J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 519–522

Effects of Current Profile on Global Ideal MHD Stability in a Compact Quasi-Axisymmetric Stellarator

SUZUKI Chihiro, OKAMURA Shoichi, ISOBE Mitsutaka, NISHIMURA Shin, SHIMIZU Akihiro, AKIYAMA Tsuyoshi, MATSUOKA Keisuke, NAKAJIMA Noriyoshi, COOPER Wilfred A.[1] and NÜHRENBERG Carolin[2]

National Institute for Fusion Science, Toki 509-5292, Japan

[1]Centre de Recherches en Physique des Plasmas, Association Euratom-Suisse, Ecole Polytechnique Fédérale de Lausanne, CH-1015, Lausanne, Switzerland

[2]Max-Planck-Institut für Plasmaphysik, D-17491, Greifswald, Germany

(Received: 9 December 2003 / Accepted: 26 February 2004)

Abstract

The global ideal magnetohydrodynamic (MHD) stability for a proposed compact quasi-axisymmetric stellarator CHS-qa has been investigated taking the effect of bootstrap current into account. Assuming experimentally achievable density and temperature profiles, the stability properties of global low-*n* modes have been studied by using three-dimensional numerical codes based on fixed boundary MHD equilibria including self-consistent bootstrap current for the CHS-qa reference configuration. Consequently it has been shown that values of edge rotational transform play a crucial role in triggering external kink instability. Concerning a lot of other possibilities in experimental practice to change the total parallel current, we have also studied equilibria with increased or decreased parallel current, but fixed profile. The onset of external kink modes depends on rotational transform or current profile, and we found a stable equilibrium in spite of the edge rotational transform above 0.5. The results imply the possibility of stabilizing external kink modes through current and/or pressure profile control in high beta equilibria.

Keywords:

quasi-axisymmetric stellarator, CHS-qa, bootstrap current, ideal MHD stability, external kink mode, rotational transform, current profile

1. Introduction

It is intrinsically unavoidable that relatively large amount of bootstrap current (BSC) flows so as to increase rotational transform in a quasi-axisymmetric (QA) stellarator according to its tokamak-like magnetic field structure. Therefore the role of the BSC on ideal magnetohydrodynamic (MHD) stability properties in a QA configuration is more important than those in other stellarators. Accordingly ideal MHD stability analysis including plasma current is essential for exploring beta limit and avoiding serious instabilities in the physics design process of a QA device.

In this study we discuss the effects of current profile on ideal MHD stability properties in a QA stellarator CHS-qa proposed as a candidate for the next step compact stellarator in Japan. In order to assess ideal MHD instabilities driven by the BSC, three-dimensional global low-*n* mode analyses based on calculations of eigenmode structure are necessary in addition to local mode analyses. Recent progress in supercomputer performance makes it possible to calculate immediately ideal MHD modes in such complicated three dimensional configurations.

Since basic information of CHS-qa has already been published [1], only a brief explanation is given here. Average major and minor radii of CHS-qa are 1.5 and 0.4 m, respectively. The maximum magnetic field strength generated by 20 modular coils is 1.5 T. The current reference configuration of CHS-qa is called "2b32", which means the number of field periods is N_p = 2, aspect ratio is 3.2, and it has been optimized to stabilize local ballooning modes on a single flux surface. We discuss this reference configuration only throughout this article.

2. Numerical procedure

The entire procedure to solve finite beta equilibria including self-consistent BSC has already been described elsewhere [2]. We have employed the SPBSC code which executes calculations of fixed boundary MHD equilibrium and analytical BSC formula [3] alternately until convergence. The assumed density and temperature profiles in this study are

Corresponding author's e-mail: csuzuki@nifs.ac.jp

$n(s) = n_0(1 - 0.8s + 1.3s^2 - 1.5s^3)$ and $T(s) = T_0(1 - s)$, respectively, where s is the normalized toroidal flux and the subscript "0" means central value. These profiles are chosen to be compatible with experimentally achievable ones in existing devices.

Since it is difficult to get good convergence of BSC in the high beta equilibria investigated in this study, only axisymmetric components of the magnetic field are included in the BSC calculation, while all components are used in the equilibrium calculation. This implies that the calculated BSC should correspond to the upper limit in that equilibrium.

