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Conceptual Approach to Steady State Reversed Field Pinch Reactor with Low Aspect Ratio

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Abstract

According to a simplified costing algorithm of steady state fusion reactor power plant, the relative attractiveness of advanced physics modes mainly depends on the stability and non-inductive current drive of the equilibrium. The high stability beta and the good alignment of current profile with plasma self-induced current profile are compatible for the low aspect ratio neoclassical RFP equilibrium solved self-consistently considering the bootstrap current. The plasma stability is due to the hollow current profile making the magnetic shear increase locally and globally and the force-free region wide. The good alignment of current profile reduces significantly the requisite power for non-inductive current drive to generate the steady state magnetic configuration. As the result, the fusion power plant based on the neoclassical reversed field pinch (RFP) equilibrium with low aspect ratio offers the lowest cost of electricity.

Keywords:

RFP, low aspect ratio, high beta, plasma self-induced current, alpha particle finite banana-width effect, steady state, reactor concept, D-T and D-^{3}He Fusion, *COE*

1. Introduction

In our previous studies, the benefit of radiofrequency wave current drive to widen the force-free field region had been theoretically demonstrated by the significant reduction of nonlinearly turbulent level associated with the dynamo activity in RFPs [1,2]. Recently the current profile modification experiments have confirmed to enhance the energy confinement time up to tokamak-like scaling at higher beta and ten times smaller strength of surface magnetic field [3-5]. In order to realize its steady state with less requirements of wave power, we are concerning with low aspect ratio equilibrium where the neoclassical viscosity has a divergent tendency for aspect ratio $A \sim 1$, which means the enhancement of bootstrap current.

2. Target equilibrium

Two types of target magnetohydrodynamic (MHD) equilibria for RFP with low aspect ratio are considered. The first is the classical one described by partially relaxed state model (PRSM), which is reasonably close to dynamo-free, stable minimum energy state at finite beta. The second is the neoclassical one solved self-consistently considering the bootstrap current, which enhances the plasma stability owing to the current profile making the magnetic shear increase locally and globally and the force-free region wide although it does not satisfy the PRSM condition [6]. The local and global magnetic shear in the neoclassical equilibrium is significantly large compared with that in the classical (PRSM) equilibrium where the Robinson's stability criterion of ideal kink modes is satisfied for the volume averaged beta value of $\langle\beta\rangle$ ($\equiv 2\mu_0\langle p/B^2\rangle$) = 66 % or the toroidal beta value of β_t ($\equiv 2\mu_0\langle p\rangle/B_0^2$) = 59 % giving the stability beta limit against Mercier's localized mode.

In addition to the requisite rf power for the generation of steady state configuration, an additional rf power for the stabilization of resonant kink modes at distant conducting wall is required for the current profile control near the plasma boundary to enhance the magnetic shear there and then to decrease the dependence of RFPs on a close fitting conducting wall.

The ideal externally nonresonant resistive wall kink modes (RWM) stability in the plasma surrounded by an external shell of finite conductivity should be investigated to develop the steady state reactor concept since it determines

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the wall ratio. According to the linear theory on RWM [7], the stability windows increase with the velocity of toroidal plasma rotation (v_0) together with the viscosity and/or the ion sound wave damping effect for finite β plasma, depending on other equilibrium parameters so that the wall position displaces further from the plasma for the shallowly reversed RFPs and is closer to the plasma for higher beta obviously. A further nonlinear study is necessary. It is noteworthy that the viscosity increases with lowering aspect ratio and the increasing viscosity would contribute also to the stabilization of neoclassical resistive g mode driven by perturbed Pfirsch-Schluter (P-S) current in negative magnetic shear as in RFPs [8,9], as well as the stabilization of RWM.

