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Bootstrap Current Coefficients in Stellarators

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Abstract

A method to derive analytical expressions for bootstrap current coefficients is studied. The drift kinetic equation is divided into two parts corresponding to the effects of local and global structures of magnetic fields. The divided transport coefficients can be approximated by connecting the results of only three types of asymptotic expansions of the divided equations. The current coefficients obtained by adding these two parts approximately reproduce results of a direct numerical calculation of the drift kinetic equation.

Keywords:

neoclassical transport, bootstrap current, neoclassical viscosity, plasma flow, drift kinetic equation, non-symmetric toroidal configuration

1. Introduction

The analytical theory of bootstrap currents in non-symmetric toroidal plasmas [1-5] has been constructed based on the moment equation approach [6]. In this approach, parallel (to the magnetic field) momentum balance equations satisfying particle, momentum, and energy conservation are used together with neoclassical parallel viscosities given by solutions of the linearized drift kinetic equation (DKE). The viscosity forces include two parts; damping force against the parallel flows and driving force for the flows due to radial gradient forces. In stellarators, both of contributions due to ripple and banana trapped orbits are included in the driving force. These trapped orbits generate the driving forces in opposite directions to each other while their contributions to the damping force have same direction. Here we discuss a method to express analytically this driving force part of the viscosity under the co-existence of two types driving mechanisms. The treatments of the damping force part in general non-symmetric toroidal plasmas [7] are substantially identical to those in symmetric plasmas such as tokamaks and thus details of them are out of scope of this paper. However, we should note that, even in the symmetric plasmas, the linearized DKE including the complete parts of the Vlasov and collision operators with the conservative properties could not be solved analytically. Therefore, for the damping and driving forces in tokamaks [6], a connection formula was used to smoothly connect results of three types of asymptotic expansions of the DKE; (1) banana regime expansion giving the viscosity of $\propto \nu/v$, (2) plateau regime expansion giving that of $\propto(\nu/v)^0$, and (3) Pfirsch-Schlueter regime expansion giving that of $\propto(\nu/v)^{-1}$, where v/ν is the collision mean free path. The collision frequency regime boundary determined by this connecting method corresponds to the bounce frequency of the banana-trapped orbits [6]. Exactly speaking, the parallel viscosity due to the existence of banana trapped orbits includes also a direct contribution of circulating particles in the banana regime [1-6]. For general collisionality regimes in stellarators, we have to express the co-existence of two types of the forces with opposite directions and with different bounce frequencies of the corresponding trapped orbits by unifying these previously established methods.

On the other hand, various numerical methods to directly solve the DKE without any asymptotic approximations have been developed to study ripple diffusions in non-symmetric toroidal plasmas. However, these codes often employed the pitch-angle-scattering collision operator instead of momentum conserving operators, since the effect of parallel momentum transferring parts in the collision term was negligible in the calculation of the ripple diffusions [7]. The relation of the numerical solutions given by these codes with the analytical theories for the flows based on the moment equations had not been clarified [3,8,9]. Motivated by recent design activities for advanced stellarators, a method was developed recently to interpret the direct solution of the linearized DKE with the pitch-angle-scattering collision operator within the framework of the moment method [7]. In this method, the formal expression of the viscosities based on the numerical solution directly coincides with that in the previous analytical theories. The results given by this method clarified that the driving forces do not change their dependence on the collisionality ($\propto \nu/v$, $\propto(\nu/v)^0$, $\propto(\nu/v)^{-1}$) and

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their polarity simultaneously. This driving force cannot be expressed analytically by appropriate connections of only three types of asymptotic results as the previous tokamak theory used. However, this numerical result also suggests that if the total driving force can be divided into appropriate two parts with different polarity and collision frequency dependence, the divided parts can be approximated by combining previously established analytical methods. This dividing qualitatively corresponds to the separation of two types driving mechanisms discussed above. Because of difference of the pitch-angle variables (banana : the adiabatic invariant of the drift orbits μ, plateau and Pfirsch-Schlueter : the pitch-angle parameter $\xi \equiv \upsilon_{\|}/\upsilon$ [10]) used in analytical calculations, we have to divide here the linearized DKE itself having a non-linearity with respect to the magnetic field strength modulation δB on the magnetic flux surface. This separation of the DKE seems to be uniquely determined by one requirement from the banana regime calculation in which the treatments of trapped and circulating particle distributions must be separated. Here we extend a dividing method for the DKE developed in a previous banana regime theory [2-4] to the collisional regimes. It is shown that the resulting divided equation set not only includes the previous banana regime theory, but also is a practically useful separation of the two types driving mechanisms in general collisionality regimes. Comparisons with the numerical results to confirm a validity of a derived formula are also shown.

