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About the Possibility of Impurity Ion Accumulation in the Island Region in Helical Plasma

SHISHKIN Alexander

Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", Academicheskaya str.1, Kharkov-108, 61108, Ukraine and Kharkov "V.N.Karazin" National University, Svobody sqr. 4, Kharkov-77, 61077, Ukraine

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Abstract

The possibility for ions to be accumulated in the island region of the toroidal plasma is shown here on the basis of magnetohydrodynamics (MHD) equations. There are analyzed the mass ion flow trajectories, which are the solution of the equation $\mathrm{d}\boldsymbol{r} \times \boldsymbol{u}_\alpha = 0$, where $\boldsymbol{u}_\alpha$ is the velocity of the plasma species α. There are used analytical expressions for two MHD variables: the velocity $\boldsymbol{u}_\alpha$ and the thermal flux $\boldsymbol{q}_\alpha$ of the same species as the functions of the plasma pressure p_α, plasma temperature T_α and the electric field in plasma $\boldsymbol{E}$. The island structure is described with the analytical expression for the function Ψ, which gives us the cross-sections of the magnetic surfaces in the vertical plane.

Keywords:

impurity ions, magnetic islands, accumulation of ions, MHD transport

1. Motivation of study

One of the methods, which can be used for the protection plasma from the penetration of impurity ions, is the producing the magnetic island structure with the use of the externally applied magnetic field perturbations. The magnetic island diverters on heliotron Large Helical Device and stellarator Wendelstein −7 AS are examples of such possibility. Can the islands effect on the plasma and impurity ion flows and how can we observe these phenomena?

In order to answer these questions there are solved the equations for the mass flow trajectories, namely $\mathrm{d}\boldsymbol{r} \times \boldsymbol{u}_\alpha = 0$, where $\boldsymbol{u}_\alpha$ is the velocity of the plasma species α. There are used analytical expressions for the velocity $\boldsymbol{u}_\alpha$ of the plasma species α and the thermal flux $\boldsymbol{q}_\alpha$ of the same specie as the functions of the plasma pressure p_α, plasma temperature T_α and the electric filed field $\boldsymbol{E}$ in plasma. These plasma characteristics entered the MHD equations through the products ($\boldsymbol{B} \times \nabla p_\alpha$), ($\boldsymbol{E} \times \boldsymbol{B}$), ($\boldsymbol{B} \times \nabla T_\alpha$). The other new issue here is the introduction of the magnetic surface function Ψ in order to describe the magnetic island structure. Another very important question considered here is the possibility to change the island geometry, particularly, to excite the second chain of islands (adjacent resonance) in addition to the first chain of islands in order to control the impurity ions more effectively.

2. Main equations

2.1 Starting equations

For our analysis the starting equations are chosen in the following form

$$-\nabla p_\alpha + e_\alpha n_\alpha \boldsymbol{E} + e_\alpha n_\alpha (\boldsymbol{u}_\alpha \times \boldsymbol{B}) - \boldsymbol{F}_{\alpha 1} = 0 \qquad (1)$$

$$-\frac{5}{2} n_\alpha \nabla T_\alpha + e_\alpha n_\alpha \frac{1}{p_\alpha} (\boldsymbol{q}_\alpha \times \boldsymbol{B}) - \boldsymbol{F}_{\alpha 2} = 0 \qquad (2)$$

where

$$\boldsymbol{F}_{\alpha 1} = \sum_\beta l_{11}^{\alpha\beta} \boldsymbol{u}_\beta - \frac{2}{5} l_{12}^{\alpha\beta} \frac{\boldsymbol{q}_\beta}{p_\beta},$$

$$\boldsymbol{F}_{\alpha 2} = \sum_\beta l_{21}^{\alpha\beta} \boldsymbol{u}_\beta - \frac{2}{5} l_{22}^{\alpha\beta} \frac{\boldsymbol{q}_\beta}{p_\beta}. \qquad (3)$$

Here $l_{ij}^{\alpha\beta}$ are the matrix elements, which depend on the plasma parameters [1,2]. The friction forces are taken into account in eqs. (1)–(3). From these equations the linear analysis can be carried out. The variables $\boldsymbol{u}_\alpha$ and $\boldsymbol{q}_\alpha$ can be expressed through the main magnetic field $\boldsymbol{B}$ and profiles of the density n_α, the temperature T_α and the electric field $\boldsymbol{E}$. The effects of inertia and viscosity should be studied separately.

