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Development of the Robust MHD Code Using CIP Scheme

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Abstract

A robust MHD code was developed to study the behavior of hot plasma with steep density and/or pressure gradient. MHD equations included in thermal conductivity, viscosity and resistivity which are strongly dependent on plasma parameters had to be solved; using splitting fractional method an advection (convection) equations were solved by the Constrained Interpolation Profile (explicit-CIP), and Poisson equations in the non-advection terms were solved by Alternating Direction Implicit scheme (ADI). Total time step was marching by Runge-Kutta method. The code was examined for simple MHD shock tube problem. The differences of the magnetic compression and the density ratio ahead and behind the shock front from the calculation results were within a few percents when the density and/or pressure ratios were less than 10^3 and 10^5, respectively.

Keywords:

steep density gradient, steep pressure gradient, robustness, CIP scheme, MHD code

1. Introduction

Recently pellet injection and/or super-sonic gas jet are used as fuelling methods of hot plasma in large experimental device. These fuelling may change the characteristics of plasma confinement. Therefore, it is interesting to study the behavior of hot plasma under these fuelling conditions. To solve these phenomena numerically the code has to include in the complex atomic processes and it can treat non-thermal equilibrium.

As already known, fluids model such as MHD only treats quasi-thermal equilibrium states. Therefore, MHD code has the limitation to solve these drastic states, and it cannot treat the transition phenomena like plasma just after thick fuelling. However, in general the duration of the transition processes are shorter than that of interested macroscopic plasma behavior *e.g.* a "snake" after pellet injection. Therefore, the macroscopic density diffusion process in whole plasma after transition processes can be treated as MHD phenomena. For this purpose a MHD code has to have the robustness in a large density and/or pressure gradient (ratio) within a few calculation cells.

We have already developed a single fluid MHD code [1] using normal explicit constrained interpolation profile scheme (explicit-CIP) [2]. CIP has robustness under severe conditions such as density ratio of 10^3 within a few cells. This code could treat ideal MHD, however, thermal conductivity, resistivity and viscosity were constant in space and time, and viscosity and thermal conduction were solved by an explicit method. Therefore, it could not be applied to the problem with wide temperature range and it had the limitation to use.

In this paper, we have developed new code to treat the variable plasma properties. Using splitting fractional method we introduce a linear interpolation at the shock front to solve the advection equation stable and we apply semi-implicit method to solve the pressure equation, also full implicit method to solve the viscosity and thermal conduction equations because of numerical stability.

2. Governing equations for a single fluid model

The governing equations of a single fluid for MHD model are as follows,

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \boldsymbol{u}) = 0,$$

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} + \rho\boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \boldsymbol{j}\times\boldsymbol{B} + \nabla\cdot\tau_{ij},$$

$$\frac{\partial p}{\partial t} = -\nabla\cdot(\rho\boldsymbol{u}) - (\gamma-1)[p\nabla\cdot\boldsymbol{u} + \eta\boldsymbol{j}\cdot\boldsymbol{j} + \phi] - \gamma\nabla\cdot\boldsymbol{q},$$

$$\tau_{ij} = 2\mu_p\left(\mathbf{e}_{ij} - \frac{1}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right),\ \phi = 2\mu_p\left(\mathbf{e}_{ij}\mathbf{e}_{ij} - \frac{1}{3}(\nabla\cdot\boldsymbol{u})^2\right),$$

$$\mathbf{e}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),\ p = \rho R_p T,\ \boldsymbol{q} = -\lambda_p\nabla T,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla\times\boldsymbol{E},\ \ \boldsymbol{E} = -\boldsymbol{u}\times\boldsymbol{B} + \eta\boldsymbol{j},\ \ \mu_0\boldsymbol{j} = \nabla\times\boldsymbol{B},$$

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(ρ, p: density and pressure, $\boldsymbol{u}$: velocity [vector], τ_{ij}: stress tensor, i, j and k are indices of space, μ_p: viscosity of plasma, R_p: gas constant for plasma = $2R_{gas}$, γ: ratio of specific heats η: resistivity, $\boldsymbol{q}$: thermal flux [vector], λ_p: thermal conductivity, Φ: dissipation function due to stress in a fluid).

