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Turbulence Spectrum and Transport Scaling

VLAD Madalina, SPINEANU Florin, MISGUICH Jacques[1], REUSS Jean-Daniel[1], BALESCU Radu[2], ITOH Kimitaka[3] and ITOH Sanae[4]

National Institute for Laser, Plasma and Radiation Physics,
Association Euratom-MEC, P.O.Box MG-36, Magurele, Bucharest, Romania
[1]Association Euratom-CEA sur la Fusion, CEA-Cadarache, 13108 Saint-Paul-Lez-Durance, France
[2]Association Euratom-Etat Belge sur la Fusion, Université Libre de Bruxelles, Belgium
[3]National Institute for Fusion Science, Toki 509-5292, Japan
[4]Research Institute for Applied Mechanics, Kyushu University, Kasuga 816-8580, Japan

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Abstract

The effect of the shape of the spectrum of an electrostatic turbulence on the scaling of the diffusion coefficient is studied using the decorrelation trajectory method. We show that a strong influence appears at large Kubo numbers due to trajectory trapping.

Keywords:

plasma turbulence, statistical approach, test particle transport

1. Introduction

The amplitudes and spectra of plasma turbulence are measured in a very large number of experiments on tokamak devices. These experimental data can be used in a test particle approach to determine the statistical properties of the trajectories and in particular the transport coefficients. For slowly varying or large amplitude turbulence a process of dynamical trapping of the trajectories around the extrema of the stochastic potential appears and strongly influences the transport. Important progresses in the study of this nonlinear process were recently obtained. New statistical methods were developed [1,2] that permitted to determine the asymptotic diffusion coefficient and also the Lagrangian correlation of particle velocity and the time dependent diffusion coefficient. It was shown that the trapping process determines the decrease of the diffusion coefficient and the change of its scaling in the parameters of the stochastic field.

The aim of the present study is to determine the effect of the turbulence spectrum on the diffusion coefficient. It is already known that in the quasilinear case the shape of the spectrum does not influence the asymptotic diffusion coefficient D, which is determined only by the average wave length and frequency. We show that in a turbulence with slow time variation or large amplitude the shape of the spectrum has a strong influence on the diffusion coefficient. This is a nonlinear effect produced by trajectory trapping.

2. The model and the statistical method

We consider a constant confining magnetic field directed along z axis $\boldsymbol{B}_0 = B_0\boldsymbol{e}_z$ (slab geometry) and an electrostatic turbulence represented by an electrostatic potential $\phi^e(\boldsymbol{x},t)$, where $\boldsymbol{x} \equiv (x_1,x_2)$ are the Cartesian coordinates in the plane perpendicular to $\boldsymbol{B}_0$. The test particle approach of the turbulent transport relies on the following Langevin equation:

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{v}\,[\boldsymbol{x}(t),t], \qquad \boldsymbol{x}(0) = \boldsymbol{0} \tag{1}$$

where $\boldsymbol{x}(t)$ represents the trajectory of the particle guiding center. The velocity $\boldsymbol{v}(\boldsymbol{x},z,t)$ is the $\boldsymbol{E}\times\boldsymbol{B}$ drift

$$\boldsymbol{v}(\boldsymbol{x},t) \equiv -\nabla\phi(\boldsymbol{x},t)\times\boldsymbol{e}_z \tag{2}$$

where ∇ is the gradient in the (x_1,x_2) plane and $\phi(\boldsymbol{x},t) = \phi^e(\boldsymbol{x},t)/B_0$ is the electrostatic potential normalized with B_0. The electrostatic potential $\phi(\boldsymbol{x},t)$ is considered to be a stationary and homogeneous Gaussian stochastic field, with zero average. Such a stochastic field is completely determined by the two-point Eulerian correlation function (EC), $E(\boldsymbol{x},t)$ or equivalently by the spectrum $S(\boldsymbol{k},\omega)$. These two quantities are related by the Fourier transform

$$S(\boldsymbol{k},\omega) = \frac{1}{(2\pi)^{3/2}}\iint \mathrm{d}\boldsymbol{x}\,\mathrm{d}t\,E(\boldsymbol{x},t)\exp(-i\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t). \tag{3}$$

