boldsymbol{E}]_k^+)$$
$$= \int \frac{\delta n_{k-k_1}}{n_0} \frac{(\boldsymbol{k}\cdot\boldsymbol{k}_1)}{|\boldsymbol{k}||\boldsymbol{k}_1|} E_{k_1}^+ \mathrm{d}k_1 \quad (16)$$

From the other hand, we have (see, e.g., [2])

$$(\varepsilon_0)_k \boldsymbol{E}_k^+ = 2\int \Sigma_{k_1,k_2,k_3}^{eff} E_{k_1}^+ E_{k_2}^+ E_{k_3}^-$$
$$\delta(k - k_1 - k_2 - k_3)\mathrm{d}k_1 \mathrm{d}k_2 \mathrm{d}k_3, \quad (17)$$

where $\Sigma_{k_1,k_2,k_3}^{eff}$ is the effective nonlinear third-order response in the fields of LH waves. The derivation of the response $\Sigma_{k_1,k_2,k_3}^{eff}$ for the case $\omega_{pe} < \omega_{Be}$ is presented, e.g., in [2]. Deriving this response for the case $\omega_{pe} < \omega_{Be}$ we use the same procedure as in [2]. We find

$$\Sigma_{k_1,k_2,k_3}^{eff} = -\frac{e^2}{2m_e^2}\frac{|\boldsymbol{k}_2 + \boldsymbol{k}_3|^2}{|\boldsymbol{k}||\boldsymbol{k}_1||\boldsymbol{k}_2||\boldsymbol{k}_3|}$$
$$\frac{\varepsilon_{k2+k3}^{(i)}}{\varepsilon_{k2+k3}}(\varepsilon_{k2+k3}^{(e)} - 1)$$
$$\times\left[\frac{k_z k_{1z}}{\omega^2} + \frac{\mathrm{i}(\boldsymbol{k}_\perp \times \boldsymbol{k}_{1\perp})_z}{\omega\omega_{Be}} - \frac{(\boldsymbol{k}\cdot\boldsymbol{k}_1)}{\omega_{Be}^2}\right]$$
$$\left[\frac{k_{2z}k_{3z}}{\omega^2} - \frac{\mathrm{i}(\boldsymbol{k}_{2\perp}\times\boldsymbol{k}_{3\perp})_z}{\omega\omega_{Be}} - \frac{(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)}{\omega_{Be}^2}\right], \quad (18)$$

where $\varepsilon^{(e)}$ and $\varepsilon^{(i)}$ are the electron and ion parts of the unperturbed dielectric permittivity ε (the subscript 0 is omitted).

Using the explicit values of $\varepsilon^{(e)}$ and $\varepsilon^{(i)}$ in the low-frequency limit $\tilde{\omega} = |\omega - \omega_1| = |\omega_2 + \omega_3| \ll \tilde{k}_z v_{Te}(\tilde{\boldsymbol{k}} = \boldsymbol{k} - \boldsymbol{k}_1 = \boldsymbol{k}_2 + \boldsymbol{k}_3)$, the fact that under the experimental conditions [1] $\omega_{pe}^2 \gg \omega_{Be}^2$, and the condition (13), we find that the main contribution to the effective response is made by the terms which do not contain the gyrotropic nonlinearity (proportional to vector products). Then Eq. (17) can be presented in the form (16), where

$$\frac{\delta n}{n_0} = -\frac{|\boldsymbol{E}|^2}{4\pi n_0(T_e + T_i)}. \quad (19)$$

Comparing (19) and (14), we find

$$\frac{\delta n}{n_0} \propto \frac{2}{\beta}\frac{\omega_0^2}{\omega_{pe}^2}\frac{\delta B_z}{|\boldsymbol{B}_0|}\sec^2\theta_0, \quad (20)$$

where $\beta = 8\pi n_0(T_e + T_i)/|\boldsymbol{B}_0|^2$, $T_{e(i)}$ is the electron (ion) temperature. For the typical data of the experiments [1] ($\omega_{LH}/2\pi = 55$ MHz, $\omega_{pe}/2\pi = 15$ GHz, $\beta \sim 0.001$, $\delta B_z/|\boldsymbol{B}_0| \sim 0.01$, $\cos\theta_0 \sim \sqrt{m_e/m_i}$) this estimate gives the value of $\delta n/n_0 \sim 0.5$. So this is in accordance with the experimental data $\delta n \sim 0.5n_0$ measured by the microwave interferometer when strong electric field bursts give rise to enhanced electron density modulation [6]. From Eq. (19) we see that the electron density modulation should be negative, i.e. this is a case corresponding to "density wells".

The above results were obtained under the condition (13). Actually, it is not necessary to require the exact fulfillment of this condition in order to consider the magnetic field perturbations in the paramagnetic sense. For this, only the inequality $|\delta\boldsymbol{B}_\perp| \ll |\delta B_z|$ should be valid. There are two cases: 1) when $|\delta\boldsymbol{B}_\perp| \ll |\delta B_z|(\omega_{Be}/\omega_{pe})^2$; and 2) when $|\delta\boldsymbol{B}_\perp| \gg |\delta B_z|(\omega_{Be}/\omega_{pe})^2$ while $|\delta\boldsymbol{B}_\perp| \ll |\delta B_z|$. In the first case the main terms in the square brackets in the right hand side of Eq. (18) are those not containing the vector products. It is this case that corresponds to the above calculations and the formulas (19) and (20). In the second case the dominant terms in the square brackets in the right hand side of Eq. (18) are those containing the vector products. In this case the result for $\delta n \sim (\boldsymbol{E} \times \boldsymbol{E}^*)_z$ coincides with that found in Ref. [7]. The sign of δn is not determined. This means that in this case both the density "wells" and "humps" can arise. This is in a good agreement with the data of the experiments [1] using high-voltage linear plasma discharge.

5. Summary

Thus the theory developed here allows us to explain the main experimental results [1] under the assumption that LH wave spectrum consists of the waves propagating (approximately) in one plane. The most probable reason for the excitation of the magnetic field perturbations in these experiments is the development of the magneto-modulational processes.

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