These equations are derived from the mass, momentum and energy conservation laws, and Maxwell equations. Using splitting fractional method the advection equations are solved first by CIP method [1], and Poisson equations in the non-advection terms are solved by ADI method. ADI is unconditional stable in two dimensions [3]. The time evolution of one full step is marching by Runge-Kutta method. The order of time accuracy is chosen 1 to 4 as an input parameter; usually we choose second or fourth order. Thermal conductivity depends on the magnetic field direction (anisotropy), other properties are assumed to be only a function of plasma parameters (isotropy). The plasma properties used in the code are described in ref. [4].

3. Numerical test

The CIP and ADI routine were tested independently, and after that we combined them to make full-MHD code. The completed code was two-dimensional. For numerical example, we choose a Riemann problem of ideal MHD shock tube, which was introduced by G. Sod [5] firstly. The solution of this problem consisted various MHD shocks and it was already analyzed numerically by some groups [6-8]. The initial value of this problem is as follows,

The dimensionless initial left and right states, which are denoted by l and r are
$\rho_l = 1$, $u_l = 0$, $p_l = 1$, $B_x = 0.75$, $B_y = 1$, $\gamma = 2$, and
$\rho_r = 0.125$, $u_r = 0$, $p_r = 0.1$, $B_x = 0.75$, $B_y = -1$, $\gamma = 2$, respectively.

Initial discontinuity is located in the middle of computational domain at the direction of X, and all states are uniform in the direction of Y. Using this 1D initial states, numerical solution, which were obtained for $Nx = 800$ and $Ny = 5$ grid

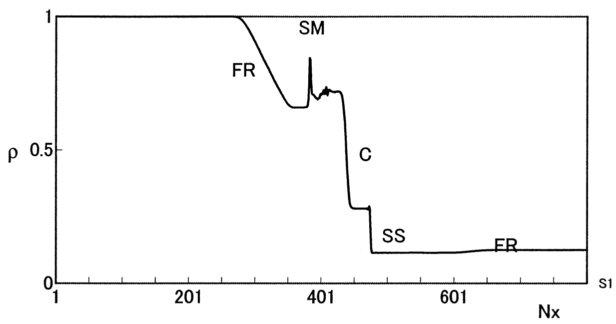

Fig. 1(a)

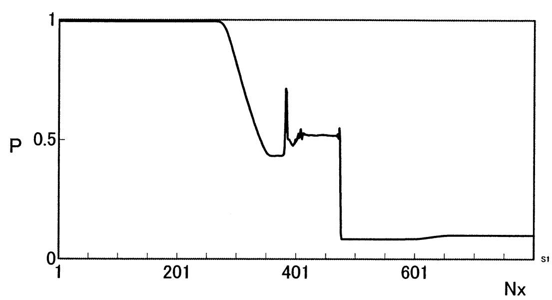

Fig. 1(b)

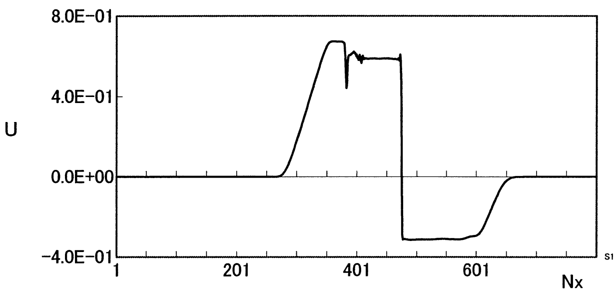

Fig. 1(c)

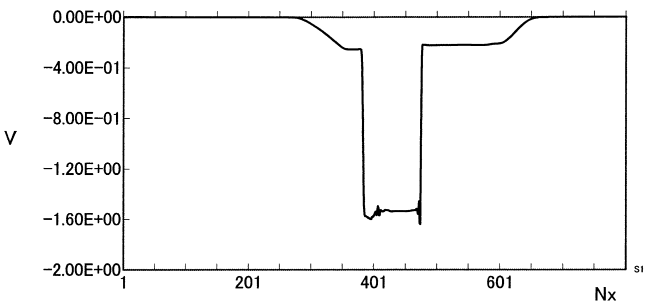

Fig. 1(d)

