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Global Plasma Equilibrium in a Helical System with Ideally Conducting Wall

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Abstract

The global plasma shift is calculated analytically for a helical system with an ideal wall. The derived expression for the plasma shift, incorporating both the finite-β plasma expansion and the opposing reaction of the nearby ideal wall, can be used for interpreting the observable high-β equilibrium effects in LHD and other helical devices.

Keywords:

stellarator, plasma equilibrium, finite-β effects, plasma shift, ideal wall

1. Introduction

Analysis of free-boundary plasma equilibrium in a conventional stellarator shows [1] that the pressure-induced shift Δ_β of the plasma column must be fairly large at high β:

$$\frac{\beta}{2\beta_{eq}^0} \leq \frac{\Delta_\beta}{b} \leq \frac{\beta_0}{2\beta_{eq}^0}. \tag{1}$$

Here $\beta_{ep}^0 = \mu_b^2 b/R$, μ_b is the rotational transform at the plasma edge, b is the averaged minor radius of the plasma, R is the major radius, β is the volume-averaged ratio $2p/B_0^2$ with p being the plasma pressure and B_0 the toroidal magnetic field at $r = R$, and $\beta_0 = 2p(0)/B_0^2$ is the β value at the magnetic axis.

For Large Helical Device (LHD) with $R = 3.9$ m, $b = 0.6$ m, $\mu_b = 1$ [2] we have $\beta_{eq}^0 = 0.15$, and the lower bound in (1) is 0.1 for $\beta = 3\%$, which gives 6 cm outward shift.

If such a large shift would appear in LHD, it certainly could be observed, for example, when the Shafranov shift was measured with soft X-ray CCD camera [3]. However, there is no mentioning of observations of large 'global' plasma shift in [3]. Therefore a question arises why the plasma column shift in LHD is actually smaller than the above estimate.

The pressure-induced plasma shift can be suppressed by the vertical field $B_\perp$ as described by the formula (see [1,4] and references therein)

$$\Delta_b = \Delta_\beta + \Delta_\perp, \tag{2}$$

where Δ_b is the observed shift of the plasma boundary or 'global' plasma shift, the expressions for Δ_β and $\Delta_\perp$ are given below. The field $B_\perp$ can be produced by the currents in the poloidal windings, which can be controlled, and by the currents induced in the conducting structures around the plasma when the plasma column tends to expand toroidally with increasing β, as described by Δ_β. Such a case can be realized in LHD which is a superconducting device operating now with high β [2,5,6].

Theory of current-carrying plasma equilibrium in a tokamak with an ideally conducting casing is described in [7,8]. These studies have been stimulated long ago by the early experimental results showing the urgent need of providing the plasma equilibrium along the major radius in a tokamak [9]. However, the same problem for stellarators has not yet been analyzed. This is so because until recently the stellarators operated at rather low β, and the plasma equilibrium in stellarators is provided by the original magnetic configuration which is not strongly distorted by the equilibrium plasma currents at low β.

The problem of global plasma equilibrium in a stellarator was discussed in [10,11], see also the reviews [1,4]. The present analysis is based on the models described there. The new element introduced here is the ideal wall at some distance from the plasma. In [1,4,10,11], the vertical field $B_\perp$ was considered as a free parameter. Now we must find it under the constraint of flux conservation due to the ideally conducting wall.

2. Brief introduction to the model and definitions

The model is based on the modified stellarator approximation allowing description of tokamaks and stellarators within the unified approach [1,4]. We consider a 'conventional stellarator' with a circular planar axis and helical fields, a device like LHD or CHS. Within the model, the plasma boundary is described as a circular torus perturbed by the helical field. Large-aspect-ratio expansion is used in ana-

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lytical calculations here. The shift Δ_b of the plasma boundary is assumed to be small: $\Delta_b/b \ll 1$. For derivation of the basic relations from the general equilibrium equations and more detail see [1,10,11].