2.2 Analytical expressions for $\boldsymbol{u}_\alpha$ and $\boldsymbol{q}_\alpha$

From the eqs. (1)–(3) the analytical expressions for variables $\boldsymbol{u}_\alpha$ and $\boldsymbol{q}_\alpha$ can be obtained and they have the following form:

$$\boldsymbol{u}_\alpha = \frac{1}{D_{u_\alpha}} \left\{ \frac{5}{2} n_\alpha (\boldsymbol{B} \times \nabla T_\alpha) + \frac{B_{q_\alpha}}{A_{q_\alpha}} [(\boldsymbol{B} \times \nabla p_\alpha) - e_\alpha n_\alpha (\boldsymbol{B} \times \boldsymbol{E})] \right.$$

Corresponding author's e-mail: shishkin@vlink.kharkov.ua

$$+e_\alpha n_\alpha \boldsymbol{B}\Big[L_1(\boldsymbol{B}\nabla p_\alpha)+L_2 e_\alpha n_\alpha(\boldsymbol{BE})+L_3 n_\alpha(\boldsymbol{B}\nabla T_\alpha)$$

$$+\sum_{\beta\neq\alpha}\Big(L_{4\beta}(\boldsymbol{Bu}_\beta)+L_{5\beta}(\boldsymbol{B}\frac{\boldsymbol{q}_\beta}{p_\beta})\Big)\Big]$$

$$+L_6(\nabla p_\alpha-e_\alpha n_\alpha \boldsymbol{E})+L_7\frac{5}{2}n_\alpha\nabla T_\alpha$$

$$+\sum_{\beta\neq\alpha}\Big((L_6 l_{11}^{\alpha\beta}-L_7 l_{21}^{\alpha\beta})\boldsymbol{u}_\beta+(-\frac{2}{5}L_6 l_{12}^{\alpha\beta}+\frac{2}{5}L_7 l_{22}^{\alpha\beta})\frac{\boldsymbol{q}_\beta}{p_\beta}$$

$$+L_{8\beta}(\boldsymbol{B}\times\boldsymbol{u}_\beta)+L_{9\beta}(\boldsymbol{B}\times\frac{\boldsymbol{q}_\beta}{p_\beta})\Big)\Big\}, \quad (4)$$

$$\frac{\boldsymbol{q}_\alpha}{p_\alpha}=\frac{1}{D_{q_\alpha}}\Big\{(\boldsymbol{B}\times\nabla p_\alpha)-e_\alpha n_\alpha(\boldsymbol{B}\times\boldsymbol{E})-\frac{A_{u_\alpha}}{B_{u_\alpha}}\frac{5}{2}n_\alpha(\boldsymbol{B}\times\nabla T_\alpha)$$

$$+e_\alpha n_\alpha \boldsymbol{B}\Big[J_1(\boldsymbol{B}\nabla p_\alpha)+J_2 e_\alpha n_\alpha(\boldsymbol{BE})+J_3 n_\alpha(\boldsymbol{B}\nabla T_\alpha)$$

$$+\sum_{\beta\neq\alpha}\Big(J_{4\beta}(\boldsymbol{Bu}_\beta)+J_{5\beta}(\boldsymbol{B}\frac{\boldsymbol{q}_\beta}{p_\beta})\Big)\Big]$$

$$+J_6(\nabla p_\alpha-e_\alpha n_\alpha \boldsymbol{E})+J_7\frac{5}{2}n_\alpha\nabla T_\alpha$$

$$+\sum_{\beta\neq\alpha}\Big((J_6 l_{11}^{\alpha\beta}-J_7 l_{21}^{\alpha\beta})\boldsymbol{u}_\beta+(-\frac{2}{5}J_6 l_{12}^{\alpha\beta}+\frac{2}{5}J_7 l_{22}^{\alpha\beta})\frac{\boldsymbol{q}_\beta}{p_\beta}$$

$$+J_{8\beta}(\boldsymbol{B}\times\boldsymbol{u}_\beta)+J_{9\beta}(\boldsymbol{B}\times\frac{\boldsymbol{q}_\beta}{p_\beta})\Big)\Big\}. \quad (5)$$

These are the general expressions, which can be useful for the linear analysis, particularly to compare the MHD velocity of the background ions $\boldsymbol{u}_i$ and the MHD velocity of the impurity ions $\boldsymbol{u}_I$. In order to carry out such analysis it is necessary to induce the expression for $\boldsymbol{q}_\alpha$ into the expressions for $\boldsymbol{u}_i$ and $\boldsymbol{u}_I$. The matrix elements mentioned above are expressed through the coefficients [1], which are obtained as moments of the collision operators from the kinetic description [2].