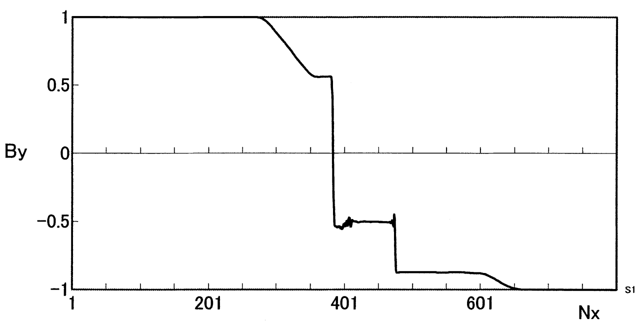

Fig. 1(e)

Fig. 1 Calculational results of a coplanar MHD Riemann problem, (a) density, (b) pressure, (c) velocity normal to the shock front, (d) velocity parallel to the shock front and (e) magnetic field parallel to the shock front. Grid numbers of calculation domain are $Nx = 800$, $Ny = 5$, and horizontal axis shows the grid of X-direction. The results of 1000 time steps at CFL = 0.4 are shown.

points with $\Delta x = 1$, $\Delta y = 1$, CFL = 0.4, are shown in Fig. 1 after 1000 time steps; it is necessary to meet Courant-Friedrihs-Lewy (CFL) condition due to explicit-CIP scheme. The solution consists of following waves; the waves moving to the left are a fast refraction wave (denoted by FR in the figure), and a compound wave (SM), which consists of a intermediate shock and attached to it a slow refraction wave; the waves moving to the right includes a discontinuity (C), a slow shock (SS), and a fast refraction wave (FR). The obtained result is very similar to that of ref [6-8]. Using Lax-Wendroff and/or MacCormack without artificial viscosity or TVD we could not solve these waves precisely. SM and SS are sharply resolved by a few points, however, the discontinuity is resolved by more than 10 points, and between SM and C some numerical oscillations are found in this solution. It seems that the numerical oscillations are due to the pressure term, which is solved by semi-implicit method this time. To avoid these oscillations we have to remake this using a full implicit method such as CIP-combine and unified procedure (C-CUP) [9]. This is very near future work.

To simulate dense plasma diffusion such as MHD behavior after pellet ablation, the robustness for high density/pressure gradient of this code has to be demonstrated. We have solved high Mach number problems as above. This shock waveform is the same as the shock tube solution of neutral gas. Choosing $\gamma = 5/3$, $Bx = 0$, and $By = 1.0$, we can compare magnetic compression with the density ratio ahead and behind the shock front; two rates should be the same value at the ideal perpendicular magnetic shock. Various initial states to create the shocks are shown in a Table 1. Also, the density ratio and magnetic compression ahead and behind the shock front from calculation results are shown in the right column. The magnetic compression is limited to the range $1 < B_2 / B_1 < (\gamma + 1)/(\gamma - 1)$, where B_1 and B_2 are the magnetic field strength ahead and behind the shock front, respectively, *i.e.* $B_1, B_2 = \sqrt{B_x^2 + B_y^2}$. For $\gamma = 5/3$ the upper limit of this compression rate is 4. We tried to confirm the effect of the plasma properties on the shock formation. Calculating thermal conductivity, viscosity and resistivity, which depend on the plasma parameters, we can also compare the density ratio with the magnetic compression ahead and behind the shock front. These cases are denoted by "yes" in each plasma properties of Table 1, while the other cases are denoted by "no", in which thermal conductivity, viscosity and resistivity are constant in time and space.

Usually Mach number represents the shock strength in neutral gas, however the magnetic compression also shows the shock strength in MHD case. Comparing the results of no.3 and no.4 in the Table 1, it is found that the shock depends on not only the upper stream plasma parameters but also downstream plasma parameters because of the sound speed of downstream region. In each case the magnetic compression is good agreement with the density ratio, when the magnetic compression is below than 1.5–2.0. These results are denoted by "good" in the status column of Table 1. However the discrepancy between the density ratio and the magnetic compression becomes large when the magnetic compression is large. Numerical oscillations appear to follow behind shock as the magnetic compression is approaching the upper limit 4; moreover the magnetic compression is over 4 in one case. Those results are denoted by "poor" and "bad" in the status of Table 1, respectively. Although the calculation results were broken in those cases, the numerical results have no negative density and pressure value and the errors in the Table 1 are also within a few percent. The limitation of valid results region of this code would be as follows,

With the adjacent grid the density ratio is below 10^3, and the pressure ratio is than 10^5 corresponding to the density ratio.

There is no need to concern above limitation as far as we treat typical experimental plasma parameters of magnetic confinement device. Therefore, it is expected that this code can treat the behavior of plasma with high density locally such as after pellet ablation and/or supersonic gas injection.