Magnetic surfaces ψ = const in stellarators (or tokamaks) are described by the function

$$\psi = \psi_v + \psi_{ext} + \psi_{pl}, \tag{3}$$

where ψ_v is the poloidal flux of the helical magnetic field, ψ_{ext} is the poloidal flux due to the external axisymmetric field, and ψ_{pl} is the plasma-produced poloidal flux. At the first step of our analysis we need the function ψ outside the plasma, in the vacuum region between the plasma and the wall.

The plasma-produced poloidal magnetic flux outside the plasma in a stellarator was calculated in [11]:

$$\psi_{pl} = 2\pi[f_0(l) + f_1(l)\cos\alpha + \ldots], \tag{4}$$

where (l, α) are the polar coordinates with origin at the center of the plasma cross-section,

$$f_0(l) = bRB_J\left(\ln\frac{8R}{l} - 2\right), \tag{5}$$

$$f_1(l) = \frac{b}{l}C - \frac{bl}{2}B_J\left(\ln\frac{8R}{l} - 1\right), \tag{6}$$

$B_J = J/(2\pi b)$ is the magnetic field of the current J, and C is a constant which can be related to plasma parameters by matching the solutions for the magnetic field at the plasma boundary.

The poloidal flux ψ_v of the helical magnetic field $\tilde{B}$ through the magnetic surface is calculated by the formula [1]

$$\psi_v = 2\pi r^2 < \tilde{B}_z \hat{\tilde{B}}_r / B_t >_\zeta, \tag{7}$$

where r, ζ, z, are the cylindrical coordinates associated with the main geometrical axis, $\hat{f}$ is the oscillating part of the integral of f over ζ, B_t is the toroidal magnetic field, and brackets $\langle\ldots\rangle_\zeta$ denote the averaging over ζ. The most simple approximation for ψ_v is obtained in coordinates (ρ, u) related to the geometrical axis: $\psi_v = \psi_v(\rho)$.

For our purposes, in the geometry assumed (circular averaged cross-sections of the plasma and the conducting wall), it is sufficient to keep in ψ_{ext} a term describing the external homogeneous vertical magnetic field:

$$\psi_{ext} = \psi_0(t) + \pi B_\perp(r^2 - R^2). \tag{8}$$

Next we must find the constraints from the boundary conditions for the function ψ.

3. Boundary conditions

By definition, the function ψ must be constant at the plasma boundary and at the ideal wall. Thus, at these two boundaries of the vacuum region we must satisfy the conditions

$$\psi = \text{const}. \tag{9}$$

We describe the plasma boundary as $l = b$. If ψ is given as a function of l and α,

$$\psi = A(l) + B_1(l)\cos\alpha, \tag{10}$$

the condition ψ = const at the plasma boundary is satisfied by $B(b) = 0$.

The plasma-generated part of ψ has been already calculated in such a form, eq. (4). We have to express two other functions in (3) in variables l and α.

The poloidal flux of the external vertical field, eq. (8), can be rewritten as

$$\psi_{ext} = \pi B_\perp[(R+\Delta_b)^2 - 2(R+\Delta_b)l\cos\alpha + l^2\cos^2\alpha]. \tag{11}$$

Here we use the relations between the coordinates:

$$r = R - \rho\cos u = R + \Delta_b - l\cos\alpha, \quad \rho\sin u = l\sin\alpha. \tag{12}$$

These relations give

$$\rho \approx l - \Delta_b\cos\alpha, \tag{13}$$

which is valid for $l \gg \Delta_b$. In this case, in linear approximation in Δ_b/b,

$$\psi_v(\rho) \approx \psi_v(l) - \psi_v'(l)\Delta_b\cos\alpha. \tag{14}$$

Combining (4), (11) and (14) and comparing the result with (10), we obtain

$$B(l) = 2\pi f_1(l) - 2\pi RlB_\perp - \psi_v'(l)\Delta_b, \tag{15}$$

and the boundary condition $B(b) = 0$ gives

$$f_1(b) - RbB_\perp + R\Delta_b B^* = 0. \tag{16}$$

Here f_1 is the function defined by (6), and

$$B^* \equiv -\frac{\psi_v'(b)}{2\pi R} = \mu_b\frac{b}{R}B_0. \tag{17}$$

In the model, the ideal wall is prescribed by $\rho = a_c$, which allows a shift of the plasma relative to the wall. To apply the constraint (9) at $\rho = a_c$, we must transform ψ to the form

$$\psi = D(\rho) + E(\rho)\cos u, \tag{18}$$

so that the condition ψ = const at the wall is satisfied by $E(a_c) = 0$.

