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Quasi-isodynamic Configuration with Small Number of Periods

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Abstract

The possibility to fulfill the quasi-isodynamicity condition [1] for all reflected particles for moderate (N=6) [2] and high (N=9) [3] numbers of periods was shown earlier by numerical optimization. It was demonstrated that the fulfillment of the quasi-isodynamicity condition leads to significant improvement of long-time collisionless fast particle confinement. In the present report, the same task is considered for a small number of periods, N=2. It is shown that for sufficiently high three-dimensionality of the magnetic axis (deviation of the magnetic axis from a plane), it is possible to satisfy the quasi-isodynamicity condition along with the Mercier- and resistive-mode stability criteria.

Keywords:

stellarator, numerical optimization, poloidal pseudo-symmetry, quasi-isodynamicity, collisionless fast particle confinement

1. Introduction

As has been shown earlier (see, e.g., Refs. [1-3]), stellarator configurations with poloidal direction of the contours of the magnetic field strength B on magnetic surfaces can be optimized with respect to poloidal pseudosymmetry [4] and quasi-isodynamicity [1], so that for all reflected particles, the second adiabatic invariant $\mathcal{J} = \int V_{\|}\, \mathrm{d}l$ becomes a function of the flux label s of the magnetic surface to a very good approximation. Approaching these conditions guarantees good long-time collisionless confinement of fast particles. It was shown also that for moderate (N=6) and high (N=9) numbers of periods, the requirement of good confinement of fast particles is well compatible with the stability conditions. It is interesting to study the possibility to realize such kind of configurations for compact systems with smaller number of periods. Here some preliminary results for a two-period configuration are presented. The configuration considered has $\langle\beta\rangle = 2\langle p\rangle/\langle B^2\rangle \approx 0.03$($\langle\ldots\rangle$ means the averaging over plasma volume, p is the plasma pressure) and is assumed to have zero net toroidal current.

2. Choice of the initial configuration and the target functions

The optimization for N=2 case was performed on the NEC SX-5 super-computers himiko (Germany) and prometeo (Switzerland) with the VMEC code [5] for equilibrium calculations, the JMC code [6] for the transition to magnetic (Boozer) coordinates and the MCT code [7] for direct calculation of particle drift orbits following the same procedure as that considered early for the N=6 and N=9 configurations.

The optimization was performed within the class of configurations with so-called "stellarator symmetry", for which, in Fourier space, the coordinates R and Z of cylindrical coordinate system R,Z,ϕ can be represented as sums of cosine (for R) and sine (for Z) terms. Also, it was assumed that the longitudinal magnetic field on the magnetic axis has only one minimum and one maximum within one period. In this case it follows from the condition of quasi-isodynamicity that the magnetic axis is a spatial (non-planar) curve. The initial geometry of the boundary magnetic surface was prescribed in the same manner as for the configurations considered earlier: the cross-sections were chosen to be triangular in the region with minimal B on the magnetic axis and bean-shaped in the region with maximal B.

Compact configurations have some peculiarities in comparison with systems with larger number of periods. Firstly, because of the small number of periods the gaps in the rotational transform between low-order resonances are narrower. Here, from the possible gaps (4/10 – 4/9), (4/9 – 4/8) and (4/8 – 4/7) the last one was chosen, so that the

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rotational transform was required to be in the region $0.5 < \iota < 0.57$ for all plasma radii. In this case there are no resonant magnetic surfaces inside the plasma column with poloidal numbers $m \leq 10$. Secondly, the transition to a small number of periods and small aspect ratio leads to an increase of the curvature of the magnetic axis. That means that the poloidal variation of the magnetic field strength increases, so that to fulfill the pseudo-symmetry condition the mirror component in the field strength must be increased, too. This increase leads to a widening of the Fourier spectrum of B, that makes the VMEC calculations and the optimization procedure itself more difficult.