The global low-n ideal MHD stability is calculated mainly with the TERPSICHORE code [4]. Since the fluid compression term in the energy integral is ignored for simplicity, the calculated eigenvalue does not strictly agree with the physical growth rate. However, the magnitude of the eigenvalue is still valid for a rough estimation, and the marginal point for stability is correct. We have just started calculation with the CAS3D3 code [5] including the fluid compression term for obtaining physical growth rates, which will be studied extensively in the future.

As is well known, global ideal MHD mode structures in non-axisymmetric configurations constitute "mode families" due to mutual mode coupling. In CHS-qa (N_p = 2), even toroidal mode numbers make up the N = 0 family, while odd numbers the N = 1 family. In this paper, we display the results of the N = 1 family only, because the N = 0 family has a similar tendency to the N = 1 family and it is meaningless to display the results of both families. The mode selection table for the N = 1 family is shown in Fig. 1, where 81 perturbation Fourier modes included in the calculation are indicated by black dots. We consider all modes resonant with rotational transform from 0.3 to 0.8 and coupling with the principal equilibrium helical component (m,n) = (1,2) to cover transform profiles of all equilibria examined in this study. The toroidal perturbation mode numbers up to 7 are considered to meet the scope of this study (low-n mode instability) and to save the calculation time.

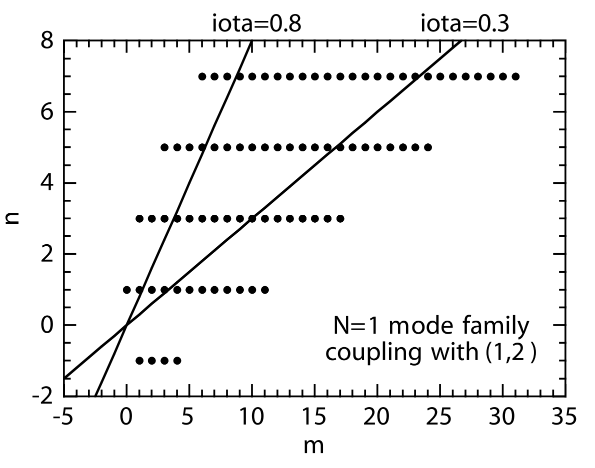

Fig. 1 Perturbation mode selection table for N = 1 family. Modes between the two solid lines are resonant with rotational transform from 0.3 to 0.8. In addition, the mode coupling with the equilibrium component (m,n) = (1,2) is considered.

3. Effect of bootstrap current

In this study, three representative cases of plasma parameter sets are investigated as shown in Table 1. The central electron density and temperature, volume average beta, and total calculated BSC per toroidal magnetic field of 1 T are listed in Table 1. The ion temperature is assumed to be three quarters of the electron temperature. The profiles of self-consistent BSC and the resulting rotational transform are illustrated in Fig. 2. Though the volume average beta in cases A and B is identical ($\langle\beta\rangle$ = 3 %), the plasma density in A is

Table 1 Plasma parameters of three representative cases for stability calculations. The ion temperature is assumed to be three quarters of the electron temperature. The average beta and the BSC are calculated per toroidal magnetic field of 1 T.

	n_e (10^{20} m^{-3})	T_e (keV)	$\langle\beta\rangle$ (%)	I_{BSC} (kA)
A	1.0	1.04	3.0	103
B	0.2	5.2	3.0	168
C	1.5	1.04	4.6	122

