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Kinetic Theory for Spontaneous Magnetic Field in Laser Plasma Interaction

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Abstract

In our previous paper [Phys. Plasmas **10**, 4166 (2003)], we pointed out that the experimental results on the axial spontaneous magnetic field given by Najmudin *et al.* [Phys. Rev. Lett. **10**, 215004 (2001)] can be explained by our kinetic model [Phys. Plasmas **8**, 329 (2001)]. We also found that the azimuthal magnetic field B_θ given by our kinetic model is much weaker than the axial spontaneous magnetic field, which is not consistent with particle-in-cell simulation results. In the present paper, we reinvestigate the generation mechanism of the spontaneous magnetic fields from relativistic Vlasov-Maxwell equations. It is found that the azimuthal spontaneous magnetic field B_θ can be same order as the axial spontaneous magnetic field if the laser beam self-focus effect is considered. It is also obtained that the magnitude of the spontaneous magnetic field is proportional to the plasma density for the case of low plasma density, but is independent of the plasma density for the case of high plasma density.

Keywords:

fast ignition, collisionless plasma, laser plasma interaction, Vlasov-Maxwell equations, spontaneous magnetic field

1. Introduction

Attention to the fast ignition scheme for inertial confinement fusion [1] have generated worldwide interest in interaction of an intense short-pulse laser with plasma [2,3]. Among the various nonlinear effects which may occur in plasma interacting with intense short-pulse laser, the generation of spontaneous magnetic field is one of the most interesting and significant problems because the fields could have considerable influence on nonlinear plasma dynamics, especially on the collimation of relativistic electron beam. The extremely high spontaneous magnetic field was observed by particle simulation in the interaction of ultraintense short laser pulse with an overdense plasma target [3,4]. Recently, experiment measurement of the spontaneous magnetic field are reported by Fuchs *et al.* [5] and Najmudin *et al.* [6]. So far, a number of theoretical models for generation of the spontaneous magnetic field are suggested [7-16]. It should be noted that most of works on the spontaneous magnetic fields is on the basis of fluid scheme. A kinetic generation mechanism for the spontaneous magnetic field driven by the beat interaction between two electromagnetic field was proposed by He [17] in 1983. One of the present authors [18] got an equation for the spontaneous magnetic field in collisionless plasma by the same method as Ref. [17], but under different approximation. In Ref. [19], we discussed the spontaneous magnetic field from relativistic Vlasov-Maxwell equations.

For real intense laser plasma interaction, electrons are heated by laser beam. One part of electrons are accelerated into relativistic velocity region, the other part of electrons are thermal ones. In the other word, there exist two groups of electron, thermal one with temperature T_{te} and relativistic one with temperature T_{re}. So, the electrons satisfy the distribution function $f_{\alpha 0} = \xi f_{\alpha M}(T_{te}) + (1-\xi) f_{\alpha M}(T_{re})$, where $f_{\alpha M}(T_{te})$ and $f_{\alpha M}(T_{re})$ are nonrelativistic and relativistic distribution function respectively, ξ is the ratio of the number of thermal electrons to the total number of electrons. According to above ideal and including the effect of finite laser beam, in Ref. [20] we generalized our kinetic model proposed in Ref. [19] and found that the experimental results on the axial spontaneous magnetic field given by Najmudin *et al.* [6] can be explained by our kinetic model. In fact, there is only the axial spontaneous magnetic field measured in Ref. [6].

We pointed out in Ref. [20] that the azimuthal magnetic field B_θ given by our kinetic model is much weaker than the axial spontaneous magnetic field, which is not consistent with particle-in-cell simulation results. In the present paper, we reinvestigate the generation mechanism of the spontaneous magnetic fields from relativistic Vlasov-Maxwell equations. We got an equation for the spontaneous magnetic field by the same method as Ref. [19], but under different parameter region. It is found that the azimuthal spontaneous

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magnetic field B_θ can be same order as the axial spontaneous magnetic field if the laser beam self-focus effect is considered. It is also obtained that the magnitude of the spontaneous magnetic field is proportional to the plasma density for the case of low plasma density, but is independent of the plasma density for the case of high plasma density.