Similar to (13), for $\rho \gg \Delta_b$ we can express l from (12) as

$$l \approx \rho + \Delta_b\cos u. \tag{19}$$

After transformation of the functions ψ_{pl} and ψ_{ext} to variables (ρ, u) we obtain

$$E(\rho) = 2\pi[f_0'(\rho)\Delta_b + f_1(\rho)] - 2\pi R\rho B_\perp, \tag{20}$$

and, finally, from $E(a_c) = 0$:

$$(b/a_c)[f_1 + f_0'(a_c)\Delta_b] - RbB_\perp = 0. \tag{21}$$

Relation (16) is a part of the equilibrium conditions for a plasma. This follows directly from $\nabla p = \boldsymbol{j} \times \boldsymbol{B}$ and must be satisfied in any case. Equation (21) is an additional constraint resulting from the requirement that the wall is ideal.

4. Equation for Δ_b

Without the wall, we have eq. (16) which gives Δ_b as a function of $B_\perp$ and f_1 with $B_\perp$ being a parameter. With ideal wall, we have two equations: (16) and (21). Subtracting one from another to eliminate $B_\perp$, we obtain

$$f_1(b) - \frac{b}{a_c} f_1(a_c) = \frac{b}{a_c} f_0'(a_c)\Delta_b - R\Delta_b B^*, \quad (22)$$

which, by using (5) and (6), can be transformed into

$$C\left(\frac{b^2}{a_c^2} - 1\right) + \frac{b^2}{2} B_J \ln\frac{a_c}{b} = R\Delta_b\left(B^* + \frac{b^2}{a_c^2} B_J\right). \quad (23)$$

The constant C, first appeared in (6), is given by [11]

$$C = \frac{bR}{2}\left(H_1 - \frac{\Delta_b}{b} B^* - \frac{b}{2R} B_J\right). \quad (24)$$

So, the equality (23) can be rewritten in a form convenient for further use:

$$\Delta_b\left(B_J + B^*\frac{a_c^2}{b^2}\right) = \frac{a_c^2}{2R}\left\{B_J \ln\frac{a_c}{b} + \left[\frac{1}{2}B_J - \frac{R}{b}\left(H_1 - \frac{\Delta_b}{b}B^*\right)\right]\left(1 - \frac{b^2}{a_c^2}\right)\right\}, \quad (25)$$

5. Two opposite limits: tokamak and current-free stellarator

Since no restriction has been imposed on B_J and B^* in the above derivation, expression (25) must be valid for any ratio between B^* and B_J. In other words, it can be applied either to tokamaks, $B^* = 0$, or to stellarators without current, $B_J = 0$, which are two limiting cases. It describes, as well, any intermediate configuration (stellarators with current).

For a tokamak, $B^* = 0$, relation (25) is reduced to

$$\Delta_b = \frac{a_c^2}{2R}\left[\ln\frac{a_c}{b} + \left(\Lambda + \frac{1}{2}\right)\left(1 - \frac{b^2}{a_c^2}\right)\right], \quad (26)$$

where $\Lambda \equiv -RH_1/(bB_J)$. This is a well-known Shafranov's result [7] for a tokamak plasma.

The tokamak expression (26) cannot be used for current-free plasma in stellarators (more precisely, for equilibrium configurations with small B_J/B^*) because Λ is not defined for $B_J = 0$ when H_1 is finite. Since B^* and B_J enter eq. (25) in different ways, it is impossible therefore to draw a close analogy between tokamaks and stellarators in this case.

