J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 256–259

Chaotic Oscillations via Quasi-Periodicity Caused by Applying External Modulation in Ionization Waves

FUKUYAMA Takao, KOZAKOV Ruslan[1,2], TESTRICH Holger[1,2], WILKE Christian[1,2] and KAWAI Yoshinobu

Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Fukuoka 816-8580, Japan
[1]Institut für Physik, Ernst-Moritz-Arndt-Universität Greifswald, Domstrasse 10a, D-17489 Greifswald, Germany
[2]Max-Planck-Institut für Plasmaphysik, Teilinstitut Greifswald, Wendelsteinstrasse 1, D-17491 Greifswald, Germany

(Received: 9 December 2003 / Accepted: 23 February 2004)

Abstract

Dynamical behavior of nonlinear ionization waves excited in positive columns of glow discharge is investigated with external modulation. It is well known that competition of two frequencies leads the system into a state of chaotic motion via quasi-periodicity as mentioned in Ruelle and Takens scenario [D. Ruelle and F. Takens, Commun. Math. Phys. **20**, 167 (1971)]. Here, quasi-periodic route to chaos is investigated experimentally in ionization waves system. Quasi-periodicity with two frequencies is realized by competition between self-sustained wave and external periodic modulation. According to the change of the rate of discharge current modulation, the system comes to show chaotic oscillations via quasi-periodicity.

Keywords:

chaos, quasi-periodicity, ionization waves in glow discharge

1. Introduction

In dissipative physical systems, such as in plasmas, fluids, chemicals etc., it is often observed that the system settles into a state of sustained chaotic motion [1-5]. Chaos has been observed in various physical systems and several routs to chaos have been identified. Ionization waves in a glow discharge are of interest as a medium for testing the universal characteristics of chaos such as the low-dimensional behavior and certain route to chaos [6]. It is well known that competition of two frequencies leads the system into a state of chaotic motion via quasi-periodicity as mentioned in Ruelle and Takens scenario [7].

Here, quasi-periodic route to chaos is investigated experimentally in ionization waves system.

2. Chaos via quasi-periodicity

There will be a spectrum consisting of one frequency (f_1), then two (f_1 and f_2), and sometimes three frequencies (f_1, f_2 and f_3). As soon as the third frequency arrives, the broad-band noise characteristic of chaos should start to appear. This shows a system with small number of degrees of freedom can engender a chaotic regime. Now it should be attempted to justify the necessity of having a quasi-periodic regime with three frequencies before a strange attractor can appear by this route. Quasi-periodicity is quite a common route to chaos. A stable fixed point turns into a limit cycle via a Hopf bifurcation. The limit cycle goes through a second Hopf bifurcation into a two-frequency torus. Under further changes of the parameter the torus becomes unstable and may either develop into a three-torus via another Hopf bifurcation, or into a chaotic attractor with a dimension slightly larger than two, which still can have some overall structure resembling the torus. In particular, the two frequencies of the limit cycle can dominate the chaotic motion.

The wave property can be mainly controlled by discharge current and pressure, and it can be classified into three types of coherent striations, which are Pomalý wave (P wave), Současný wave (S wave), and Rychlý wave (R wave) according to "Novak constant" [8]. Novak constant N is defined as follow: $N[\mathrm{V}] = E[\mathrm{V/cm}] \times \lambda[\mathrm{cm}]$. It shows 9[V], 20[V] and 13[V] for P wave, S wave and R wave, respectively. Periodic modulation is superimposed to the discharge current, when the system is in R wave regime which shows coherent striation. Quasi-periodicity with two frequencies is realized by competition between self-sustained R wave and external periodic modulation. According to the change of the rate of discharge current modulation, the system comes to show chaotic oscillations via quasi-periodicity.

3. Experimental setup

The experiments are performed using a glass tube with dimensions of 2 cm diameter and 75 cm length, as shown in Fig. 1.

Corresponding author's e-mail: fukuyama@aees.kyushu-u.ac.jp

High direct current voltage is applied to anode and the plasma is produced by a glow discharge between anode and cathode. Unstable ionization waves propagate from cathode to anode, which is "traveling waves". The experiments are done under the condition of electrostatic without magnetic effects. The chamber is evacuated to vacuum, and then neon gas is introduced into the chamber at a pressure of 0.83 Torr. Typical electron temperature T_e is about 10 eV. The current source is used in order to adjust discharge current. Here, discharge current is fixed at 5.4 mA and sinusoidal wave f_1 is superimposed to discharge current as an external force using a function generator. Time series signals for analysis are obtained from the photo diodes and sampled with a digital oscilloscope.