2. Spontaneous magnetic fields

Consider the relativistic Vlasov-Maxwell equations

$$\left(\frac{\partial}{\partial t}+\frac{c\vec{p}_\alpha}{p_{0\alpha}}\bullet\frac{\partial}{\partial\vec{x}}+ e_\alpha\vec{G}(\vec{x},t)\bullet\frac{\partial}{\partial\vec{p}_\alpha}\right)f_\alpha(\vec{x},\vec{p}_\alpha,t)=0 \tag{1}$$

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t},\quad \nabla\times\vec{B}=\frac{1}{c}\frac{\partial\vec{E}}{\partial t}+\frac{4\pi}{c}\vec{j} \tag{2}$$

$$G(\vec{x},t)=\vec{E}(\vec{x},t)+\frac{\vec{p}_\alpha\times\vec{B}}{p_{0\alpha}} \tag{3}$$

$$\vec{j}=\sum_\alpha\int e_\alpha\frac{c\vec{p}_\alpha}{p_{0\alpha}}f_\alpha(\vec{x},\vec{p}_\alpha,t)\mathrm{d}^3\vec{p}_\alpha \tag{4}$$

where the index $\alpha(\alpha=i,e)$ represents ions and electrons respectively, $P_{0\alpha}=\sqrt{\vec{P}_\alpha^2+m_\alpha^2c^2}$, $\vec{P}_\alpha$ is the momentum, m_α is the rest mass of particle, c is the light speed. The other notations in Eqs. (1)-(4) are standard.

We split the particle distribution functions up into their background part $n_\alpha f_{\alpha 0}$, slow-time-scale part $f_{\alpha s}$ (time scale for spontaneous magnetic field) and fast-time-scale part $f_{\alpha f}$(time scale for laser oscillation):

$$f_\alpha=n_\alpha f_{\alpha 0}+f_{\alpha s}+f_{\alpha f}, \tag{5}$$

where $f_{\alpha 0}=\xi f_{\alpha M}(T_{te})+(1-\xi)f_{\alpha M}(T_{te})$ is the background distribution function and n_α is the particle number density. Substituting Eq. (5) into Vlasov equation, splitting the electromagnetic fields up into their slow-time-scale and fast-time-scale parts, and using Fourier-transform technique, we have the slow-time-scale electric fields $\vec{E}_S^{\sigma_S}(q)$ (the subscript "s" means slow-time-scale field, the superscript "σ_S "($\sigma_S = l,t$) represents longitudinal and transverse fields, respectively) and the spontaneous magnetic fields $\vec{B}_S(q)$ in the Fourier representation:

$$\vec{E}_s^{\sigma_s}(q)=-\frac{\mathrm{i}4\pi n_{e0}e^3}{\Omega\Theta_s^{\sigma_s}(q)}\sum_{\sigma_1,\sigma_2(s)}\int\Pi_e^{\sigma_s\sigma_1\sigma_2}(q,k_1,k_2)\times E_f^{\sigma_1}(k_1)E_f^{\sigma_2}(k_2)\delta^4(q-k_1-k_2)\frac{\mathrm{d}^4k_1\mathrm{d}^4k_2}{(2\pi)^4} \tag{6}$$

$$\vec{B}_s(q)=\frac{c}{\Omega}\vec{q}\times\vec{E}_s^t(q) \tag{7}$$

$$\Pi_\alpha^{\sigma_s\sigma_1\sigma_2}(q,k_1,k_2)\equiv\frac{1}{2}\int\left[\frac{c\vec{e}_s^{\,\sigma_s}(q)\cdot\vec{p}_\alpha}{p_{0\alpha}\left(\Omega-\dfrac{c\vec{q}\cdot\vec{p}_\alpha}{p_{0\alpha}}\right)}\vec{e}'^{\,\sigma_1}_f(k_1)\cdot\frac{\partial}{\partial\vec{p}_\alpha}\frac{\vec{e}'^{\,\sigma_2}_f(k_2)\cdot\dfrac{\partial f_{\alpha 0}}{\partial\vec{p}_\alpha}}{\left(\omega_2-\dfrac{c\vec{k}_2\cdot\vec{p}_\alpha}{p_{0\alpha}}\right)}+(1\leftrightarrow 2)\right]\mathrm{d}^3\vec{p} \tag{8}$$