For a current-free plasma, eq. (25) gives

$$\frac{\Delta_b}{b}B^* = -\frac{1}{2}\left(H_1 - \frac{\Delta_b}{b}B^*\right)\left(1 - \frac{b^2}{a_c^2}\right). \quad (27)$$

The quantity H_1 can be found from the magnetic measurements outside the plasma [11]. The shift Δ_b itself can be measured by the magnetic loops and probes [11, 12]. Thus, eq. (27) relating the measurable values allows experimental verification.

Equations (25), (26) and (27) clearly show the effect of the conducting wall on the plasma shift. In all cases, for $a_c = b$ they give a natural result $\Delta_b = 0$. This illustrates a general tendency: the plasma shift Δ_b is smaller for the wall closer to the plasma.

6. Current-free stellarator with circular shifted magnetic surfaces

If the surfaces ψ = const near the plasma boundary Γ are circular (the relative shift is allowed), we obtain [11]

$$H_1 = \frac{b}{R}B_J + \frac{b}{R}B_0(\mu\Delta' + \mu_h'\Delta_b) + B^*\frac{\Delta_b}{b}, \quad (28)$$

which is valid for both the tokamaks and stellarators. If we assume that the model of circular shifted magnetic surfaces is applicable for the description of the whole plasma column, one can obtain from the equilibrium equations [1,4]

$$(\mu_h\Delta)'|_\Gamma = 2\frac{R}{b}\frac{B_\beta}{B_0}, \quad (29)$$

where

$$B_\beta = \frac{1}{B_0}\int_0^b \frac{p'(a)}{\mu(a)}\frac{a^2}{b^2} C_{PS}\, da, \quad (30)$$

and C_{ps} is the coefficient describing the reduction of the Pfirsch-Schlüter current when the plasma column is strongly shifted relative to the geometrical center of a stellarator. Usually $C_{ps} \approx 1$ in conventional stellarators. For the case considered, eq. (28) yields

$$H_1 - B^*\Delta_b/b = 2B_\beta. \quad (31)$$

This allows us to rewrite (27) in a compact form

$$\Delta_b = \Delta_\beta(1 - b^2/a_c^2), \quad (32)$$

where

$$\Delta_\beta = B_\beta/B^*. \quad (33)$$

The latter is the free-boundary pressure-incduced plasma shift disussed in the Introduction.

Expression (32) explicitly describes the ideal wall effect on the pressure-induced shift of the current-free plasma column in a stellarator. The effect is stronger for the wall closer to the plasma. However, the effect is noticeable even when the gap between the plasma and the ideal wall is rather large. For example, eq. (32) shows that for $b/a_c = 0.7$ the wall effect results in 50% reduction of the shift. This means that in the example considered in the Introduction we would obtain 3 cm plasma shift with the ideal wall instead of 6 cm for the free-boundary case.

Note that Δ_b can be also written in the form (2) with Δ_β given by (33) and with

$$\Delta_\perp = bB_\perp/B^* \quad (34)$$

(this follows from (16) for the current-free plasma). Equations (32) – (34) imply that with ideal wall $\Delta_\perp = -\Delta_\beta b^2/a_c^2$, so that in this case

$$B_\perp = B_\beta\, b^2/a_c^2. \quad (35)$$

The latter relation gives the value of the vertical field produced by the currents induced in the ideal wall.

7. Conclusion

The analysis shows that the ideal wall must strongly affect the pressure-induced plasma shift in helical devices. According to (32), the effect is strong even for $b/a_c = 1/2$. The plasma shift reduction due to the currents induced in the wall may be a reason why the Shafranov shift measured with soft X-ray CCD camera [3] was small.

According to (32), the plasma shift may be completely suppressed by the wall reaction in case only when the plasma-wall gap is negligible. In real devices, the gap is finite. For LHD, a rough estimate can be $b/a_c = 0.6/0.9$ [13]. In this case, the plasma shift must be twice smaller than it would be without the wall. For $\beta \geq 3\%$ we obtain from (32) that the shift will be several centimeters in LHD. Such a shift is large enough to be measured, which can be used for diagnostic purposes. Therefore, the problem deserves more attention.

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