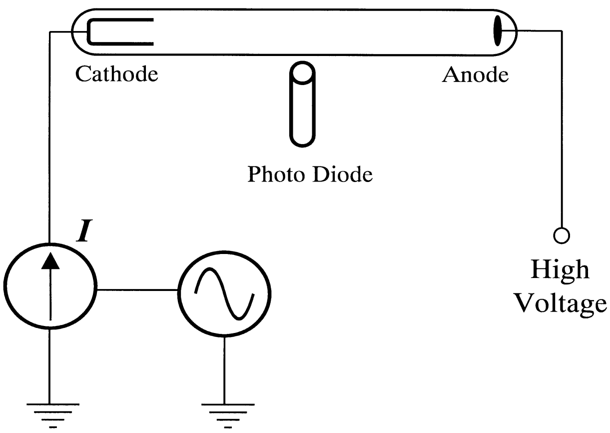


Fig. 1 Schematic diagram of the experimental apparatus.

4. Results and discussions

At firstly, dynamical behavior is studied when modulation frequency f_1 is changed. Experimental conditions are as follows: f_1 is a control parameter, the modulation rate of discharge current $\hat{I}$ = 0.41 %, fundamental frequency of the system f_0 = 20.75 kHz, wave length λ = 3.60 cm, electric field E = 3.49 V/cm, Novak constant $N = E \times \lambda$ = 12.6 V which is corresponding to "R wave" region.

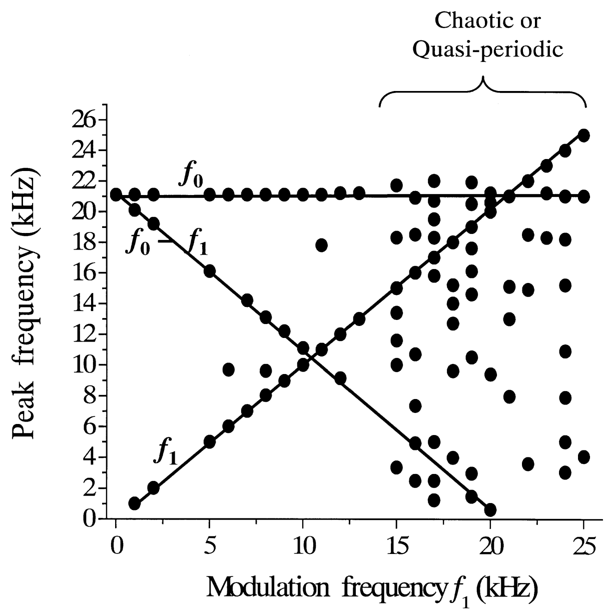


Fig. 2 Dynamical behavior when modulation frequency f_1 is changed.