The value of $<\beta>$ was chosen as a compromise: low $<\beta>$ does not create the radial dependence of $<\mathcal{J}>_\theta$ necessary for improvement of fast particle confinement, while a large value of $<\beta>$ leads to problems with convergence and stability. In the present report a configuration with $<\beta> \approx 0.03$ is considered.

In addition to the requirement mentioned above on the rotational transform, further requirements such as pseudo-symmetry, Mercier- and resistive-mode stability and restrictions on the magnetic axis shift were imposed as nonlinear conditions. As a target function the measure of quasi-isodynamicity was used. This measure was constructed in such a way that optimization leads to an increase of the area inside the closed contours of the second adiabatic invariant for a sufficiently large set of pitch angles.

3. Results of the numerical optimization

The optimization of the N=2 configuration was performed for $<\beta> = 2.7\%$. A 3D view of the boundary magnetic surface of the optimized configuration is shown in Fig. 1. The configuration obtained is similar to a figure-eight stellarator. The ratio B_{max}/B_{min} is quite large here, $B_{max}/B_{min} \approx 3.5$. Another characteristic feature of the configuration found is a large vertical excursion of the magnetic axis, so that in spite of a small toroidal aspect ratio, $R/a_0 \approx 3.6$ (here R, a_0 are averaged large and small plasma radii), the ratio $L/2\pi a_0$ is about 6 with L the magnetic axis length. The boundary magnetic surface cross-sections at the beginning, one quarter of and half a period are shown in Fig. 2, as well as radial profiles of the rotational transform, magnetic well and plasma pressure. The high elongation of the cross-section at half a period is partly due to the high slope of the magnetic axis (the cross-sections are shown for ϕ=*constant*).

Fig. 3 shows the contours of B on the magnetic surface that corresponds to 1/2 of the small plasma radius (1/4 of the toroidal flux). As in previously found quasi-isodynamic configurations [2,3], the curvature of the magnetic axis in the regions of the extrema of B on the magnetic axis is nearly zero. Fig. 4 shows the contours of the second adiabatic invariant for a set of different values of B_{ref}, the value of B from which the trapped particle is reflected, or for different pitch angles, starting from deeply trapped particles (top left, $B_{ref} = B_{min} + \Delta B/5$) to barely reflected particles (bottom right, $B_{ref} = B_{min} + 4\Delta B/5$). Here $\Delta B = B_{max} - B_{min}$ with B_{max}, B_{min} the maximal and minimal values of B on 1/2 of the minor plasma radius. It is seen that for the inner part of the plasma column the contours of $\mathcal{J}$ are closed inside the plasma for all values of pitch angles. The quality of the reflected particle confinement in the optimized configuration was verified by

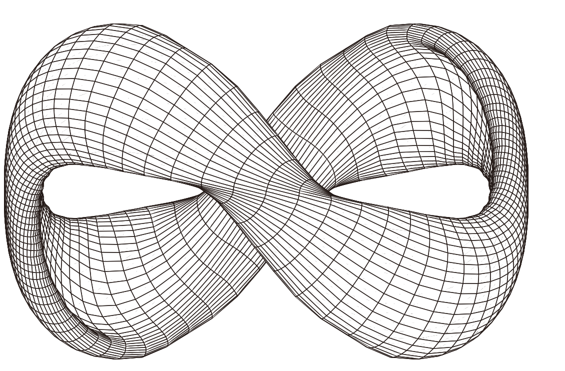

Fig. 1 Boundary magnetic surface of the optimized configuration.