2. Separation of the parallel viscosity force

In the banana regime theory in non-symmetric toroidal plasmas [2-4], a specific method was used to obtain only the parallel viscosity force directly without solving full part of the equation. The distribution function for the circulating particle can be calculated analytically using a Fourier expansion even in the banana regime where μ is used as the pitch-angle variable. On the other hand, it was also known that the contribution of the trapped particles to the driving force can be determined by incompressible particle and heat flow patterns, which are strongly affected by the local structure of the magnetic field such as the helical ripple in stellarators. Using these characteristics of distribution functions of circulating and trapped particles, Shaing, *et al.* [2-4] subtracted a part expressing the effect of this local structure from the DKE, and then calculated the circulating particle distribution based on the remaining part of the equation. Since this separation was done on the lowest order equation in the banana regime expansion, the divided equations did not include Coulomb collision effects. Here we extend this theory to collisional regimes by adding a linearized collision operator as following.

To discuss about general collisionality regimes, it is convenient to consider at first the linearized DKE defined in ref. [7]. The solution for the particle species a is determined by the perturbed distribution functions G_{Xa} and G_{Ua}. The former function G_{Xa}, which is driven by the radial gradient force, satisfies the equation,

$$V_{\|} G_{Xa} - C_a^{\ L}(G_{Xa}) = \sigma_{Xa} . \tag{1}$$

Here, $V_{\|} \equiv \upsilon\xi(\boldsymbol{B}/B)\cdot\nabla + V_M$ is the linearized Vlasov operator, where $V_M \equiv -(\upsilon/2)[(\boldsymbol{B}\cdot\nabla \ln B)/B](1-\xi^2)\partial/\partial\xi$ is the mirror force operator. The linearized Coulomb collision operator $C_a^{\ L}$ includes only the pitch-angle-scattering operator [7] and the energy scattering Krook collision operator for $l = 2$ spherical harmonic perturbation [6]. The particle velocity υ and the pitch-angle parameter ξ are used as the velocity space variables here. In discussions on the plateau and the Pfirsch-Schlueter asymptotic calculations, we show here the obtained equations and numerical examples for the Boozer coordinates. The magnetic field strength is assumed to be given by the Fourier expression $B = \Sigma B_{mn}^{(Boozer)}\cos(m\theta_B - n\zeta_B)$, in the flux coordinates (s,θ_B,ζ_B), where θ_B and ζ_B are the poloidal and toroidal angles, respectively, and s is an arbitrary label of a flux surface. By applying the method in refs. [2-4] to subtract the part corresponding to the local structure effect, the source term σ_{Xa} corresponding to the radial gradient forces can be written as $\sigma_{Xa} = \sigma_{Xa}^{(sym)} + \sigma_{Xa}^{(asym)}$ by using

$$\sigma_{Xa}^{(sym)} \equiv -\sigma_{Ua}\frac{c}{2e_a\chi'\psi'}\left[\frac{\psi' B_\zeta - \chi' B_\theta}{\langle B^2\rangle} + \frac{V'}{4\pi^2}H_2\right], \tag{2}$$

$$\sigma_{Xa}^{(asym)} \equiv \frac{m_a c}{2e_a\chi'\psi'}\frac{B}{\langle B^2\rangle}\upsilon^2 P_2(\xi)\left[\chi'(1-H_2)\frac{\partial B}{\partial\theta_B} - \psi'(1+H_2)\frac{\partial B}{\partial\zeta_B}\right] + \frac{m_a c}{e_a}\frac{B}{\langle B^2\rangle}\upsilon^2 P_2(\xi)\left[\frac{\partial G}{\partial\zeta_B}\frac{\partial B}{\partial\theta_B} - \frac{\partial G}{\partial\theta_B}\frac{\partial B}{\partial\zeta_B}\right], \tag{3}$$