where e is the charge of proton, $k=(\vec{k},\omega)$, $q=(\vec{q},\Omega)$, the symbol $\int_{(s)}$ refers to a slow-time-scale motion caused by the difference beat of two fast-time-scale motions. In Eq. (8), $(1\leftrightarrow 2)$ represents the preceding term with the indices 1 and 2 interchanged. The amplitude of the fast-time-scale electric field $E_f^\sigma(k)$ is defined as follows:

$$\vec{G}_f(k)=\sum_{\sigma=t,l}E_f^\sigma(k)\vec{e}'^{\,\sigma}_f(k), \tag{9}$$

$$\vec{e}'^{\,\sigma}_f(k)=\left[1-\frac{c\vec{k}\cdot\vec{p}_\alpha}{\varpi p_{0\alpha}}\right]\vec{e}'^{\,\sigma}_f(k)+\left[\frac{c\vec{e}_f^{\,\sigma}(k)\cdot\vec{p}_\alpha}{\varpi p_{0\alpha}}\right]\vec{k} \tag{10}$$

We have the same definition for the slow-time-scale field. In Eq. (6), $\Theta_S^{\sigma_S}(q)$ is the dielectric functions

$$\Theta_s^l(q)=1+\sum_\alpha\sum_{\sigma_s}\frac{\omega_{p\alpha}^2m_\alpha}{\Omega|\vec{q}|}\int\frac{c(\vec{q}\cdot\vec{p}_\alpha)\vec{e}'^{\,\sigma_s}_s\cdot\dfrac{\partial f_{\alpha 0}}{\partial\vec{p}_\alpha}}{p_{0\alpha}\left(\Omega-\dfrac{c\vec{q}\cdot\vec{p}_\alpha}{p_{0\alpha}}\right)}\mathrm{d}^3\vec{p}_\alpha \tag{11}$$

$$\Theta_s^t(q)=1-\frac{c|\vec{q}|^2}{\Omega^2}+\sum_\alpha\sum_{\sigma_s}\frac{\omega_{p\alpha}^2m_\alpha}{\Omega}\int\frac{c\vec{e}_s^{\,t}(q)\cdot\vec{p}_\alpha\vec{e}'^{\,\sigma_s}_s\cdot\dfrac{\partial f_{\alpha 0}}{\partial\vec{p}_\alpha}}{p_{0\alpha}\left(\Omega-\dfrac{c\vec{q}\cdot\vec{p}_\alpha}{p_{0\alpha}}\right)}\mathrm{d}^3\vec{p}_\alpha \tag{12}$$

where $\omega_{p\alpha}$ is the plasma frequwncy. The definition of the notations in Eqs. (6)-(11) can be found in Ref. [19].

For a laser beam with wave number $k_0=2\pi/\lambda_0$ and intensity $I=I_0\exp[-2\frac{x^2+y^2}{[r(z)]^2}-2\frac{(z-z_0)^2}{L^2}]$, where $r(z)$ and L are the transverse and logitudinal dimensions of laser beam, the laser electric field is usually expressed as

$$\vec{E}_L(\vec{x},t)=E_0(\vec{x},t)(\vec{e}_x+\mathrm{i}\varepsilon\vec{e}_y)\exp(\mathrm{i}k_0z-\mathrm{i}\varpi_0t)+c.c.$$

$$E_0(\vec{x},t)=\sqrt{\frac{I_0}{2(1+\alpha^2)}}\exp\left[-\frac{x^2+y^2}{[r(z)]^2}-\frac{(z-z_0)^2}{L^2}\right] \tag{13}$$

where $\varepsilon = 0, \pm 1$ for linearly polarized (LP) and circularly polarized (CP) laser respectively. It should be noted that this $\vec{E}_L(\vec{x},t)$ is not correct because it does not satisfy the Poisson equation $\nabla \cdot \vec{E}_L = 0$. Here we write the field in the form

$$\vec{E}_L(\vec{x}, t) = [E_0(\vec{x}, t)(\vec{e}_x + \mathrm{i}\varepsilon\vec{e}_y) + (E_{zr} + \mathrm{i}E_{zi})\vec{e}_z] \exp(\mathrm{i}k_0 z - \mathrm{i}\varpi_0 t) + c.c. \tag{14}$$

