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Cross-Field Electron Transport and Dynamo Effect Induced by Unstable Localized Electrostatic Waves in a Magnetized Plasma

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Abstract

Cross-field electron transport and cross-field electric field induced by linearly or nonlinearly unstable localized electrostatic waves in an electron beam-plasma system were investigated theoretically and numerically. It was verified that the parallel and perpendicular electron transport generates a large and abrupt dip of the electron density and a strong peak of the electron temperature in the radial direction. At the same time the cross-field electric field is produced partially by the cross-field transport and partially by the perpendicular pressure gradient.

Keywords:

cross-field electron transport, cross-field electric field, dynamo effect, localized electrostatic waves, pressure gradient

1. Introduction

Cross-field electron transport and cross-field electric field were investigated theoretically and numerically on the basis of the transport equations and magnetohydrodynamic equations [1,2]. They are induced by linearly or nonlinearly unstable localized electrostatic waves in an electron beam-plasma system. The transport equations are derived from the θ-dependent quasilinear velocity-space diffusion equation [1-4]. It was verified that the parallel and perpendicular electron transport generates a large and abrupt dip of the electron density and a strong peak of the electron temperature in the radial direction. At the same time the cross-field electric field is produced partially by the cross-field electron transport and partially by the perpendicular pressure gradient, that is, it is given by $\boldsymbol{E}_\perp = \boldsymbol{B}_0 \times \boldsymbol{v}_{e\perp}/c - (\nabla_\perp p_e)/en_e$ ($p_e = n_e k_B T_e$). The small cross-field electron transport is predicted to create the strong cross-field electric field. On the other hand, the parallel electric field $\boldsymbol{E}_\| = -(\nabla_\| p_e)/en_e + m_e \nu_{en} \mathbf{v}_{e\|}/e$ is created by the parallel transport and pressure gradient. Here, $\boldsymbol{B}_0 = (0, 0, B_0)$ is the external uniform magnetic field in the z direction, $\boldsymbol{v}_{e\perp} = (v_{ex}, v_{ey}, 0)$ is the cross-field drift velocity of plasma electrons, $\boldsymbol{v}_{e\|} = (0, 0, v_{ez})$ is the parallel drift velocity of plasma electrons, c is the light velocity, e is the electron charge, n_e is the electron density, T_e is the electron temperature, k_B is the Boltzmann constant and ν_{en} is the electron-neutral collision frequency. The radial and axial profiles of the electron density and temperature and the electrical potential were obtained numerically and they can explain well the experimental observation in an electron beam-plasma system qualitatively and quantitatively [5,6].

2. Transport equations

We consider the electron transport arising from the θ-dependent quasilinear velocity-space diffusion due to spatially localized unstable electrostatic waves propagating almost perpendicularly to the magnetic field. The transport equations and magnetohydrodynamic equations for plasma electrons are given by

$$\frac{\partial U_e}{\partial t} = -2\gamma_{\boldsymbol{k}}^{(e)} U_{\boldsymbol{k}} , \tag{1}$$

$$\frac{\partial \boldsymbol{P}_e}{\partial t} = -\frac{2\gamma_{\boldsymbol{k}}^{(e)} \boldsymbol{k}}{\omega_{\boldsymbol{k}}} U_{\boldsymbol{k}} - en_e \boldsymbol{E}_\| - \nabla_\| p_e - m_e n_e \nu_{en} \boldsymbol{v}_{e\|} , \tag{2}$$

$$n_e \boldsymbol{v}_{e\|} - \mu_e n_e \boldsymbol{E}_\| - \nabla_\| (D_e n_e) = 0 , \tag{3}$$

$$-en_e \boldsymbol{E}_\perp - \frac{e}{c} n_e \boldsymbol{v}_{e\perp} \times \boldsymbol{B}_0 - \nabla_\perp p_e = 0 , \tag{4}$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma_e = 0 , \tag{5}$$

$$\Gamma_e = n_e \boldsymbol{v}_e - \mu_e n_e \boldsymbol{E}_\| - \nabla_\| \left(D_e n_e + \frac{1}{\nu_{en}} n_e v_{ez}^2 \right) , \tag{6}$$