Even in MHD stable, dynamo-free RFPs, the energy confinement time is determined by micro-instability. There exists a favorable stability for both micro- and macro-instabilities in low aspect ratio magnetic configuration with a wide region of good curvature which results in high pitch magnetic lines of force and enhances central q value [10]. Furthermore the equilibrium with a modest poloidal beta ($\beta_p \equiv (2\mu_0 \langle p \rangle / B_\theta^2(a) < 1$, $B_\theta(a)$: poloidal magnetic field at plasma boundary), which is realized in low aspect ratio RFPs and tokamaks, is essentially force-free, that is, a paramagnetism with the current density nearly parallel to the magnetic line of force [11]. Since the magnetic lines of force have a high pitch, a large poloidal current component is produced to enhance the toroidal magnetic field strength at the plasma axis and then to lead the near-omnigeneity [11,12]. In the omnigenous systems the bounce-averaged drift remains within a flux surface. The maximum of longitudinal adiabatic invariant (J) at the magnetic axis or $\nabla p \nabla J > 0$ is attained in low aspect ratio RFPs with strong paramagnetism, which improves the stability for the ion mode of trapped particle micro-instabilities, the toroidal ion temperature gradient (ITG) mode and probably the confinement degradation of high energy particle due to the non-closure magnetic field strength resulting from a small Shafranov shift in tokamak with negative magnetic shear [13]. The toroidicity-induced Alfven eigenmodes (TAEs) can be expected to be reduced for the RFPs with small safety factor and strong poloidal magnetic field at the plasma boundary.

3. Plasma self-induced current

The total toroidal current in an equilibrium is given by $I_p = -(2\pi) \int (\mathrm{d}p/\mathrm{d}\psi)(\langle B_\theta^2 \rangle / \langle B^2 \rangle)\mathrm{d}V + (1/2\pi) \int F(\langle \boldsymbol{j}\cdot\boldsymbol{B} \rangle / \langle B^2 \rangle) \langle 1/R^2 \rangle \mathrm{d}V$, where $\langle\ \rangle$ denotes the flux surface (ψ = const.) average, V the volume of the region enclosed by a flux surface and F the toroidal field function given by $F = RB_\varphi$. The first term in above equation represents contribution from perpendicular current, that is the diamagnetic and the Pfirsch-Schluter current, giving a substantial contribution to the equilibrium current for usual RFPs with B_θ (poloidal field) $> B_\varphi$ (toroidal field). Hereafter it is called "P-S current (I_{ps})", which is the diamagnetic current and the P-S current. The second term represents the contribution from parallel current necessary to maintain the equilibrium parallel momentum balance along the magnetic field. Since the bootstrap current is a parallel current, only the second term is kept for a total bootstrap current, giving $I_{bs} = 2\pi \int \mathrm{d}\psi q(\psi) \langle \boldsymbol{j}\cdot\boldsymbol{B} \rangle_{bs} / \langle B^2 \rangle$, where $q(\psi)$ is the safety factor and $\langle \boldsymbol{j}\cdot\boldsymbol{B} \rangle_{bs}$ the bootstrap parallel current density.

The bootstrap current obtained by the conventional theory described above has been shown to be modified by the finite banana-width effects around the magnetic axis satisfying the condition $\varepsilon^{1/2} < \delta_a^{1/3}$, where ε is the inverse aspect ratio and $\delta_a = 2q_0\rho_a/\kappa R_0$ with the central safety factor q_0, the Larmor radius ρ_a for species a, the elongation parameter κ and the major radius R_0 [14]. The modification is significant for high energy particle changing its trajectory from the banana orbit to the potato one. The expression obtained for the bootstrap current due to fusion-produced alpha particles by taking into account the finite banana-width effect is applied for the low aspect ratio RFPs. For the RFPs with a relatively small B_0, large ρ_a, small R_0 and then large δ_a, the ratio of the alpha particle-induced bootstrap current I_α to the bulk bootstrap current I_{bs} becomes relatively large depending on aspect ratio A and the profiles of pressure / temperature for a given beta value. Accordingly the plasma self-induced current is increased in the low aspect ratio RFP equilibrium.