Here, H_2 is a constant indicating the flux surface averaged effect of the local structure of the magnetic field on the incompressible flow. This constant is defined in refs. [2-3] using the Hamada coordinates. Definitions of the other quantities in eqs. (2)-(3) are those in ref. [7] so that χ' and ψ' are the radial derivatives of the poloidal and toroidal magnetic fluxes, respectively, and V' is the radial derivative of the volume enclosed by the flux surface, where the derivative is denoted by $' = \mathrm{d}/\mathrm{d}s$. In eq. (2), B_θ and B_ζ are the covariant poloidal and toroidal components of the magnetic field, respectively, and $P_2(\xi)$ in eq. (3) is the Legendre polynominal of order $l = 2$. The charge and mass of the particle species a are denoted by e_a and m_a, respectively, and $\langle\cdot\rangle$ represents the flux surface average. The source term in eq. (2) $\sigma_{Ua} \equiv -m_a V_{\|}(\upsilon\xi B) = -m_a\upsilon^2 P_2(\xi)\boldsymbol{B}\cdot\nabla\ln B$ is that for the other DKE, $V_{\|}G_{Ua} - C_a^{\ L}(G_{Ua}) = \sigma_{Ua}$ defined in ref. [7] to derive the flow-driven part of the perturbation. In eq. (3), G is the generating function related to the Hamada/Boozer coordinates conversion [7] and can be calculated using the Fourier expansion of B^{-2}. The equation for G_{Ua} is identical to that used to derive the damping force part in the previous theories for symmetric and non-symmetric toroidal plasmas [1-6]. The analytical expressions for this damping force part in non-symmetric plasmas are shown in refs. [5,7]. It should be noted that explicit expression for G_{Ua} itself is not always required in obtaining the asymptotic values of the mono-energetic parallel viscosity coefficients, especially when the $\langle B\int v_{\|}\mathrm{d}^3\boldsymbol{v}\rangle$ moments of the DKE is used to obtain the pitch-angle integrated and flux surface averaged viscosity forces [1-6].

As shown in eqs. (24)-(25) in ref. [2] and Appendix D in ref. [7], $\sigma_{Xa}{}^{(asym)}$ vanishes when the magnetic field strength has a complete symmetry such as an axisymmetry, or a helical symmetry, or poloidal symmetry. Only $\sigma_{Xa}{}^{(sym)}$ remains in these completely symmetric cases. The superscripts (asym) and (sym) denote that the part exists only in non-symmetric configurations, and the part exists even in the symmetric configurations, respectively. The solution of eq. (1) is given by the sum of two components, $G_{Xa} = G_{Xa}{}^{(sym)} + G_{Xa}{}^{(asym)}$, where $G_{Xa}{}^{(sym)}$ and $G_{Xa}{}^{(asym)}$ are the solutions of $V_{\|}G_{Xa}{}^{(sym)} - C_a{}^L[G_{Xa}{}^{(sym)}] = \sigma_{Xa}{}^{(sym)}$ and $V_{\|}G_{Xa}{}^{(asym)} - C_a{}^L[G_{Xa}{}^{(asym)}] = \sigma_{Xa}{}^{(asym)}$, respectively. The former part $G_{Xa}{}^{(sym)}$ is proportional to G_{Ua} and only its absolute value is determined by the incompressible particle and heat flows under the asymmetry. Therefore the already known results on G_{Ua} are applicable to this former part. The original purpose of this part in refs. [2-4] was the trapped particles. When we consider the case where the collision operator vanishes, the DKE in ref. [7] using eqs. (2)-(3) coincides with the lowest order equation in the banana regime expansion in refs. [2-4] and thus eqs. (1)-(3) automatically include this previous banana regime theory. In this theory, $V_{\|}G_{Xa}{}^{(sym)} = \sigma_{Xa}{}^{(sym)} \propto \sigma_{Ua}$ was used to derive the contribution of the trapped particles. The remaining part $V_{\|}G_{Xa}{}^{(asym)} = \sigma_{Xa}{}^{(asym)}$ was used to derive the remaining effect of the circulating particles without solving the trapped particle part. This contribution of the circulating particles due to the asymmetry appears in the relatively collisionless limit where the banana-trapped part of the distribution can be treated by the banana regime expansion. With this separation for treatments of the circulating and trapped particles, the total driving force due to all of the particles was easily obtained without the complete solution of the equations in the banana regime theory [2-4]. The moment of the DKE itself is used also in this step to derive viscosity force without solving full part of the DKE. Also for general collisionality regimes, this separation shown in eqs. (2)-(3) is a practically useful separation of the effects of local and global structures of the magnetic fields which make the ripple and banana trapped orbits, respectively. In many stellarators with helically twisted structures giving large values for $(\psi'/\chi')\partial B/\partial\zeta_B$, the toroidicity component B_{10} is the dominant Fourier component of $\sigma_{Xa}{}^{(asym)}$ as shown in ref. [2] while the dominant Fourier component of $\sigma_{Xa}{}^{(sym)} \propto \boldsymbol{B}\cdot\nabla B$ is the ripple component. In plateau and Pfirsch-Schlueter expansions omitting the mirror force operator [10], these Fourier compositions of the source terms $\sigma_{Xa}{}^{(asym)}$ and $\sigma_{Xa}{}^{(sym)}$ directly determine those of the response functions $G_{Xa}{}^{(asym)}$ and $G_{Xa}{}^{(sym)}$. The $n = 0$ component dominating $G_{Xa}{}^{(asym)}$ expresses the banana-trapped effects while the $n \neq 0$ component in $G_{Xa}{}^{(sym)}$ expresses the ripple-trapped effects. Therefore eqs. (2)-(3) is a practically useful separation of the local and global structure effects especially when the divided driving forces have different polarity and collision frequency dependence. As discussed later on eqs. (4)-(8), this tendency of the separation, in which the $G_{Xa}{}^{(asym)}$ and $G_{Xa}{}^{(sym)}$ dominate the collisioless and collisional regimes, respectively, is not peculiar to stellarators with helically twisted structures only, but fairly universal for various types of non-symmetric toroidal plasmas. Hereafter, we call $G_{Xa}{}^{(asym)}$ "global" part, and call $G_{Xa}{}^{(sym)}$ "local" part, respectively.