$$\nabla \cdot \boldsymbol{E} = 0 , \tag{7}$$

where $U_{\boldsymbol{k}} = \frac{1}{8\pi} (\partial(\varepsilon_{\boldsymbol{k}}' \omega_{\boldsymbol{k}})/\partial\omega_{\boldsymbol{k}}) |E_{\boldsymbol{k}}|^2$ is the wave energy density, $\boldsymbol{k} U_{\boldsymbol{k}}/\omega_{\boldsymbol{k}}$ is the wave momentum density, $U_e = \frac{1}{2} m_e n_e v_e^2 + \frac{3}{2} m_e k_B T_e$ and $\boldsymbol{P}_e = m_e n_e \mathbf{v}_e$ are the energy and momentum

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densities of plasma electrons, $\gamma_{\boldsymbol{k}}^{(e)}$ is the linear Landau and cyclotron damping rate ascribed to plasma electrons, $\mu_e = e/m_e\nu_{en}$ is the electron mobility, $D_e = k_BT_e/m_e\nu_{en}$ is the electron diffusion coefficient, $\varepsilon_{\boldsymbol{k}} = \varepsilon'_{\boldsymbol{k}} + i\varepsilon''_{\boldsymbol{k}}$ is the dielectric constant, $E_{\boldsymbol{k}}$ is the magnitude of the wave electric field, $\omega_{\boldsymbol{k}}$ is the wave frequency and $\boldsymbol{k} = (k_\perp, 0, k_\parallel)$ is the wave number. In (6), the mobility and diffusion of plasma electrons across the magnetic field is neglected because of $\nu_{en}/\omega_{ce} \ll 1$ (ω_{ce} is the electron cyclotron frequency). On the other hand, (3) means that the diffusion along the magnetic field is the ambipolar diffusion. According to (3), the electron flux Γ_e given by (6) is reduced to $\Gamma_e = n_e\boldsymbol{v}_{e\perp} = (n_ev_{ex}, n_ev_{ey}, 0)$ under the condition of $n_ev_{ez}^2/\nu_{en} \ll D_en_e$, and the transport equation (2) becomes

$$\frac{\partial \boldsymbol{P}_e}{\partial t} = -\frac{2\gamma_{\boldsymbol{k}}^{(e)}\boldsymbol{k}}{\omega_{\boldsymbol{k}}}U_{\boldsymbol{k}} - 2m_en_e\nu_{en}\boldsymbol{v}_{e\parallel}. \tag{8}$$

Equations (1) and (2) predict that the electrostatic waves create strong electron transport or acceleration along and across the magnetic field. The generated Ohm's law of (4) shows that the cross-field electric field $\boldsymbol{E}_\perp$ is produced by the dynamo effect of the cross-field electron drift and the radial pressure gradient induced by the electron transport.

3. Numerical analysis

The temporal and spatial development of the transport of plasma electrons has been investigated numerically assuming that the electrostatic waves excited in an electron beam-plasma system are localized radially such that $U_{\boldsymbol{k}} \propto \exp[-(x^2 + y^2)/a^2]$, and the background plasma is spatially uniform initially. The electrostatic waves are assumed to be governed by the following kinetic wave equations:

$$\frac{\partial U_{\boldsymbol{k}}}{\partial t} = 2\gamma_N U_{\boldsymbol{k}} \ , \tag{9}$$

$$\frac{\partial U_{\boldsymbol{k}}}{\partial z} = 2\gamma_{sN} U_{\boldsymbol{k}} \ , \tag{10}$$