The "P-S current" flows in the boundary region of plasma, then makes a good alignment with the equilibrium current profile in conjunction with the bootstrap current flowing in central and mid-radius region. The plasma self-induced current ratio F_p is defined as the ratio of total toroidal bootstrap current and "P-S current" to equilibrium plasma current I_φ^{eq}, which determines the power required (P_{CD}) for rf current drive (RFCD) to generate the steady state configuration. The RF power spectrum is selected in order to that RFCD should create the current profile of equilibrium. The RFCD equilibria calculation employs as many as three types of wave: low frequency fast wave (LFFW), high frequency fast wave (HFFW) and lower hybrid (LH) slow wave. A low frequency fast wave (LFFW) provides the seed current near the magnetic axis. The frequency of the LFFW is chosen to place the $\omega = 2\Omega_d$ (Ω_d: deuteron cyclotron frequency) resonance just outside the low field edge of the plasma; for $A = 2.0$, this places the tritium cyclotron harmonic $\omega = 3\Omega_t$ (Ω_t: tritium cyclotron frequency) well inboard of the axis, with the result that LFFW deposit most of its power on electrons in a single path through the plasma. An additional, more strongly damped wave is usually needed to provide RFCD at mid-minor radius. A high frequency fast wave (HFFW) at a high harmonic of the ion cyclotron ($\omega \sim 15\Omega_i$) is used for this role [15]. Finally, near the plasma surface the lower hybrid (LH) slow wave is ideal for the current drive. Its depth of penetration is, however, strictly limited by the usual density accessibility constraints. The total RF power P_{CD} is given by the normalized current driving efficiency which depends on the current driving system only, considering the contributions to the power spectrum from each wave with different current driving efficiency. Here is stipulated the electric-to RF power conversion efficiencies to be $\eta_{CD} = 0.62$ in view of the small variation in η_{CD} computed for advanced steady state tokamak power plants [16].

4. Economic analysis

The calculation of the cost of electricity (*COE*) at each design point is achieved by using the costing algorithms described in detail in ref. [17]. This economic model finds that the engineering power gain (Q_E; the ratio of plant gross electric power P_{ET} to circulation power P_C for sustaining reactor operation) and the mass power density (MPD; the ratio of net electric power (P_E) to the grid divided by the total mass of the fusion power core M_{FPC}) are the primary variables determining *COE*. The MPD is evaluated by the size scaling of M_{FPC}. We desire to achieve the compact power plants with high MPD as well as low circulation power (high Q_E). This goal is achieved with the low toroidal current equilibria and by optimizing the plasma self-induced current contribution, thereby minimizing the non-inductive seed current.

The design points in economic analysis for the classical and the neoclassical equilibrium are summarized in Table 1. For the RFP reactor, a factor H is defined by the ratio of energy confinement time for burning τ^E_{Re} against the empirical scaling results for the global energy confinement time τ^E_{ES}, depending on the inverse of Greenwald normalized density I_N ($\equiv I_\varphi^{eq} / \pi a^2 \langle n_e \rangle_{20}$) [18]. For the D-T-fueled plasma, $I_N \sim 2.7$, then $H \sim 2.6$ and $\tau^E_{ES} \sim 1$ sec. For the classical (PRSM) equilibrium, the alignment of bootstrap current profile with equilibrium one becomes poor although the stability beta limit increases, with flattening plasma pressure profile. The neoclassical equilibrium with a relatively flat pressure profile has a dominant (~ 100 %) plasma self-induced current, simultaneously the high stability beta of toroidal beta $\beta_t \sim 63$ % owing to hollow current profile making the magnetic shear stronger and the force-free region wide. The compatibility of the high stability beta and the good alignment of plasma self-induced current profile with equilibrium one gives an economical attractiveness of enhancing the plant gross electric power (P_{ET}) and reducing significantly the non-inductive current driving power required for the generation of steady state magnetic configuration (P_{CD}).