2.3 Physics in the expression for $\boldsymbol{u}_\alpha$

From the expressions (4) and (5), after the substitution the expression for $\boldsymbol{u}_\beta$ and $\boldsymbol{q}_\beta$, the following expression can be obtained

$$\boldsymbol{u}_\alpha=\frac{1}{D_{u_\alpha}}\Big\{(\nabla p_\alpha-e_\alpha n_\alpha\boldsymbol{E})L_{6\alpha}+\frac{5}{2}n_\alpha\nabla T_\alpha L_{7\alpha}$$

$$+(\nabla p_\beta-e_\beta n_\beta\boldsymbol{E})L_{6\beta}^*+\frac{5}{2}n_\beta\nabla T_\beta L_{7\beta}^*$$

$$+\frac{5}{2}n_\alpha(\boldsymbol{B}\times\nabla T_\alpha)+\frac{B_{q_\alpha}}{A_{q_\alpha}}[(\boldsymbol{B}\times\nabla p_\alpha)-e_\alpha n_\alpha(\boldsymbol{B}\times\boldsymbol{E})]$$

$$+N_{1\beta}n_\beta(\boldsymbol{B}\times\nabla T_\beta)+N_{2\beta}(\boldsymbol{B}\times\nabla p_\beta)+N_{3\beta}(\boldsymbol{B}\times\boldsymbol{E})\Big\} \quad (6)$$

Coefficients $L_{6\beta}^*$, $L_{7\beta}^*$, $L_{1\beta}^*$,…are finally expressed through $l_{ij}^{\alpha\beta}$. In (6) we remain three well-known physics mechanisms, which effect on the plasma flow: combination of pressure gradients ($\propto\nabla P_\alpha$ and $\propto\nabla P_\beta$), which makes the impurity ions penetrate into the plasma core, the electric field $\boldsymbol{E}$ in plasma and the temperature gradient, which can screen plasma from the impurity ions. The terms, which are proportional to $\boldsymbol{B}\nabla p$, $\boldsymbol{BE}$, $\boldsymbol{B}\nabla T$, $\boldsymbol{Bu}_\beta$, $\boldsymbol{Bq}_\beta$ are important in the transport along the magnetic field lines but in the study below we take the impact of vector products $(\boldsymbol{B}\times\nabla p_\alpha)$, $(\boldsymbol{E}\times\boldsymbol{B})$, $(\boldsymbol{B}\times\nabla T_\alpha)$ into account. The contributions from the terms mentioned above will be studied in future paper.

2.4 Magnetic field model

In our case the magnetic field is modeled in the following simplest way

$$\boldsymbol{B}^0=B_0\left\{0,\frac{r}{R}\iota(\bar{r}^2),\frac{1}{1-\frac{r}{R}\cos\vartheta}\right\}, \quad (7)$$

the magnetic surface has the form $\Psi^0=\frac{\bar{r}^2}{2}$, where $\bar{r}=\frac{r}{a}$ (Fig. 1). The magnetic perturbation field is chosen in the form

$$\boldsymbol{B}_{m,n}^1=B_0\{b_{m,n}\bar{r}^{m-1}\sin(m\vartheta-n\varphi),$$
$$b_{m,n}\bar{r}^{m-1}\cos(m\vartheta-n\varphi),\ 0\}. \quad (8)$$

Then the modified function Ψ in the case of one resonance perturbation

$$\Psi=\int[m\iota(\bar{r}^2)-n]\bar{r}\mathrm{d}\bar{r}+b_{m,n}\frac{R}{a}\bar{r}^m\cos(m\vartheta-n\varphi). \quad (9)$$

The shape of the magnetic surface with one chain of island described with (9) is shown in Fig. 2.