where γ_N and γ_{sN} are the temporal and spatial growth rates resulting from the electron beam, respectively. The numerical analysis of Eqs. (1)-(10) was performed under the parameters of $\omega_{\boldsymbol{k}}/\omega_{ce} = 0.3$, $k_\perp v_{te0}/\omega_{ce} = 1$, $k_\perp/k_\parallel = 0.2$, $|\gamma_{\boldsymbol{k}}^{(e)}|/\omega_{ce} = 0.2$, $\gamma_N/|\gamma_{\boldsymbol{k}}^{(e)}| = 1$, $\gamma_{sN}v_{te0}/|\gamma_{\boldsymbol{k}}^{(e)}| = 0.2$, $\nu_{en}/|\gamma_{\boldsymbol{k}}^{(e)}| = 1$, $a\omega_{ce}/v_{te0} = 2$, and $U_{\boldsymbol{k}}(0)/n_{e0}k_BT_{e0} = 0.6\times10^{-3}$. Thus the temporal evolution of the three-dimensional profiles of the system was obtained. We show only the transverse profiles in the x-direction for the fixed value of $y/a = -0.4$. Figure 1 exhibits the axial evolution of transverse profiles of the normalized electron density, that is, n_e/n_{e0} versus x/a and z/z_e is shown as a parameter of $|\gamma_{\boldsymbol{k}}^{(e)}|t$ = 0.4, 0.6, 0.8 and 0.9 for the *x-z* profiles of (a), (b), (c) and (d), respectively, where $z_e = v_{te0}/|\gamma_{\boldsymbol{k}}^{(e)}|$ and $n_{e0} = n_e(0)$. Figure 2 is the axial evolution of the transverse profiles of the normalized electron temperature, that is, T_e/T_{e0} versus x/a and z/z_e is shown under the same parameters of $|\gamma_{\boldsymbol{k}}^{(e)}|t$, where $T_{e0} = T_e(0)$. It is found that the hollow profile of the electron density and the peaked profiles of the electron temperature develop temporally and axially. Simultaneously the cross-field electric field (not shown) is generated. This means that the cross-field drift of plasma electrons (not shown) induced by the spatially localized electrostatic waves produce the abrupt inhomogeneity of plasma profile in the radial direction and the strong cross-field electric field. The transport equations (1) and (2) govern the electron temperature and the drift of plasma electrons (v_{ex} and v_{ez}). The generalized Ohm's law (4) demonstrate that E_y is determined by v_{ex} and $\nabla_y p_e$. The axial electric field E_z is given by (3) and the electric fields in *y* and *z* directions produce the electric field in *x* direction so as to satisfy the Poisson's equation (7). Thus E_x and $\nabla_x p_e$ create the cross-field drift v_{ey} via $\boldsymbol{E} \times \boldsymbol{B}$ drift and diamagnetic drift.

The typical parameters of the experiment reported by the author *et al.* [5,6] are T_{e0} = 5 eV, $B_0 = 7 \times 10^{-3}$ Tesla and $a = 1.5 \times 10^{-3}$ m (beam radius), and hence $\omega_{ce}a/v_{te0} \approx 2$ is given. From the numerical analysis shown partially in Figs. 1-2, we find $\mathbf{v}_e/v_{te0} \approx (0.5, 0.5, 0.05)$ and $e\boldsymbol{E}a/k_BT_{e0} \approx (1, 1.5, 0.01)$ at $|\gamma_{\boldsymbol{k}}^{(e)}|t = 0.9$ and $z/z_e = 9.0$. Thus we find $\boldsymbol{E}a \approx (5, 7.5, 0.05)$ volt, $Ea \approx 9.0$ volt, $E_x \approx 3.3 \times 10^3$ volt/m, $E_y \approx 5 \times 10^3$ volt/m and $E_z \approx 33$ volt/m. This value is very close to the experimental value of $V_{exp} \approx 10$ volt. Also the drift velocities v_{ex}, $v_{ey} \approx 6.6 \times 10^5$ m/s (1.3 eV), $v_{ez} \approx 6.6 \times 10^4$ m/s (0.013 eV) are reasonably obtained.

4. Conclusion

It was verified theoretically and numerically that the parallel and perpendicular electron transport induced by localized unstable electrostatic waves generates large and abrupt inhomogeneities of the electron density and temperature in the radial direction. Simultaneously the strong cross-field electric field is produced by the cross-field electron transport and the pressure gradient. The obtained results can well explain the experimental observation in an electron beam-plasma system.