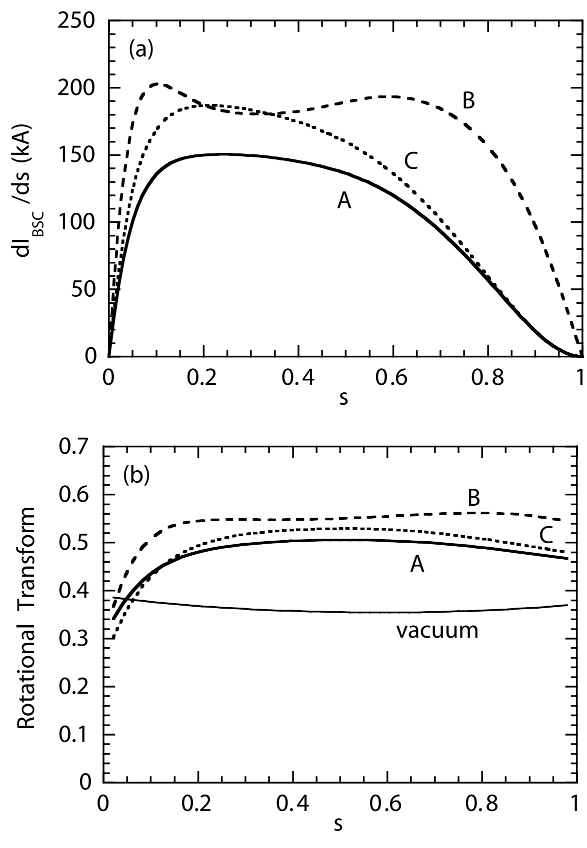

Fig. 2 The radial profiles of (a) bootstrap current density and (b) rotational transform for the three cases listed in Table 1. The rotational transform in vacuum is also displayed in (b) for reference.

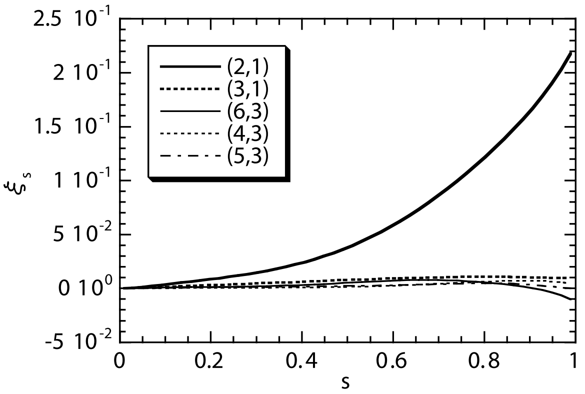

Fig. 3 Fourier components of the most unstable perturbation normal to the flux surfaces in the case B.

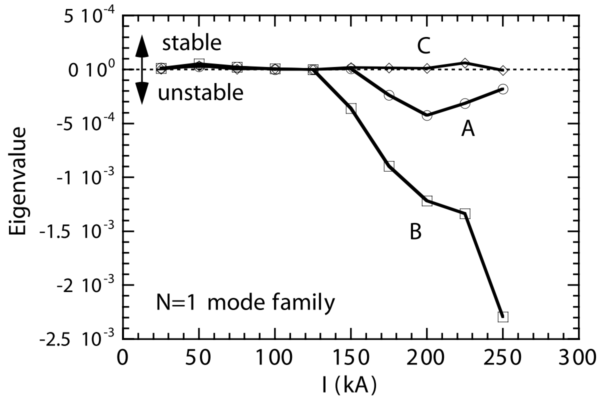

Fig. 4 The most unstable eigenvalues as functions of total current for three representative cases.

five times higher than that in B. This means that the collisional regimes are different, which results in differences in the magnitude of BSC and rotational transform as shown in Fig. 2. Note that the edge rotational transform exceeds 0.5 for B, while not for A. The plasma density in the case C corresponds to 1.5 times the case A, which results in higher average beta ($\langle\beta\rangle$ = 4.6 %). The current profile is slightly shifted inward in comparison with A, and the edge rotational transform is kept below 0.5.

Under these conditions, the global ideal MHD stability is analyzed in the manner described in Sec. **2**. As a result, instability appears only in the case B. The eigenmode structure normal to flux surfaces for the most unstable perturbation for this case is shown in Fig. 3. The amplitude of the dominant perturbation mode number (m,n) = (2,1) increases toward the edge, which clearly indicates the characteristics of an external kink instability caused by the rational surface of 0.5. This indicates that values of edge rotational transform play a crucial role in triggering external kink instability.

The calculation of this instability has also been performed with the CAS3D code including the fluid compression term (CAS3D3), and similar results have been obtained with slightly distorted eigenfunction. The resulting physical growth rate is $\omega = 1.0\times10^5$ s^{-1}.