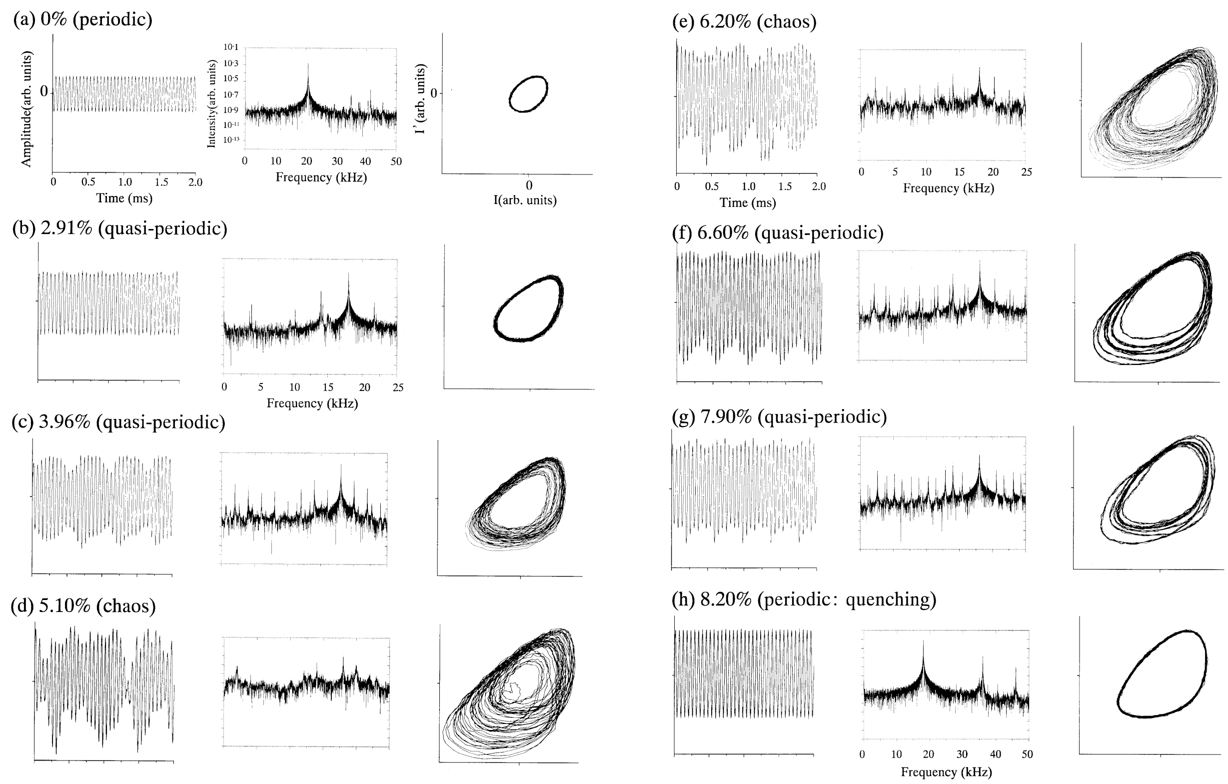
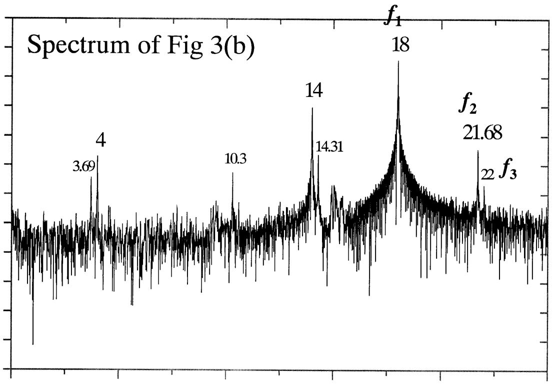
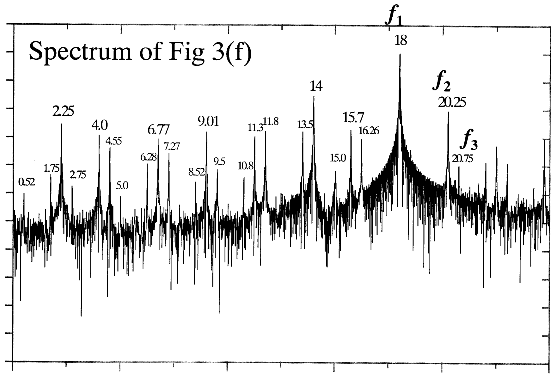


Fig. 3 Dynamical behavior when the modulation rate of discharge current $\hat{I}$ is changed.



f_1 = 18 kHz, f_2 = 21.68 kHz, f_3 = 22 kHz

3.69	=	$f_2 - f_1$
4.00	=	$f_3 - f_1$
10.30	=	$3f_1 - f_2 - f_3$
14.00	=	$2f_1 - f_3$
14.31	=	$2f_1 - f_2$



f_1 = 18 kHz, f_2 = 20.25 kHz, f_3 = 20.75 kHz

0.52 = $f_3 - f_2$	9.01 = $4f_2 - 4f_1$
1.75 = $2f_2 - f_1 - f_3$	9.5 = $3f_2 + f_3 - 4f_1$
2.25 = $f_2 - f_1$	10.8 = $6f_2 - 5f_1 - f_3$
2.75 = $f_3 - f_1$	11.3 = $5f_2 - 5f_1$
4.0 = $8f_3 - 8f_2$	11.8 = $4f_2 + f_3 - 5f_1$
4.55 = $9f_3 - 9f_2$	13.5 = $f_1 + 9f_2 - 9f_3$
5.0 = $f_2 + f_3 - 2f_1$	14.0 = $f_1 + 8f_2 - 8f_3$
6.28 = $4f_2 - 3f_1 - f_3$	15.0 = $3f_2 + 3f_3 - 6f_1$
6.77 = $3f_2 - 3f_1$	15.7 = $2f_1 - f_2$
7.27 = $2f_2 + f_3 - 3f_1$	16.26 = $2f_1 - 2f_2 + f_3$
8.52 = $5f_2 - 4f_1 - f_3$	

Fig. 4 Details of the frequency in Fig. 3(b) and (f). Every peak frequency in a quasi-periodic state can be calculated as a combination of three frequencies f_1, f_2 and f_3

The results are shown in Fig. 2. Peak frequency f_0, f_1, and $f_0 - f_1$ are observed. For $f_1 > 15$ V, many peak frequencies are observed and the system shows quasi-periodic or chaotic state.