There is a slight difference in error between the results of no.15 and no.16. Initial condition of the temperature is 100 eV, therefore the viscosity, which is proportional to the mean free path is higher than other cases. In general the effect of plasma properties on the shock formation was not found clearly in all cases, this was good while it means the calcula-

Table 1 Various initial conditions for shock calculations.

No.	Left		Right		compression		error	status	plasma properties		
	density	temperature	density	temperature	X1=ρ2/ρ1	X2=B2/B1	X1/X2		conductivity	viscosity	resistivity
1	1.00E+20	1.00E+03	1.25E+19	1.00E+01	1.01	1.01	1.00	good	No	No	No
2	1.00E+21	1.00E+03	1.25E+19	1.00E+01	1.04	1.04	1.00	good	No	No	No
3	1.00E+22	1.00E+03	1.25E+19	1.00E+01	1.12	1.12	1.00	good	No	No	No
4	1.00E+22	1.00E+03	1.25E+21	1.00E+01	1.64	1.70	0.96	good	No	No	No
5	2.00E+22	1.00E+03	2.50E+19	1.00E+01	1.20	1.20	1.00	good	No	No	No
6	3.00E+21	1.00E+03	3.00E+19	1.00E+01	1.10	1.09	1.01	good	No	No	No
7	3.00E+21	1.00E+04	3.00E+19	1.00E+01	1.57	1.62	0.97	poor	No	No	No
8	3.00E+22	1.00E+04	3.00E+19	1.00E+01	1.96	2.12	0.92	poor	No	No	No
9	3.00E+22	2.00E+04	3.00E+19	1.00E+01	2.86	2.86	1.00	poor	No	No	No
10	3.00E+22	4.00E+04	3.00E+19	1.00E+01	4.07	4.26	0.96	bad	No	No	No
11	1.00E+20	1.00E+03	1.25E+19	1.00E+01	1.01	1.01	1.00	good	YES	YES	YES
12	1.00E+21	1.00E+03	1.25E+19	1.00E+01	1.04	1.04	1.00	good	YES	YES	YES
13	1.00E+22	1.00E+03	1.25E+21	1.00E+02	1.61	1.66	0.97	poor	No	No	No
14	1.00E+22	1.00E+03	1.25E+21	1.00E+02	1.61	1.66	0.97	poor	YES	YES	YES
15	1.00E+22	1.00E+02	1.25E+19	1.00E+02	1.02	1.01	1.01	good	No	No	No
16	1.00E+22	1.00E+02	1.25E+19	1.00E+02	1.01	1.01	1.00	good	YES	YES	YES

tion of thermal conduction, viscosity and resistivity does not do anything bad. In reality the shock should disappear itself with increasing time marching due to dissipation. For example, when high temperature plasma would be selected in the shock calculation, it might be possible to confirm the effect of thermal conduction and viscosity even though we need huge computational domain and time steps.

4. Conclusion

The robust MHD code was developed. Although the pressure term was solved by semi-implicit method, this code could handle the large density and/or pressure gradient (ratio) within a few calculation cells without any negative density and pressure.

Test results of MHD shock tube calculation showed that it could calculate MHD behavior under the initial density ratio of 10^3 and pressure ratio of 10^5 within a few percent error.

It was found that there are numerical oscillations between severe states; therefore, the improvement would be necessary for this code. This is near future work.

Acknowledgements

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References

[1] N. Nishino, J. Plasma Fusion Res. Series **5**, 541(2001).
[2] T. Yabe and T. Aoki, Comput. Phys. Comm. **66** 219 (1991).
[3] W.H. Press *et al.*, *Numerical recipes in C++* (Cambridge 2002).
[4] S.I. Braginskii, *Reviews of plasma physics* **1**, 205 (Consultants Bureau, 1966).
[5] G. Sod, J. Comput. Phys. **27**, 1 (1978).
[6] M. Brio and C.C. Wu, J. Comput. Phys. **75**, 400 (1988).
[7] D. Ryu and T.W. Jones, Astrophys. J. **442**, 228 (1995).
[8] G.S. J and C.C. Wu, J. Comput. Phys. **150**, 561 (1999).
[9] T. Yabe and P.Y. Wang, J. Phys. Soc. Jpn. **60**, 2105 (1991).