J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 391–394

Production of Ultrarelativistic Positrons by an Oblique Magnetosonic Shock Wave in an Electron-Positron-Ion Plasma

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(Received: 9 December 2003 / Accepted: 23 February 2004)

Abstract

Wave propagation and particle acceleration in a plasma consisting of electrons, positrons, and ions are studied with theory and particle simulations. With one-dimension, relativistic, electromagnetic, particle simulations, it is found that a magnetosonic shock wave propagating obliquely to an external magnetic field can accelerate some positrons to ultrarelativistic energies. Also, a theory is presented which explains the energy increase rate and particle motion observed in the simulations.

Keywords:
positron, oblique shock, particle acceleration, particle simulation, multicomponent plasma

1. Introduction

Particle simulations show that a magnetosonic shock wave can accelerate particles with various nonstochastic mechanisms. It can accelerate some ions by reflection, which is caused by rapid increase in the electric potential or the magnetic field strength in the shock transition region [1-5]. In a multi-ion-species plasma with the hydrogen the major component, a shock wave accelerates all the heavy ions with the transverse electric field; their maximum speeds are nearly the same (independent of the particle species) [6,7]. Also, it can accelerate some electrons to ultrarelativistic energies; electrons that are reflected near the end of the main pulse are trapped in the pulse and gain energies from the wave fields [8,9]. If there exist nonthermal energetic ions, a shock wave can incessantly accelerate them with the transverse electric field [10-13]. Furthermore, in a plasma containing positrons as well as electrons and ions, it can accelerate positrons to ultrarelativistic energies [14].

Positrons could be found, for instance, in pulsar magnetospheres, astrophysical jets, or laboratory laser-plasmas [15-18]. Electron-positron plasmas have thus been studied by many authors [19-23]. In this paper, we investigate wave propagation and positron acceleration in electron-positron-ion (e-p-i) plasmas. In Sec. **2**, we describe the dispersion relations of three low frequency waves propagating obliquely to an external magnetic field; one of them is the magnetosonic wave. In Sec. **3**, we theoretically discuss positron acceleration caused by a magnetosonic shock wave. In Sec. **4**, by using relativistic, electromagnetic, particle simulations, we show that a shock wave can accelerate positrons to ultrarelativistic energies; in the simulation, the maximum positron energy was $\gamma \sim 600$, where γ is the Lorentz factor.

2. Dispersion relations

From the three-fluid model for a cold e-p-i plasma, we have three different modes in the frequency regime lower than the electron gyrofrequency $|\Omega_e|$. Figure 1 shows dispersion curves for these modes. Here, the electron gyrofrequency is $|\Omega_e|/\omega_{pe} = 3.0$, where ω_{pe} is the electron plasma frequency, and the angle between the wave normal and the external magnetic field $\boldsymbol{B}_0$ is taken to be $\theta = 42°$. The ion-to-electron mass ratio is $m_i/m_e = 100$; we have chosen this value to compare with simulations. In the upper panel, the positron-to-electron density ratio is $n_{p0}/n_{e0} = 0.02$, while in the lower panel, it is $n_{p0}/n_{e0} = 0.5$. In the frequency regime $\omega \gtrsim |\Omega_e|$, we find three modes, which we are not concerned with in this paper. In the low frequency regime $\omega \lesssim |\Omega_e|$, we have high-frequency mode (line H), magnetosonic mode (line M), and Alfvén mode (line A).

The high-frequency mode has a cutoff frequency $\omega_{\rm hf0}$. Its value approaches the electron gyrofrequency, $\omega_{\rm hf0} \sim |\Omega_e|$, as the positron density goes to zero, $n_{p0}/n_{e0} \to 0$, while it approaches the ion gyrofrequency, Ω_i, as $n_{p0}/n_{e0} \to 1$. The resonance frequency of this mode is given by the electron gyrofrequency $|\Omega_e|$.

The Alfvén mode has a resonance at $\omega \sim \Omega_i$. Its dispersion relation in the long-wavelength regime is given by

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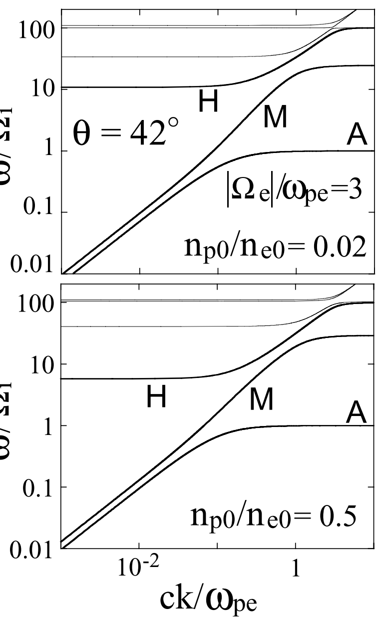

Fig. 1 Dispersion relations of low frequency waves in magnetized e-p-i plasmas with $\theta = 42°$ for two different positron densities.

$$\omega \simeq v_A (1 + v_A^2/c^2)^{-1/2} k \cos\theta, \qquad (1)$$

where v_A is the Alfvén speed in the e-p-i plasma defined as $v_A = B_0/(4\pi \sum_j n_j m_j)^{1/2}$, c is the speed of light, and k is the wave number.