By Poisson equation, we have approximately

$$E_{zr} = -\frac{1}{k_0}\left[\varepsilon\frac{\partial E_0}{\partial y} + \frac{1}{k_0}\frac{\partial^2 E_0}{\partial x \partial z}\right] \tag{15}$$

$$E_{zi} = \frac{1}{k_0}\left[\frac{\partial E_0}{\partial x} - \frac{\varepsilon}{k_0}\frac{\partial^2 E_0}{\partial y \partial z}\right] \tag{16}$$

From Eqs. (13) and (14), considering the expression for the laser electric fields, after a lengthy calculation, we find the spontaneous magnetic field for the approximate region of $v_{ti}|\vec{q}| \ll \Omega \ll v_{te}|\vec{q}|$ ($v_{t\alpha}$ is the thermal velocity of particle.),

$$\vec{B}(\vec{x}, t) = n_e[\xi + 2(1-\xi)\beta_3] (\varepsilon E_0 E_{zr}\vec{e}_x + E_0 E_{zi}\vec{e}_y - \varepsilon E_0^2 \vec{e}_z), \tag{17}$$

and the spontaneous magnetic field under the condition of $v_{te}|\vec{q}| \ll \Omega \ll c|\vec{q}|$,

$$(\xi n_e - \nabla^2)\vec{B}(\vec{x}, t) = 2n_e\left[\frac{1}{2}\xi + (1-\xi)\beta_3\right] \nabla \times [\nabla \times (\varepsilon E_0 E_{zr}\vec{e}_x + E_0 E_{zi}\vec{e}_y - \varepsilon E_0^2\vec{e}_z)]. \tag{18}$$

In Eqs. (17) and (18), the subscript "s" has been omitted for simplicity, and

$$\beta_3 = m_e^2 c^2 \int \frac{1}{p_{0e}^2}\left(\frac{1}{2} - \frac{|p_e|^2}{3p_{0e}^2}\right) f_{eM}(T_{re})\mathrm{d}^3 p_e, \tag{19}$$

and the normalized transform is introduced as

$$t \to t/\varpi_0, \vec{x} \to c\vec{x}/\varpi_0,$$
$$\vec{E} \to \vec{E}\frac{m_e c \varpi_0}{e}, \vec{B} \to \vec{B}\frac{m_e c\varpi_0}{e}. \tag{20}$$

Equation (17) had been obtained in our previous paper. From Eq. (17), we can find that the magnitude of the spontaneous magnetic field is proportional to the plasma density. But Eq. (18) shows that the magnitude of the spontaneous magnetic field is proportional to the plasma density in the case of low plasma density and is independent of he plasma density for the case of relative dense plasma density. It should be noted that Eqs. (17) and (18) are suitable for both LP and CP lasers.

3. Solutions of spontaneous magnetic field equations

In Ref. [20], we calculated the spontaneous magnetic field given by Eq. (17) using the experimental parameters in Ref. [6] and found that the theoretical expectation has a good agreement with the experiment results given in Ref. [6]. We here do not pay more attention to the solution of Eq. (17) and focus our attention to the solution of Eq. (18).