As the result, $P_E \sim 778.5$ MWe, $MPD \sim 310.5$ kWe/tonne, $P_{CD} \sim 11.0$ MW and $Q_E \sim 14.1$, then the lowest cost of electricity of $COE \sim 47$ mill/kWe·h for the D-T-fueled steady state fusion power plant based on the neoclassical RFP equilibrium with the low aspect ratio of $A = 2.0$ under the wall ratio of $r_w/a = 1.35$ as predicted by the linear theory on RWM while $r_w/a = 1.55$ for the perfect conducting shell. Generally the smaller the wall ratio is, the higher the *MPD* is, and the *COE* is the lower but not so sensitive to the wall ratio for a given Q_E. However the surface average neutron wall loading achieves $W_n \sim 7.55$ MW/m^2. For the reduction of W_n, the D-^{3}He-fueled reactor design operating at a higher temperature (volume average electron temperature of $T_e = 50$ keV) and a stronger toroidal field ($B_0 = 6$ T) for a given beta value might be desired. In this case, $P_E \sim 385$ MWe, $MPD \sim 381$ kWe/tonne, $P_{CD} \sim 3.5$ MW and $Q_E \sim 15.7$, then $COE \sim 60$ mill/kWe·h with the lunar-^{3}He fusion supply contributing ~ 20 % to the *COE*. The net electric power decreases by a factor of ~ 2.0 without changing significantly Q_E and *COE*, but the neutron wall loading is reduced to $W_n = 0.18$ MW/m^2. For the D-T-fueled fusion power plant based on the classical (PRSM) equilibrium, $P_E \sim 1215$ MWe, $MPD \sim 339$ kWe/tonne, $P_{CD} \sim 421$ MW and $Q_E \sim 3.0$, then $COE \sim 170$ mill/kWe·h and $W_n \sim 9.0$ MW/m^2. Comparing with the D-T-fueled fusion power plant based on the neoclassical equilibrium, the net electric power and the neutron wall loading become large by a factor of ~ 1.6 with a smaller Q_E and a higher *COE*.

5. Conclusion

The plasma self-induced current in the neoclassical RFP equilibrium with low aspect ratio has a hollow current profile, which enhances the stability beta making the magnetic shear increase locally and globally and the force-free region wide, and a good alignment with equilibrium current profile, which reduces the requisite power of non-inductive current drive to generate the steady state magnetic configuration. The compatibility of high stability beta and small amount of RF current driving power allows the economical design of compact fusion power plant with the lowest cost of electricity. The D-^{3}He-fueled reactor designed has a net electric power

Table 1 Design points for RFP equilibria in the economic analysis; A is the aspect ratio, κ the cross sectional ellipticity, β_t the toroidal beta, F the field reversal parameter, R_0 the major radius, B_0 the magnetic field strength, n_{e0} and T_{e0} the electron density and temperature on magnetic axis, I_φ^{eq} the total toroidal current in equilibrium, over bar denotes volume average, n_{e20} the electron density in unit of 10^{20} /m^3, F_p the plasma self-induced toroidal current fraction to I_φ^{eq}, Z the effective charge number. CL and NEC denote operating modes for the classical and the neoclassical equilibrium, respectively; (1) D-T-fueled fusion, (2) D-^{3}He-fueled fusion.

	A	κ	β_t [%]	β_p [%]	q_0	F	R_0 [m]	B_0 [T]	I_φ^{eq} [MA]	$n_{e0}/\langle n_e\rangle$	$T_{e0}/\langle T_e\rangle$	$\langle n_e\rangle_{20}$ [m^{-3}]	$\langle T_e\rangle$ [keV]	F$_p$	Z
CL	2	2.0	59	72	0.68	-0.21	2.8	3.0	24.2	1.04	1.63	2.42	22.9	1.29	2.0
NEC(1)	2	1.4	63	45	~1.0	-0.10	2.8	3.0	30.8	1.10	1.26	1.88	29.6	0.96	2.0
NEC(2)	2	1.4	63	178	~1.0	-0.10	2.8	6.0	30.8	1.10	1.26	4.40	50.0	0.96	2.0

less by a factor of ~ 2.0 without changing significantly an economical attractiveness but with reducing the neutron wall loading, comparing with D-T-fueled reactor designed. The problem remained to be solved is to optimize the design point parameters meeting to the stability window of wall ratio on RWM and the engineering technology of reactor.

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