Then the transport coefficient $N_a(K)$, which expresses the parallel viscosity as the driving force, defined in ref. [7] is obtained from this solution. We show here the expressions of "global" part of the normalized mono-energetic coefficient $N^* \equiv N_a(K)/[(c/e_a)m_a v_{Ta}K^{3/2}]$ due to the "global" part of the perturbation $G_{Xa}{}^{(asym)}$. Here, v_{Ta} is the thermal velocity and $K \equiv (v/v_{Ta})^2$. From the relation of $\sigma_{Xa}{}^{(sym)}$ and σ_{Ua} shown in eq. (2), the remaining part of the coefficient $N^{*(sym)}$ due to the "local" part of the perturbation is directly obtained from the normalized mono-energetic parallel viscosity coefficient M^* describing the damping force by the relation $N^{*(sym)} = -M^*[(\psi'B_\zeta - \chi'B_\theta)/\langle B^2\rangle + H^2V'/4\pi^2]/(2\chi'\psi')$. The analytical expressions for M^* are shown in ref. [7], and M^* is always positive in general toroidal configurations. By combining the analytical expression of the parallel viscosity force using these coefficients with the parallel momentum balance eqs. [5,7,11], the bootstrap current coefficients of the neoclassical transport matrix can be obtained.

The detail of the banana regime expansion for the "global" part of the equation is shown in refs. [2-4], and the resulting mono-energetic coefficient $N^{*(asym)}$ is given in the form of a pitch-angle-integral as

$$N^{*\,(asym)}_{(banana)} = \frac{\langle B^2\rangle}{2\chi'\psi' f_c}\frac{V'}{4\pi^2}\frac{\nu_D^a(K)}{v}\int_0^1 \mathrm{d}\lambda\,\frac{\lambda W(\lambda)}{\langle |v_{\|}|/v\rangle}$$

$$\equiv N_B\frac{\nu_D^a(K)}{v}\,, \tag{4}$$

where $\nu_D{}^a$ is the pitch-angle-scattering collision frequency [6,7] and f_c is the fraction of circulating particles [1-7]. The integral $\int \mathrm{d}\lambda$ in the range of $0 \leq \lambda \leq 1$ indicates the sum of the contributions of the circulating particles, and $W(\lambda)$ is the Fourier expanded perturbation of the circulating particle distribution. The explicit expression of this integral in the Hamada coordinates is given in ref. [3] and that for the Boozer coordinates is in ref. [4]. In the plateau and Pfirsch-Schlueter expansions neglecting the mirror force operator, the perturbed distribution functions are obtained by using a Fourier expansion for the dependence on (θ_B,ζ_B) in full range of ξ. The connected result given by these expansions is