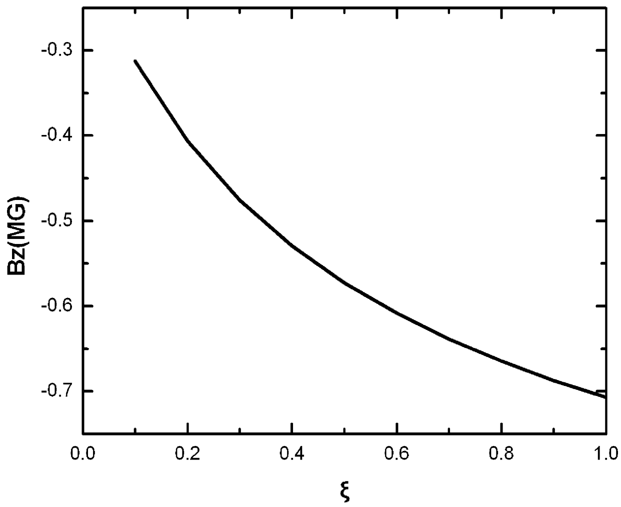

Fig. 1 The dependence of $B_z(x = 0, y = 0, z = 0)$ on the ratio of the number of thermal electron to the total electrons ξ, where CP laser with $\lambda = 1.05$ μm, $I_0 = 6.7 \times 10^{18}$ W/cm^2 is used and the other parameters is given as $r(z) = 5\lambda$, $L = \infty$, plasma density $n_e = 0.028n_c$ and $T_{te} = 5$ keV.

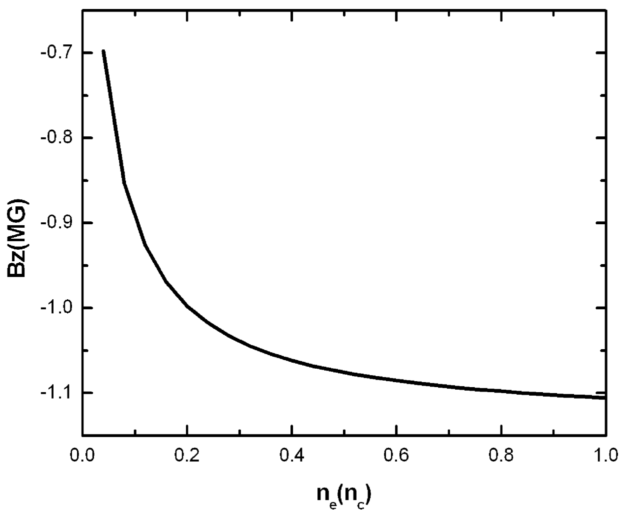

Fig. 2 The dependence of $B_z(x = 0, y = 0, z = 0)$ on the plasma density n_e, where CP laser with $\lambda = 1.05$ μm, $I_0 = 6.7 \times 10^{18}$ W/cm^2 is used and the other parameters is given as $r(z) = 5\lambda$, $L = \infty$, $\xi = 0.6$ and $T_{te} = 5$ keV.

Figure 1 shows the dependence of the magnitude of the spontaneous magnetic field on the ratio of the number of thermal electrons to the total number of electrons ξ, where CP laser with $\lambda = 1.05$ μm, $I_0 = 6.7 \times 10^{18}$ W/cm^2 is used and the other parameters is given as $r(z) = 5\lambda$, $L = \infty$, plasma density $n_e = 0.028n_c$ (n_c is the critical density) and $T_{te} = 5$ keV. The temperature of relativistic electrons T_{re} is determined by Wilks's scaling law $T_{re} = 511(\gamma - 1)$ keV. It can be seen that the absolute magnitude of the spontaneous magnetic field

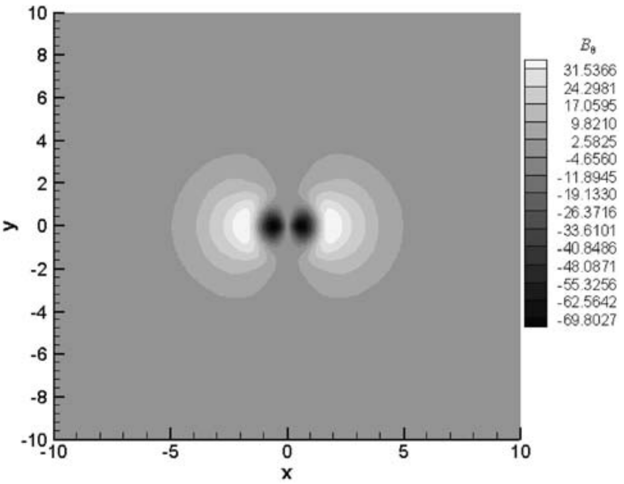

Fig. 3 The spatial profile of $B_\theta(x = 0, y = 0, z = 1.96)$, where LP laser with $\lambda = 1.05$ μm, $I_0 = 4.7 \times 10^{18}$ W/cm^2 is used and the other parameters is given as $r(z) = r_0(1 + a_r \sin(k_r z))/(1 + a_r)$, $r_0 = 4\lambda$, $a_r = 0.5$, $k_r = 12/\lambda$, $L = 5\lambda$, plasma density $n_e = 0.2n_c$, $\xi = 0.6$ and $T_{te} = 5$ keV. The magnitude of magnetic field is in the unit of MG.