2.5 The description of the island structure

The function, which can describe the magnetic surfaces

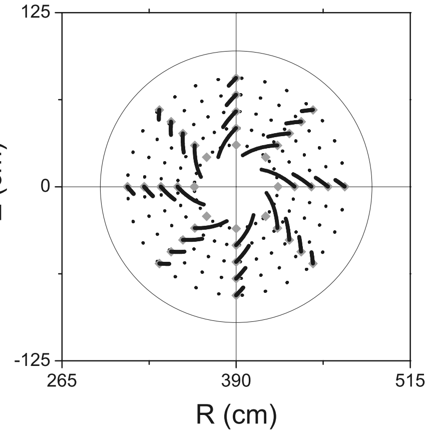

Fig. 1 Flow trajectories in the magnetic configuration without islands.

with islands with the "wave" numbers: m,n and m',n', has the following form:

$$\Psi_{m,n,m',n'} = \frac{1}{r_{m,n}^6}\int (r^2 - r_{m,n}^2)(r^2 - r_{m',n'}^2) r\mathrm{d}r$$
$$+ \frac{1}{2}\left(\frac{\Delta r_{m,n}}{r_{m,n}}\right)^2 \left(\frac{r}{r_{m,n}}\right)^m \frac{r^2 - r_{m',n'}^2}{r_{m,n}^2}\cos(m\vartheta - n\varphi + \delta_{m,n})$$
$$+ \frac{1}{2}\left(\frac{\Delta r_{m',n'}}{r_{m',n'}}\right)^2 \left(\frac{r}{r_{m',n'}}\right)^{m'} \left(\frac{r_{m',n'}}{r_{m,n}}\right)^4 \frac{r^2 - r_{m,n}^2}{r_{m,n}^2}$$
$$\times \cos(m'\vartheta - n'\varphi + \delta_{m',n'}) \qquad (10)$$

Here $r_{m,n}$ and $r_{m',n'}$ are the radii of the magnetic surfaces with the rotational transform values $\iota(r_{m,n}) = n/m$ and $\iota(r_{m',n'}) = m'/n'$; $\Delta r_{m,n}$ and $\Delta r_{m',n'}$ are the half widths of the magnetic islands. The half width is $\Delta \bar{r}_{m,n} = \sqrt{\dfrac{2b_{m,n}\frac{R}{a}(\bar{r}_{m,n})^m}{m\frac{\mathrm{d}\iota}{\mathrm{d}(\bar{r}^2)}(\bar{r}_{m,n}^2)}}$. The parameter $\frac{r_{m',n'}}{r_{m,n}}$ is the ratio of the rational magnetic surface radii. The shape of the cross-sections of the magnetic surfaces with two chains of islands are shown on Fig. 3.

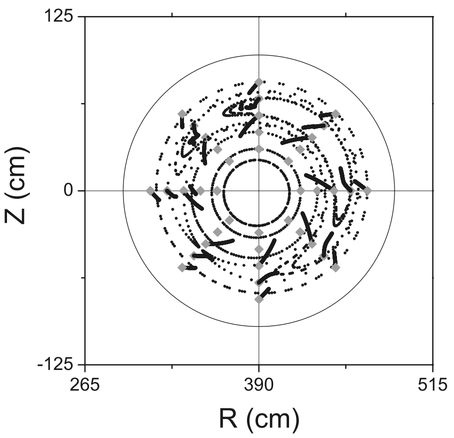

Fig. 2 Flow trajectories in the magnetic configuration with $m/n = 1/1$ island.

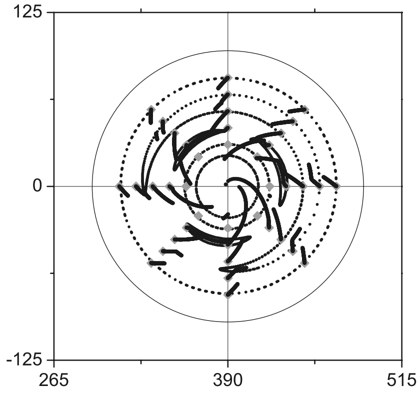

Fig. 3 Flow trajectories in configuration with magnetic island chains: $m/n = 1/1$ and $m/n = 2/3$.