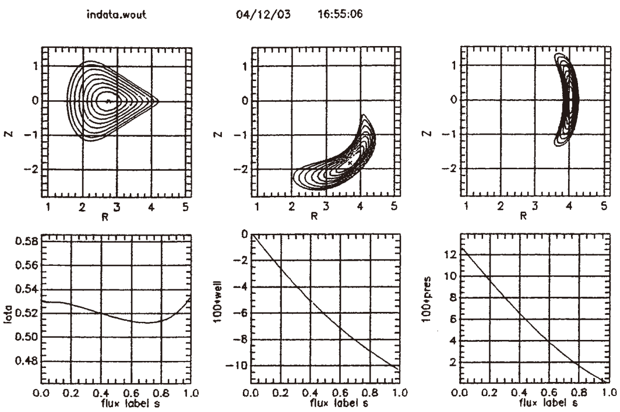

Fig. 2 Magnetic surface cross-sections at the beginning, quarter of and half a period and radial profiles of the rotational transform, magnetic well and plasma pressure.

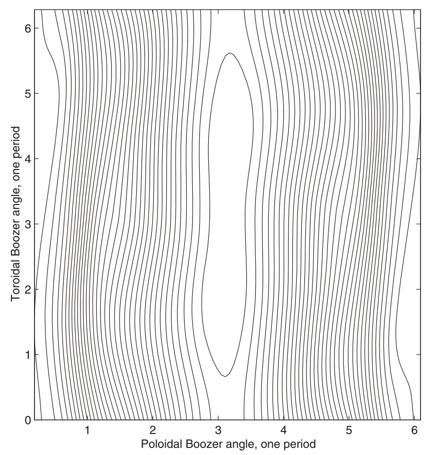

Fig. 3 Contours of B on the magnetic surface with s = 0.25 for the optimized configuration.

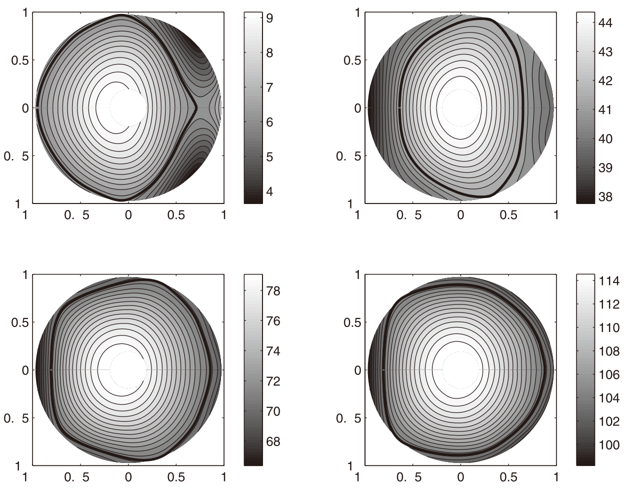

Fig. 4 $\mathcal{J}$ contours for increasing values of B_{ref} in polar-coordinate representation $\sqrt{s}$, θ with s the flux label.

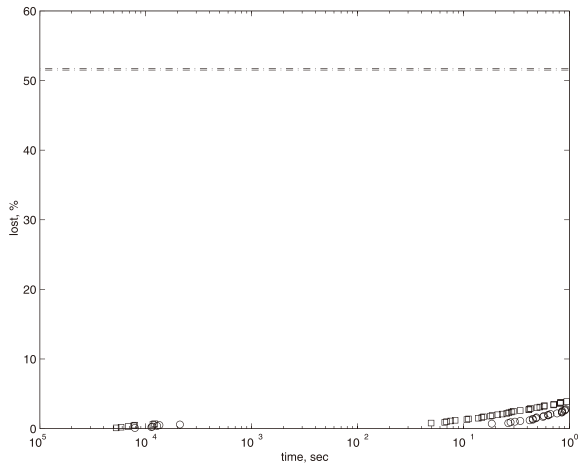

Fig. 6 Collisionless α-particle confinement in the optimized configuration as a function of the time of flight. Particles are started at 1/2 of the plasma radius. Normalizations: plasma volume 1000 m^3, B_0 = 5 T (circles) and B_0 = 3 T (squares).