$$N^{*\,(asym)}_{P\text{-}P} = -\frac{1}{2\chi'\psi'\langle B^2\rangle^{1/2}}$$

$$\sum_{m,n}\frac{[\chi'(1-H_2)m + \psi'(1+H_2)n](\chi'm-\psi'n)[B_{mn}^{(Boozer)}]^2}{\frac{8}{\pi}\left|\chi'm-\psi'n\right| + 5\,\frac{\nu_T^a(K)}{v}\langle B^2\rangle^{1/2}\frac{V'}{4\pi^2}}$$

$$+\left[\frac{\pi}{4\langle B^2\rangle^{1/2}}I_{p1}+\frac{2}{5\langle B^2\rangle}\frac{4\pi^2}{V'}I_{PS}\frac{\nu_T^a(K)}{\upsilon}\left(\frac{5\pi}{8}\frac{\upsilon_{Ta}}{\omega_{Ta}}\right)^2\right]$$

$$\left(1+\frac{5\pi}{8}\frac{\nu_T^a(K)}{\upsilon}\frac{\upsilon_{Ta}}{\omega_{Ta}}\right)^{-2}, \qquad (5)$$

where I_{p1} and I_{PS} are defined by

$$I_{p1}\equiv\frac{1}{4\pi^2}\int \mathrm{d}\theta_B\int \mathrm{d}\zeta_B\left\{\sum_{m,n}\frac{\chi'm-\psi'n}{\left|\chi'm-\psi'n\right|}B_{mn}^{(Boozer)}\right.$$

$$\left.\sin(m\theta_B-n\zeta_B)\right\}\left(\frac{\partial G}{\partial\zeta_B}\frac{\partial B}{\partial\theta_B}-\frac{\partial G}{\partial\theta_B}\frac{\partial B}{\partial\zeta_B}\right), \qquad (6)$$

and

$$I_{PS}\equiv-\frac{1}{4\pi^2}\int \mathrm{d}\theta_B\int \mathrm{d}\zeta_B\left(\chi'\frac{\partial B}{\partial\theta_B}+\psi'\frac{\partial B}{\partial\zeta_B}\right)$$

$$\left(\frac{\partial G}{\partial\zeta_B}\frac{\partial B}{\partial\theta_B}-\frac{\partial G}{\partial\theta_B}\frac{\partial B}{\partial\zeta_B}\right), \qquad (7)$$

respectively. The transit frequency ω_{Ta} in eq. (5) corresponding to the boundary between the plateau and the Pfirsch-Schlueter regimes is defined in refs. [6,11]. The relation of the anisotropy relaxation collision frequency $\nu_T{}^a$ for the Pfirsch-Schlueter regime with $\nu_D{}^a$ is given by $\nu_T{}^a = 3\nu_D{}^a + \nu_E{}^a$ using the energy scattering collision frequency $\nu_E{}^a$ [6,7]. In calculations in next section, $\nu_T{}^a$ is replaced by $3\nu_D{}^a$, and subscripts and superscripts denoting the types of collisions and the particle species are omitted following ref. [7]. Connecting eq. (4) and eq. (5) at a collision frequency satisfying $|C_a{}^L G_{Xa}{}^{(asym)(banana)}| \approx |V_M G_{Xa}{}^{(asym)(plateau)}|$ gives a good approximation since these terms are treated as higher order terms in the banana and plateau expansions. Since dominant functional components in both of these terms are odd functions of ξ, we use l = 1 Legendre components of them: $[C_a{}^L G_{Xa}{}^{(asym)(banana)}]^{(l=1)} = 3\nu^a{}_D m_a c f_c \upsilon\xi B N_B/(2e_a\langle B^2\rangle)$ and $[V_M G_{Xa}{}^{(asym)(plateau)}]^{(l=1)} = 3m_a c\upsilon^2\xi N_P/(2e_a\langle B^2\rangle^{1/2})$, where N_P is an appropriate constant expressing a representative value for the mirror force term. The detail of this constant in general multi-helicity stellarator is out of scope here and will be reported in a separated article. However, in so-called single-helicity stellarator model magnetic fields such as examples in the next section, a method to use the dominant Fourier mode term in Σ of the plateau value of $N^{*(asym)}$ is practically valid. This simplified method is used in the next section. Using what we have noted above, the "global" part of the mono-energetic coefficients is given by a banana/plateau connection formula

$$N^{*(asym)} = [\nu^a{}_D(K/\upsilon)][N_B + N_{P\text{-}P}{}^{*(asym)}(N_B/N_P)^2\nu^a{}_D(K)/\upsilon]/$$

$$[1 + |N_B/N_P|\nu^a{}_D(K)/\upsilon]^2. \qquad (8)$$

Here, the transition collision frequency is multiplied by f_c to take the effect of the parallel momentum exchange term ($H_a{}^{(l=1)}$ in ref. [7]) into account for the banana regime viscosity force.