2.6 Flow trajectory equations

The mass flow trajectories can be obtained as the solution of the equations

$$\mathrm{d}\boldsymbol{r} \times \boldsymbol{u}_\alpha = 0. \qquad (11)$$

Plasma parameters are the functions of the magnetic surface function, *i.e.* $p = p(\Psi)$, $T = T(\Psi)$, $\Phi = \Phi(\Psi)$, where Φ is the electric field potential ($\boldsymbol{E} = -\nabla\Phi$), and Ψ is the magnetic surface function. Functions $p_\alpha(\Psi)$, $T_\alpha(\Psi)$, $\Phi(\Psi)$ enter in $u_{\nabla\Psi}$ and $u_{\boldsymbol{B}\times\nabla\Psi}$, those give $\boldsymbol{u}_\alpha$ in the form:

$$(\boldsymbol{u}_\alpha)_r = u_{\nabla\Psi}\frac{\partial\Psi}{\partial r} + u_{\boldsymbol{B}\times\nabla\Psi}\left(B_\vartheta \frac{1}{R}\frac{\partial\Psi}{\partial\varphi} - B_\varphi \frac{1}{r}\frac{\partial\Psi}{\partial\vartheta}\right)$$
$$(\boldsymbol{u}_\alpha)_\vartheta = u_{\nabla\Psi}\frac{1}{r}\frac{\partial\Psi}{\partial\vartheta} + u_{\boldsymbol{B}\times\nabla\Psi}\left(B_\varphi \frac{\partial\Psi}{\partial r} - B_r \frac{1}{R}\frac{\partial\Psi}{\partial\varphi}\right)$$
$$(\boldsymbol{u}_\alpha)_\varphi = u_{\nabla\Psi}\frac{1}{R}\frac{\partial\Psi}{\partial\varphi} + u_{\boldsymbol{B}\times\nabla\Psi}\left(B_r \frac{1}{r}\frac{\partial\Psi}{\partial\varphi} - B_\vartheta \frac{\partial\Psi}{\partial r}\right) \qquad (12)$$

Here the parabolic functions of the equilibrium quantities $p = p(\Psi)$, $T = T(\Psi)$, $\Phi = \Phi(\Psi)$ are determined. In that case the magnitudes $u_{\nabla\Psi}$ and $u_{\boldsymbol{B}\times\nabla\Psi}$ are independent on Ψ to underline the effect of the island structure. The more general cases and the effect of the sign of the electric field on the transport of ions through the island region will be considered further.

3. Physics results

One can see the results of the solution of the eq. (11) on the Figs. 1-3. The flow trajectories are shown for the configuration without islands (Fig. 1), in the configuration with one m/n = 1/1 island (Fig. 2) and in the configuration with two chains of magnetic islands: m/n = 1/1 and m/n = 3/2 (Fig. 3). The principal thing is the following: the flow trajectories change in the presence of islands. Flow trajectories concentrate in the region with the islands. It means that the islands can be the transport barriers on the way of impurity ions into the plasma core. However in the vicinity of X-points the flows are directed in the core of plasma (Fig. 3).

4. Discussion

Some words about the restriction of the theory and calculation results. The equations for $\boldsymbol{u}_\alpha$ and $\boldsymbol{q}_\alpha$ are obtained in the case of the axisymmetric main magnetic field $\boldsymbol{B}^0$ and magnetic field perturbations $\boldsymbol{B}^1_{m,n}$ and parabolic profiles of the density n_α, the temperature T_α and the electric field $\boldsymbol{E}$. The effects of inertia and viscosity should be studied separately.

The considered here configuration is similar to the helical field configuration in the magnetic coordinates. The helical field can change the picture: the shape of the magnetic cross-section from circular to elliptical, for example. Nevertheless the principal features in the behavior of ions, when the islands are present, should be the same.

In the experiments on Large Helical Device it is observed the accumulation of the test impurity in the region, where the m/n = 1/1 island is produced [3]. The possible physics explanation of this effect on the base of MHD approach is proposed in this paper.

5. Conclusions

The theory of the accumulation of impurity ions in the island region of the helical plasmas is given on the basis of the MHD approach. The mass flow trajectories as the solution of the equation $d\boldsymbol{r} \times \boldsymbol{u}_\alpha$ are obtained for the configurations *with* and *without* islands. One can see the strong effect of the islands on the mass flows.

Magnetic islands can be the transport barriers on the way of the impurity ions into the plasma core.

The change of the island geometry: the excitation of the additional islands (adjacent resonance) can help to control the impurity flows in the helical plasmas.

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