The dispersion relation of the magenetosonic wave is approximated by

$$\omega \simeq v_A (1 + v_A^2/c^2)^{-1/2} k, \qquad (2)$$

in the long-wavelength regime, $ck/\omega_{pe} \ll 1$. Also, it has a resonance at $\omega \sim |\Omega_e| \cos\theta$. The shock wave studied in this paper is the magnetosonic mode.

3. Acceleration theory

3.1 Positron reflection

In magnetohydrodynamic waves, the electric field parallel to the magnetic field is quite weak. The ions are thus rarely reflected along the magnetic field in shock waves. They are reflected across the magnetic field. On the other hand, positrons can be easily reflected by the shock wave along the magnetic field because their mass is small.

Suppose that a shock is propagating in the x direction with a propagation speed $v_{\rm sh}$ in an external magnetic field $\boldsymbol{B}_0 = B_0 (\cos\theta, 0, \sin\theta)$, where the subscript 0 denotes equilibrium quantities. Then, in the wave frame, the electric field is given as $\boldsymbol{E}_w = (E_{wx}, -(v_{sh}/c)B_{wz0}, 0)$, where the subscript w refers to quantities in the wave frame. From the equation of motion for the positrons, one obtains the following equation [8,14],

$$\frac{1}{2} m_p (v_{w\parallel} - v_{rv})^2 = -e(F_w - F_{w0}) - K - \mu_{Bw} B_w + \frac{1}{2} m_p v_{w0}^2 - \frac{1}{2} m_p v_{wd}^2 - m_p c \frac{E_{wy0}}{B_{x0}} (v_{\rm wdz} - v_{wz0}), \qquad (3)$$

where $v_{w\parallel}$ is the velocity parallel to the magnetic field, v_{wdz} is the z component of the drift velocity v_{wd}, and μ_{Bw} is the magnetic moment, $m_p v_{w\perp}^2/(2B_w)$ with $v_{w\perp}$ the gyration speed. Also, $v_{rv} = -cE_{wy0} B_{wz}/(B_{x0} B_w)$, and $K = -m_p v_{rv}^2/2$. The quanty F is defined by

$$F_w = -\int E_{w\parallel} \, {\rm d}s \ , \qquad (4)$$

where ds is the infinitesimal length along the magnetic field. This quantity F_w has large positive values in the shock region. Positrons cannot penetrate regions where F_w has large positive values, because the left-hand side of eq. (3) must be positive. Noting that all the terms except F_w in eq. (3) are proportional to m_p and that $v_{w\parallel 0}$ and v_{wd} are of the order of v_A, we find that positrons can be reflected by a shock wave.

This reflection can occur in oblique shock waves. Another important effect of oblique propagation is that it enables some positrons to move with the shock wave for long periods of time. In the next subsection, we will discuss these positrons.

3.2 Positron acceleration

Here, we consider positrons staying in the shock transition region and assume that their velocities satisfy the relations $v \approx c$ and $v_x \approx v_{sh} \gg |v_y|$. We will derive the energy increase rate for these particles.

Becuase γ is quite large, we neglect $\gamma {\rm d}v_\sigma/{\rm d}t$ compared with $v_\sigma {\rm d}\gamma/{\rm d}t$, where $\sigma = x, y$, or z. Then, using the relations $E_y = (v_{sh}/c)(B_z - B_{z0})$ and $E_z = -(v_{sh}/c)B_y$, we may write the zeroth-order equation of motion as

$$m_p v_{sh} \frac{{\rm d}\gamma}{{\rm d}t} = eE_x(\xi) + e\frac{v_y}{c} B_z(\xi) - e\frac{v_z}{c} B_y(\xi), \qquad (5)$$

$$0 = e\frac{v_z}{c} B_{x0} - e\frac{v_{sh}}{c} B_{z0}, \qquad (6)$$

$$m_p v_z \frac{{\rm d}\gamma}{{\rm d}t} = -e\frac{v_y}{c} B_{x0}, \qquad (7)$$

where ξ is the particle position, $\xi = x - v_{sh} t$. Equation (6) gives

$$v_z/v_{sh} = B_{z0}/B_{x0} = \tan\theta, \qquad (8)$$

which indicates that the velocity of an accelerated positron is nearly parallel to the external magnetic field. Also, substituting eqs. (7) and (8) in (5) yields the time rate of change of γ

$$\frac{{\rm d}\gamma}{{\rm d}t} = \frac{eB_{x0}^2}{m_p v_{sh}} \left(\frac{E_x(\xi) - v_{\rm sh} B_{z0} B_y(\xi)/(cB_{x0})}{B_{z0} B_z(\xi) + B_{x0}^2} \right). \qquad (9)$$

This shows that γ would grow linearly with time if the particle position ξ is constant.