By neglecting the higher order of δB in eq. (5) and by using the definition of H_2 [2-3], it can be shown that $N^{*(asym)}$ ($\nu_T{}^a\to\infty$) $\approx$ 0. This is a result of a characteristic of the "global" part in the banana regime. Although eq. (4) implicitly includes a residual contribution of the trapped particle part via the incompressible particle and energy conservation [2], the contribution of the deeply trapped particle part almost vanishes in the pitch-angle integral and the flux surface averaging. The transit mean free path for the plateau/banana transition of the "global" part eq. (8) is usually longer than that for the "local" part $N^{*(sym)} \propto M^*$ since the "global" part in the banana regime expresses the effects of circulating particles while the transitions of the "local" part is strongly affected by the ripple trapped effects [7].

3. Numerical examples

We assumed the magnetic configurations to be $B = B_0\,[1 - \varepsilon_t\cos\theta_B + \varepsilon_h\cos(l\theta_B - n\zeta_B)]$, l = 2, n = 10, B_0 = 1 T, χ' = 0.15 T·m, ψ' = 0.4 T·m, B_θ = 0, B_ζ = 4 T·m and investigated dependence of the driving force on toroidicity ε_t and helical ripple amplitude ε_h. In Fig. 1, we illustrate the procedure to calculate the mono-energetic coefficients N^* based on the dividing method in a quasi-helically symmetric configuration with ε_t = 0.01 and ε_h = 0.05. The "local" part $N^{*(sym)}$ qualitatively corresponds to the driving force due to the ripple-trapped orbits while the "global" part $N^{*(asym)}$ corresponds to that due to the banana-trapped orbits in this example. The $N^{*(sym)}$ dominating the relatively collisional regimes of $\nu/\upsilon > 10^{-4}$ m^{-1} is always positive. This polarity means the driving force making the flow in the direction of the helical winding. The $N^{*(asym)}$ given by eq. (8) is negative in the collisionality range of $\nu/\upsilon < 10^{-1}$ m^{-1} but it dominates

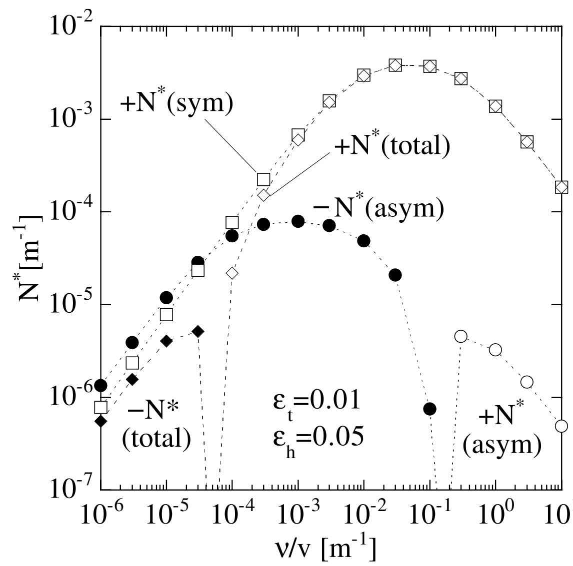

Fig. 1 The procedure to obtain the mono-energetic coefficient N^* by using two connection formulas corresponding to two types of driving mechanisms for the flows. The closed and open symbols indicate minus and plus values, respectively.