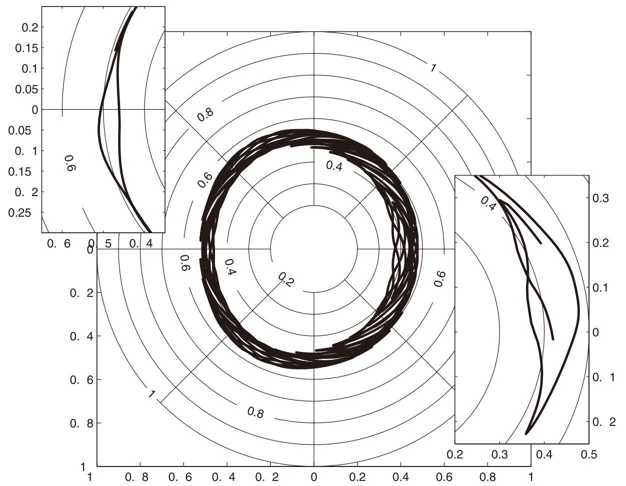

Fig. 5 Particle drift trajectory for the N = 2 configuration. In the additional frames single bananas are shown.

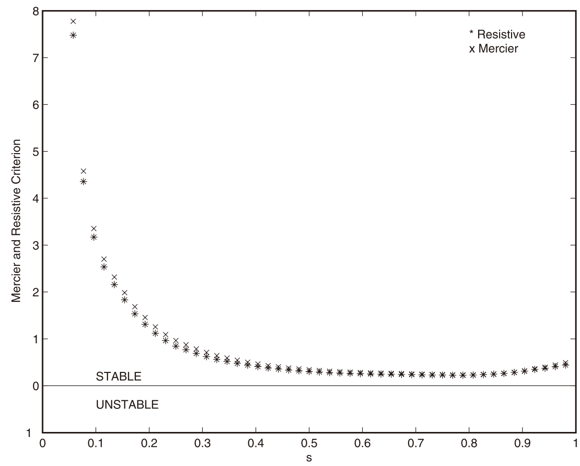

Fig. 7 Mercier- and resistive-mode stability vs. normalized flux label.

direct calculation of the drift particle orbits. Fig. 5 shows an example of the projection of the reflected α-particle trajectory for a fusion-plant-sized machine (B_0 = 5T, V = 1000 m^3). The history of losses of 1000 α-particles born at a half of plasma minor radius is shown in Fig. 6. In addition, the results of the same calculations are shown for B_0 = 3 T. It is seen that diminishing the magnetic field strength does not lead to a strong increase of losses.

Fig. 7 shows that the Mercier and the resistive modes are stable for all plasma radii for $<\beta>$ = 2.7%.

The configuration described above was examined with respect to local ballooning-mode stability. It was found that these modes are unstable for $a/a_0 > 0.3$. Some attempts were made to stabilize the ballooning modes. It was found that to improve the ballooning mode stability it is necessary to increase the vertical excursion of the magnetic axis, i.e. to make the configuration less compact. The modified configuration obtained requires further optimization of particle confinement. The possibility to stabilize Mercier modes for larger $<\beta>$ was investigated too. It was shown that for $<\beta>$ up to 5% Mercier and resistive modes can be stabilized, with retaining fast particle confinement. Some attempts were made to decrease the ratio $\mathcal{R} = B_{max}/B_{min}$. It was found that $\mathcal{R}$ can be diminished from 3.5 to 3.0 without violation of confinement.

The configuration found is not fully optimized. However, it demonstrates that the method of the optimization considered hitherto can be applied not only to configurations with high number of periods but to compact systems as well.

4. Conclusions

By computational optimization, it is shown that compact configurations with poloidal direction of the contours of B exist in which the confinement of fast particles can be significantly improved by approaching pseudo-symmetry and quasi-isodynamicity. Also, it is shown that the condition of improved confinement is well compatible with magnetic well creation and Mercier- and resistive-mode stability requirements. The optimization towards the ballooning-mode stability, diminishing of the neoclassical effective ripple and bootstrap current are still under investigation.

Acknowledgments

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