The EC of the stochastic potential is defined by

$$E(\boldsymbol{x},t) \equiv \left\langle \phi(\boldsymbol{x}',t')\,\phi(\boldsymbol{x}'+\boldsymbol{x},t'+t)\right\rangle. \tag{4}$$

The average $\langle\ldots\rangle$ is the statistical average over the realizations of $\phi(\boldsymbol{x},t)$, as is usually considered in theoretical studies although it is actually experimentally obtained as a space and time average over $\boldsymbol{x}'$ and t' of the measured potential.

Corresponding author's e-mail: madi@ifin.nipne.ro

The statistical properties of the velocity components are completely determined by those of the potential; they are stationary and homogeneous Gaussian stochastic fields like $\phi(\boldsymbol{x},t)$. The two-point Eulerian correlations of the velocity components, $E_{ij}(\boldsymbol{x},t) \equiv \langle v_i(\boldsymbol{x}',t')v_j(\boldsymbol{x}'+\boldsymbol{x}',t'+t)\rangle$, are

$$E_{ij}(\boldsymbol{x},t) = -\varepsilon_{in}\,\varepsilon_{jm}\,\frac{\partial^2 E(\boldsymbol{x},t)}{\partial x_n \partial x_m}, \qquad (5)$$

where $\varepsilon_{12} = -\varepsilon_{21} = 1$, $\varepsilon_{11} = \varepsilon_{22} = 0$. The EC of the velocity (5) evidences three parameters that characterize the (isotropic) stochastic velocity field: the amplitude $V = \sqrt{E_{11}(\mathbf{0},0)}$, the correlation time τ_c and the correlation length λ_c, which are the characteristic decay time and distance of this function. These three parameters combine in a dimensionless Kubo number

$$K = \frac{V\tau_c}{\lambda_c} = \frac{\tau_c}{\tau_{fl}} \qquad (6)$$

which is the ratio of τ_c to the average time of flight of the particles over the correlation length, $\tau_{fl} = \lambda_c/V$.

Starting from this statistical description of the stochastic potential, we will determine the Lagrangian velocity correlation (LVC), defined by:

$$L_{ij}(t) \equiv \left\langle v_i\,[\boldsymbol{x}(0),0]\,v_j\,[\boldsymbol{x}(t),t]\right\rangle. \qquad (7)$$

The mean square displacement and the running diffusion coefficient are determined by this function:

$$\left\langle x_i^2(t)\right\rangle = 2\int_0^t \mathrm{d}\tau\; L_{ii}(\tau)\,(t-\tau), \qquad (8)$$

$$D_i(t) = \int_0^t \mathrm{d}\tau\; L_{ii}(\tau), \qquad (9)$$

provided that the process is stationary.

For small Kubo numbers (quasilinear regime) corresponding to fast varying turbulence such that $\tau_c \ll \tau_{fl}$, the results are well established: the diffusion coefficient does not depend on the shape of the correlation and it is $D_{ql} = (\lambda_c^2/\tau_c)K^2$. For $K > 1$ ($\tau_c > \tau_{fl}$) the time variation of the stochastic potential is slow and the trajectories approximately follow the contour lines of $\phi(\boldsymbol{x},t)$. This produces a trapping effect: the trajectories are confined for long periods in small regions. A typical trajectory shows an alternation of large displacements and trapping events. The latter appear when the particles are close to the maxima or minima of the potential and consist of trajectory winding on almost closed small size paths. The large displacements are produced when the trajectories are at small absolute values of the potential. The most important effect of trajectory trapping consists of decreasing the diffusion coefficient and of changing its dependence on the Kubo number from the Bohm scaling [$D_B \sim (\lambda_c^2/\tau_c)K$] to a trapping scaling [$D_{tr} \sim (\lambda_c^2/\tau_c)K^\gamma$] with $\gamma < 1$. Simple heuristic arguments can be used to show that the value of γ has to be in the range [0,1]. The lower limit $\gamma = 0$ corresponds to total trapping and the upper limit $\gamma = 1$ correspond to the absence of trapping.