4. Dependence on total parallel current

Since non-axisymmetric Fourier components of the magnetic field are excluded in the BSC calculation shown in Section **3**, the magnitude of BSC is actually reduced to some extent from the values listed in Table 1. On the other hand, it would be possible to further elevate the rotational transform by external current drive techniques such as neutral beam injection, ohmic current, *etc*. Therefore it is important to investigate the dependence of the stability properties on the magnitude of the total parallel current. For this purpose, we have artificially changed the magnitude of the parallel current from 25 to 250 kA (per 1 T) while keeping its profile shown in Fig. 2 (a).

Figure 4 shows eigenvalues of the most unstable perturbation modes for the N = 1 family as functions of the total parallel current. Positive and negative eigenvalues correspond to stability and instability, respectively. The mode number of the dominant Fourier component in the unstable eigenfunction is (m,n) = (2,1) for 150–225 kA, and (5,3) for 250 kA. The edge rotational transform, which is closely associated with external kink instability, traverses 0.5 between 125 and 150 kA and 0.6 between 225 and 250 kA for all the cases studied. This implies that the onset of destabilization above 150 kA in the cases A and B clearly corresponds to the crossing of the edge rotational transform beyond 0.5 and 0.6. However, case B is apparently more unstable in comparison with A. Furthermore, if the total current is increased above 200 kA, the eigenvalue for A tends to decrease with total current, while it continues to increase for B. Since the pressure profile and average beta are completely identical for these two cases, this difference can be attributed to the difference in rotational transform profile closely associated with the BSC profile. One possible reason for this phenomenon is the fact that the edge shear is stronger and the current density near the edge is lower in A compared with those in B as shown in Fig. 2 (b).

The result of the case C is more interesting. In this case the global mode is kept stable up to 250 kA total current despite the edge rotational transform being raised above 0.5. Apparently it seems to be reasonable that it could be attributed to the difference in current profile. However, if the average beta is artificially decreased to the same level ($\langle\beta\rangle$ = 3.0 %) as the other two cases while keeping the current profile fixed, the (m,n) = (2,1) external kink mode is destabilized (not shown in the figure). Therefore, other reasons should be considered to explain this phenomenon. Since fixed boundary calculations at high beta above 4 % generate equilibrium solutions with strongly distorted flux surfaces, more realistic calculations based on free boundary VMEC equilibria are necessary to investigate this implication. Nevertheless, this study demonstrated the importance of rotational transform profile on global MHD stability in the QA configuration and the possibility of stabilization of the external kink mode by controlling the parallel current through pressure profile control or various current drive methods.

5. Conclusion

We have investigated the global ideal MHD stability properties of a proposed compact QA stellarator CHS-qa reference configuration considering a self-consistent BSC. With a reasonably achievable plasma pressure, external kink modes resonant with the $(m,n) = (2,1)$ rational surface are destabilized in the low density case at an average beta of 3.0 % because the edge rotational transform crosses 0.5. The dependence of the eigenvalue on the total parallel current has revealed that the global mode stability properties are strongly affected by the rotational transform or the parallel current profile, and it may be completely stabilized under appropriate conditions. The results imply the possibility of stabilization of external kink modes by controlling the current and/or the pressure profiles. In order to clarify the destabilizing mechanism in more detail, contributions of each destabilizing term to the overall energy integrals should be investigated with the CAS3D3 or modified TERPSICHORE codes [6]. Additionally, more realistic calculations based on free boundary MHD equilibria are required to estimate more precisely the onset of current-driven instabilities.

References

[1] S. Okamura *et al.*, Nucl. Fusion **41**, 1865 (2001).
[2] M. Isobe *et al.*, J. Plasma Fusion Res. SERIES **5**, 360 (2002).
[3] K.Y. Watanabe *et al.*, Nucl. Fusion **35**, 335 (1995).
[4] D.V. Anderson *et al.*, Int. J. Supercompt. Appl. **4**, 34 (1990).
[5] C. Nührenberg, Phys. Plasmas **6**, 137 (1999).
[6] W.A. Cooper *et al.*, “*Analysis of the MHD Instability Driving Mechanismsin 3D Heliotron and Quasi-Axisymmetric Systems*”, J. Plasma Fusion Res. SERIES **6**, P1-03 (2004).