4. Simulation

To further study the positron acceleration, we use a one dimension (three velocities), relativistic, electromagnetic, particle simulation code with full particle dynamics. The system length is $L_x = 8192\Delta_g$ with Δ_g the grid spacing. The total number of electrons is $N_e = 614,000$. The ion-to-

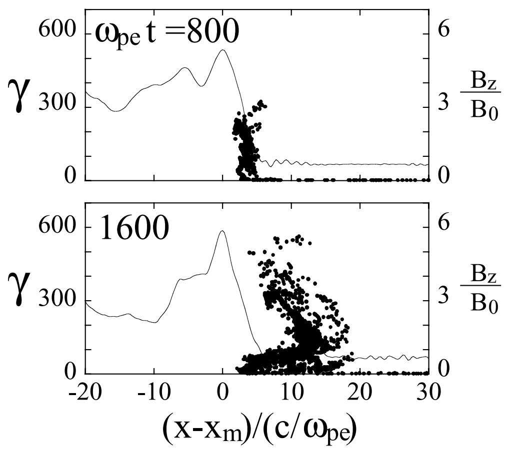

Fig. 2 Phase space plots (x, γ) of positrons. The solid lines indicate profiles of B_z.

electron mass ratio is $m_i/m_e = 100$, with charges $q_i = q_p = -q_e$. The electron skin depth is $c/\omega_{pe} = 4\Delta_g$. The magnetic field strength is $|\Omega_e|/\omega_{pe} = 3.0$ in the upstream region with $\theta = 42°$. The Alfvén speed for these parameters is $v_A/c = 0.30$. The time step is $\omega_{pe}\Delta t = 0.01$, which is much smaller than the periods of electron plasma oscillation and gyromotion.

Figure 2 shows phase space plots of positrons at two different times for a shock wave with $v_{sh} = 2.4v_A$. Here, x_m is the x position at which the magnetic field has its maximum value. The solid lines indicate profiles of B_z. Many positrons are reflected in the shock transition region and then accelerated to ultrarelativistic energies; the maximum energy in this example is $\gamma \sim 600$.

Figure 3 displays time variations of γ and velocities of a positron that has been accelerated to $\gamma \sim 600$. This particle encountered the shock wave at $\omega_{pe}t \sim 200$ and was reflected. Its speed quickly reached $v \sim c$. The panels of v_x and v_z indicate that the particle moves with the shock speed in the x direction, $v_x \simeq v_{sh}$, and that its velocity is nearly parallel to the external magnetic field, $v_z \sim v_{sh}\tan\theta$, which agrees with eq. (8). We also note that the magnitude of v_y is quite small.

We obtain time-averaged velocities $\langle v_y/c \rangle = -0.12$ and $\langle v_z/c \rangle = 0.66$ from the simulation data from $\omega_{pe}t = 250$ to $\omega_{pe}t = 1600$. Substituting these values in eq. (7), we find the energy increase rate as $d\gamma/d(\omega_{pe}t) = 0.41$. This is also in good agreement with the observed value $d\gamma/d(\omega_{pe}t) = 0.41$ for the same time period.

5. Summary

We have studied the wave propagation and positron acceleration in an electron-positron-ion plasma. It is found that an oblique magnetosonic shock wave can accelerate some positrons to ultrarelativistic energies; in our simulations, positron energy reached $\gamma \sim 600$. Particle motion and energy increase rate predicted by our theory are in good agreement with the simulation results.

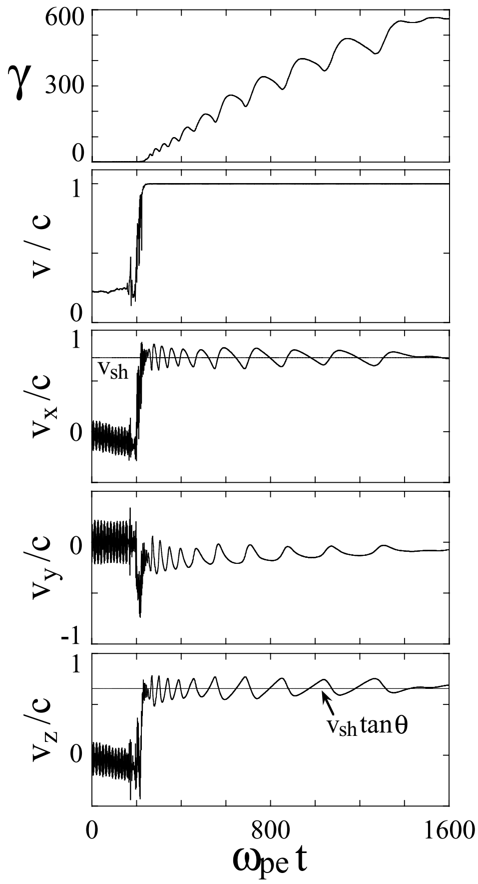

Fig. 3 Time variations of γ, v, v_x, v_y, and v_z of an accelerated positron.

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