over the $N^{*(sym)}$ only at the collisionless limit of $\nu/\upsilon < 10^{-4}$ m^{-1}. This difference of the dependence of these two parts on the collisionality qualitatively indicates the difference of the bounce frequencies and fractions of corresponding trapped orbits. By adding these two components, N^* having the polarity reversal at $\nu/\upsilon \sim 10^{-4}$ m^{-1} is obtained. Here we call the change of the sign in N^* polarity reversal. This result cannot be obtained by the connection formula in ref. [11] without dividing into two contributions with opposite directions. Figure 2 shows comparisons of the mono-energetic coefficients N^* obtained by using eq. (8), with those obtained numerically by combining the DKES (Drift Kinetic Equation Solver) code [12] with our conversion formula [7]. Following ref. [7], we plot the geometrical factor [1-5,7] $G^{(BS)} \equiv -\langle B^2\rangle N^*/M^*$ instead of N^* in this figure, where M^* and consequently $N^{*(sym)}$ are given by the DKES with the relation [7] $M^* = (\nu/\upsilon)^2 D^*_{33}/[1 - (3/2)(\nu/\upsilon)D^*_{33}/\langle B^2\rangle]$. Here, D^*_{33} is an output of the DKES. Therefore the deviation of an analytical result from the numerical one shown here indicates the error of eq. (8). In the numerical results for the relatively collisionless regimes of $\nu/\upsilon < 10^{-3}$ m^{-1}, N^* has a weak dependence on the radial electric field [3]. This dependence seems to be caused by effects of the trapped/untrapped boundary layer in the velocity space [13] combined with the bounce-averaged part of the ripple trapped particle distribution function associated with the ripple diffusions [1,7]. In Fig. 2, we show the results for the cases with the radial electric field strength of $E_r/\upsilon = 1 \times 10^{-3}$ T, 3×10^{-3} T. The dependence of the numerical results in the collisionless limit of $\nu/\upsilon < 10^{-5}$ m^{-1} on the radial electric field is saturated in this range of the electric field strength because of the suppression of the ripple diffusions. Since eq. (4) is derived neglecting this boundary layer effect [2,3], the connection formula eq. (8) is applicable for these saturated states in the collisionless regimes and for the collisional regimes where the dependence on the radial electric field disappears. The suppression of the boundary layer effect due to the radial electric field is still small around the boundary between the banana and plateau regimes at $\nu/\upsilon = 10^{-3} \sim 10^{-4}$ m^{-1} and thus a deviation of the connection formula from the numerical results remains fairly large especially for the case with $\varepsilon_t = 0.1$ and $\varepsilon_h = 0.05$. However, the polarity reversal at $\nu/\upsilon \sim 10^{-4}$ m^{-1} for $\varepsilon_t = 0.01$, $\varepsilon_h = 0.05$ and at $\nu/\upsilon \sim 10^{-2}$ m^{-1} for $\varepsilon_t = 0.1$, $\varepsilon_h = 0.05$ can be precisely predicted by the analytical calculation using eq. (8), since it occurs in the plateau or the Pfirsch-Schlueter regimes of the "asymmetric" parts where the boundary layer effect is small.

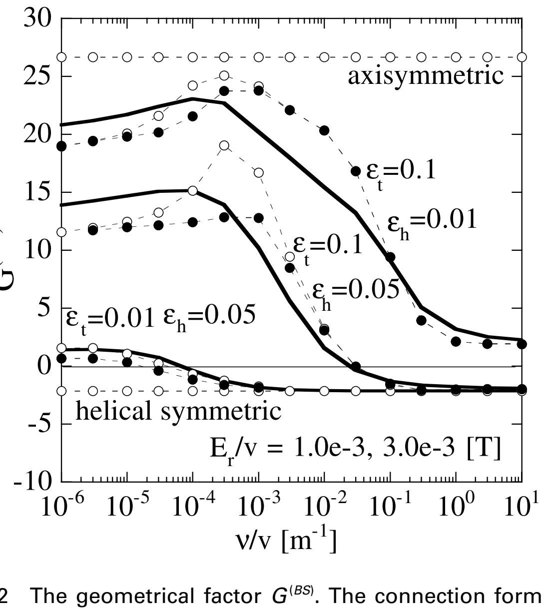

Fig. 2 The geometrical factor $G^{(BS)}$. The connection formula (solid curves), and the numerical results for the cases with $E_r/\upsilon = 1 \times 10^{-3}$ T(open circles) and 3×10^{-3} T(closed circles) are shown.

4. Concluding remarks

We have presented a method to derive analytical expressions for the bootstrap current coefficients in stellarators. Since this method separates two types driving mechanisms for the flows in stellarators, connecting the results of three types of asymptotic expansions gives a good approximation for the divided transport coefficients. We have also shown the numerical results to confirm the validity of derived formulas.

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