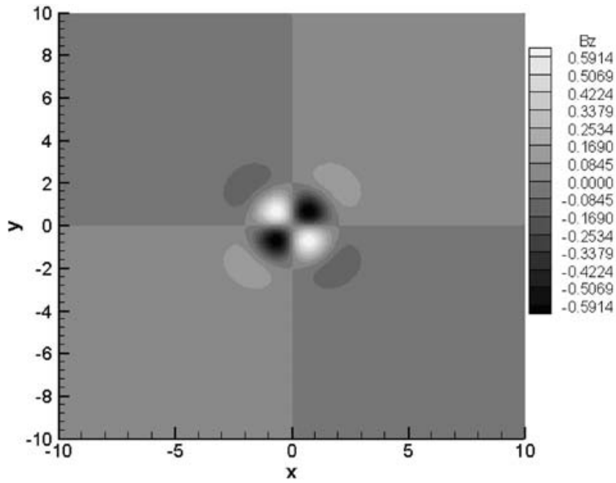

Fig. 4 The spatial profile of $B_z(x = 0, y = 0, z = 0.98)$, where LP laser with $\lambda = 1.05$ μm, $I_0 = 4.7 \times 10^{18}$ W/cm^2 is used and the other parameters is given as $r(z) = r_0(1 + a_r \sin(k_r z))/(1 + a_r)$, $r_0 = 4\lambda$, $a_r = 0.5$, $k_r = 12/\lambda$, $L = 5\lambda$, plasma density $n_e = 0.2n_c$, $\xi = 0.6$ and $T_{te} = 5$ keV. The magnitude of magnetic field is in the unit of MG.

reduces as the number of relativistic electrons increases. Figure 2 shows the dependence of the magnitude of the spontaneous magnetic field on plasma density n_e, where CP laser with $\lambda = 1.05$ μm, $I_0 = 6.7 \times 10^{18}$ W/cm^2 is used and the other parameters is given as $r(z) = 5\lambda$, $L = \infty$, $\xi = 0.6$ and $T_{te} = 5$ keV. It is clear that the magnitude of the spontaneous magnetic field increases as the plasma density grows up and is independent of the plasma density as n_e is greater than about $0.3n_c$.

The spatial profile of the spontaneous magnetic field is shown in Fig. 3 and Fig. 4, where LP laser with $\lambda = 0.5$ μm, $I_0 = 4.7 \times 10^{18}$ W/cm^2 (The parameters for laser beam is same as that given in Fuchs *et al.*'s experiment [5]) is used and the other parameters is given as $r(z) = r_0(1 + a_r \sin(k_r z))/(1 + a_r)$, $r_0 = 4\lambda$, $a_r = 0.5$, $k_r = 12/\lambda$, $L = 5\lambda$, plasma density $n_e = 0.2n_c$, $\xi = 0.6$ and $T_{te} = 5$ keV. The peak magnitude of azimuthal magnetic field B_θ is about 70 MG, which is consistent with experimental result given by Fuchs *et al.* [5]. As shown Fig. 4, the peak magnitude of B_z is about 0.7 MG and much smaller than B_θ. For the case of CP lasers where $\varepsilon \neq 0$, B_z should be enhanced and has the same order of magnitude as B_θ.

4. Summary

The generation mechanism of the spontaneous magnetic fields is reinvestigated from relativistic Vlasov-Maxwell equations. It is found that the azimuthal spontaneous magnetic field B_θ can be same order as the axial spontaneous magnetic field if the laser beam self-focus effect is considered. It is also obtained that the magnitude of the spontaneous magnetic field is proportional to the plasma density for the case of very low plasma density, but is independent of the plasma density for the case of high plasma density.

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