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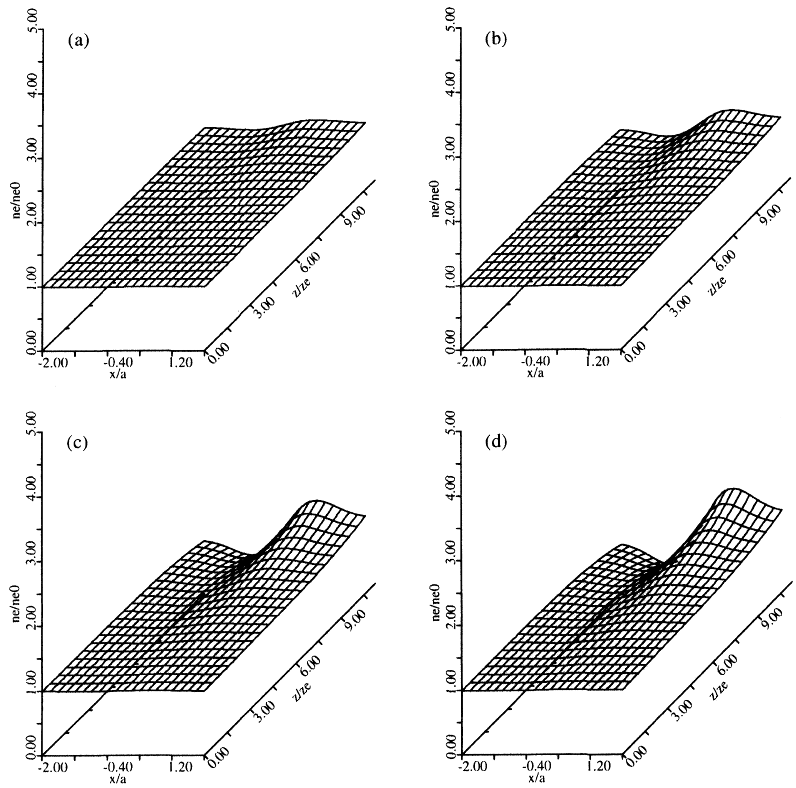

Fig. 1 Axial evolution of the transverse profiles of the electron density. n_e/n_{e0} is shown versus x/a and z/z_e as a parameter of $|\gamma_k^{(e)}|\,t$ = 0.4 for (a), 0.6 for (b), 0.8 for (c) and 0.9 for (d).

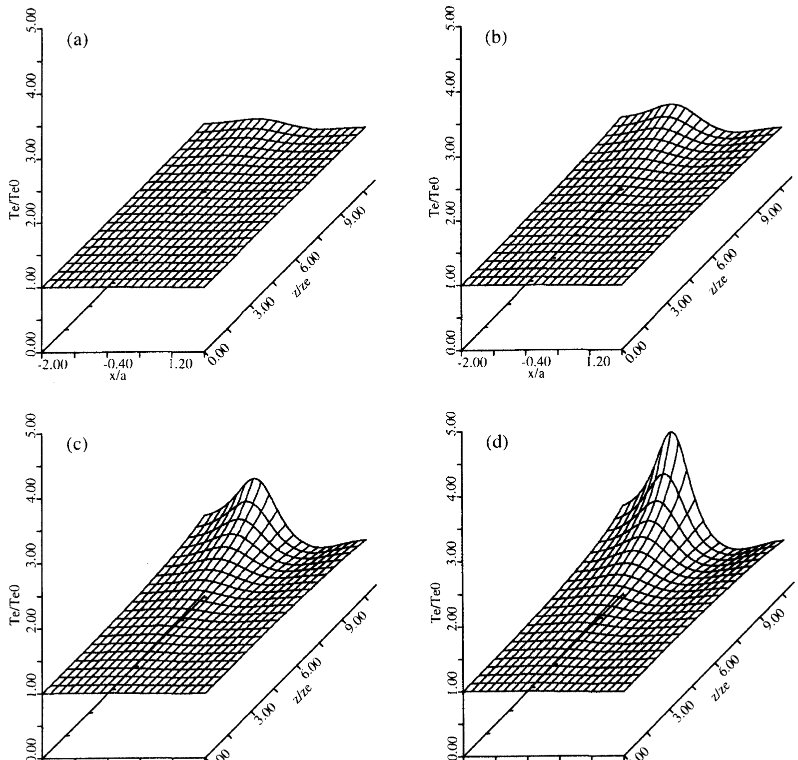

Fig. 2 Axial evolution of the transverse profiles of the electron temperature. T_e/T_{e0} is shown versus x/a and z/z_e as a parameter of $|\gamma_k^{(e)}|\,t$ = 0.4 for (a), 0.6 for (b), 0.8 for (c) and 0.9 for (d).