On the other hand, dynamical behavior is studied when the modulation rate of discharge current $\hat{I}$ is changed. Experimental conditions are as follows: f_1 is 18 kHz (here, $f_1/f_0 = 0.867\ldots$), the modulation rate of discharge current $\hat{I}$ is a control parameter, fundamental frequency of the system f_0 = 20.75 kHz, wave length λ = 3.60 cm, electric field E = 3.49 V/cm, Novak constant $N = E \times \lambda = 12.6$ V which is corresponding to "R wave" region. The results are shown in Fig. 3. For 0 % $\le \hat{I} \le$ 2.91 %, the system shows a periodic oscillation at self-sustained frequency 20.75 kHz, as shown in Fig. 3(a). For 2.91 % $\le \hat{I} \le$ 3.96 %, the system shows a quasi-periodic oscillation with an interaction between f_0 and f_1, as shown in Fig. 3(b) and (c). For 3.96 % $\le \hat{I} \le$ 5.33 %, the system shows a chaotic oscillation with an interaction between f_0 and f_1, as shown in Fig. 3(d) and (e). Here, chaotic analysis is performed using each time series for Fig. 3(d) and (e) in order to quantitatively evaluate whether two systems are chaotic or not; calculations of the correlation dimension are performed. In the calculation, the method of Grassberger and Procaccia [9] is used. As a result, it is confirmed that systems are chaotic since the value of the correlation dimension is not an integer, which is the characteristic of the chaotic state. For 5.33 % $\le \hat{I} \le$ 8.09 %, the system shows a mode-locked or chaotic oscillation, as shown in Fig. 3(f) and (g). For $\hat{I} \ge$ 8.09 %, the system shifts to the external force f_1 = 18 kHz, namely quenching occurs, as shown in Fig. 3(h). Furthermore, every peak frequency in a quasi-periodic state can be calculated as a combination of three frequencies f_1, f_2 and f_3 as shown in Fig. 4. Thus, quasi-periodic route to chaos has been experimentally observed in ionization waves system.

In low-pressure region (less than 0.75 Torr), the ion acoustic waves have been observed. Therefore, detail studies from the viewpoint of the transition to the ion acoustic waves are considered as future work.

5. Conclusion

Dynamical behavior is studied when modulation frequency f_1 is changed. Peak frequency f_0, f_1, and $f_0 - f_1$ are observed. For $f_1 > 15$ V, many peak frequencies are observed and the system shows quasi-periodic or chaotic state. Moreover, dynamical behavior is also studied when the modulation rate of discharge current $\hat{I}$ is changed. For 0 % $\le \hat{I} \le$ 2.91 %, the system shows a periodic oscillation at self-sustained frequency 20.75 kHz. For 2.91 % $\le \hat{I} \le$ 3.96 %, the system shows a quasi-periodic oscillation with an interaction between f_0 and f_1. For 3.96 % $\le \hat{I} \le$ 5.33 %, the system shows a chaotic oscillation with an interaction between f_0 and f_1. For 5.33 % $\le \hat{I} \le$ 8.09 %, the system shows a mode-locked or chaotic oscillation. For $\hat{I} \ge$ 8.09 %, the system shifts to the external force f_1 = 18 kHz, namely quenching occurs.

Furthermore, every peak frequency in a quasi-periodic state can be calculated as a combination of three frequencies f_1, f_2 and f_3. Thus, quasi-periodic route to chaos has been experimentally observed in ionization waves system.

FUKUYAMA T. is supported by JSPS Postdoctoral Fellowships for Research Abroad (Max-Planck-Institut, Germany) from April, 2004.

References

[1] A. Komori *et al.*, Phys. Fluids B **4**, 3573 (1992).
[2] N. Ohno *et al.*, Phys. Fluids B **5**, 796 (1992).
[3] P.Y. Cheung, S. Donovan and A.Y. Wong, Phys. Rev. Lett. **61**, 1360 (1988).
[4] J. Qin *et al.*, Phys. Rev. Lett. **63**, 163 (1989).
[5] T. Fukuyama, H. Shirahama and Y. Kawai, Phys. Plasmas **9**, 4525 (2002).
[6] K. Ohe, Current Topics in the Phys. Fluids **1**, 319 (1994).
[7] D. Ruelle and F. Takens, Commun. Math. Phys. **20**, 167 (1971).
[8] S. Pfau and A. Rutscher, Beitr. Plasmaphys. 8, (Germany, 1968).
[9] P. Grassberger and I. Procaccia, Physica **D9**, 189 (1983).