The first estimation of γ is based on an analogy with the percolation in stochastic landscapes [3] and yields $\gamma = 0.7$. This value appears as a critical exponent valid for any EC of the potential that decays fast enough when $\boldsymbol{x} \to \infty$. We show here that the shape of the EC can strongly influence the diffusion and in particular the value of the exponent γ. We use a new statistical approach, the decorrelation trajectory method, which is a semi-analytical approach developed in [1] and generalized to more complicated systems. In a recent work [2] the results of this method concerning the LVC were validated by developing a more accurate approach.

The main idea in this method is to study the Langevin equation (1) in subensembles (S) of realizations of the stochastic field, which are determined by given values of the potential and of the velocity in the starting point of the trajectories. The potential and the velocity considered in such a subensemble are Gaussian but non-stationary and non-homogeneous, with space and time dependent averages. This subensemble average Eulerian velocity determines an average motion (the decorrelation trajectory) which is estimated by neglecting the fluctuations of the trajectories around the average trajectory. This approximation is analyzed in [1] and [2] where it is shown that it has the important property of maintaining the invariance of the average Lagrangian potential in the case of static turbulence. The trapping process is essentially related with this invariance property of the evolution equation (1). The decorrelation trajectory method eventually yields a set of trajectories with a weighting factor for each one. It shows that the statistical properties of the complicated stochastic trajectories can be determined by performing averages over these very simple and smooth trajectories. They are determined from equations of motion that have the same Hamiltonian structure as Eq. (1) but with a simple Hamiltonian function determined by the EC of the potential.

A fast and precise code for determining the time dependent diffusion coefficient for given EC of the stochastic potential was developed [2]. It can be used as a tool in the analysis of the experimental data.

3. Results

We consider the following model for the Eulerian correlation of the potential

$$\varepsilon(|\boldsymbol{x}|) = c_1 \exp\left(-\frac{|\boldsymbol{x}|^2}{2c_0}\right) + \frac{c_2}{\left(1+|\boldsymbol{x}|^2\right)^\alpha} \qquad (10)$$

where c_0, c_1, c_2 and α are parameters. This correlation is normalized such that the amplitude of the velocity fluctuations $V^2 = -E_{11}(0) = 1$, which determines one of parameters, $c_2 = (1 - c_1/c_0)/2\alpha$. This model of the EC permits to consider several typical types of correlations. The first term is a localized space-function while the second is an extended function with algebraic behavior at large distance. The variation of the parameter c_1 between c_0 and 0 determines a continuous change of the shape of this EC from the pure Gaussian (at $c_1 = c_0$), to a Gaussian with a tail, then to an

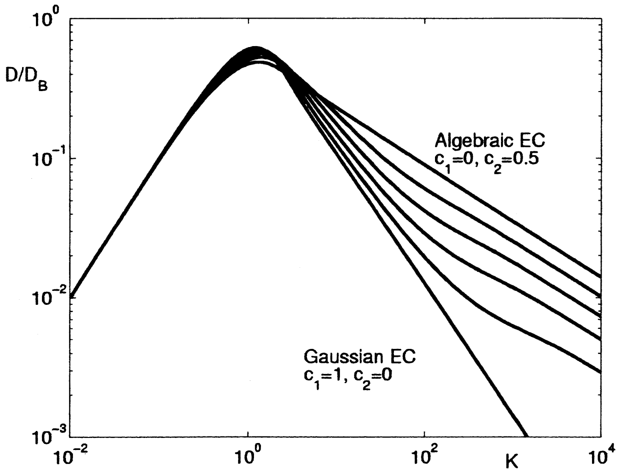

Fig. 1 The normalized diffusion coefficient $F(K) = D(K)/D_B$ obtained for the EC (10) with (c_1, c_2) having the values (0, 0.5), (0.2, 0.4), (0.4, 0.3), (0.6, 0.2), (0.8, 0.1), (1, 0), from the upper to the lower curve; $\alpha = 1$ and $c_0 = 1$.

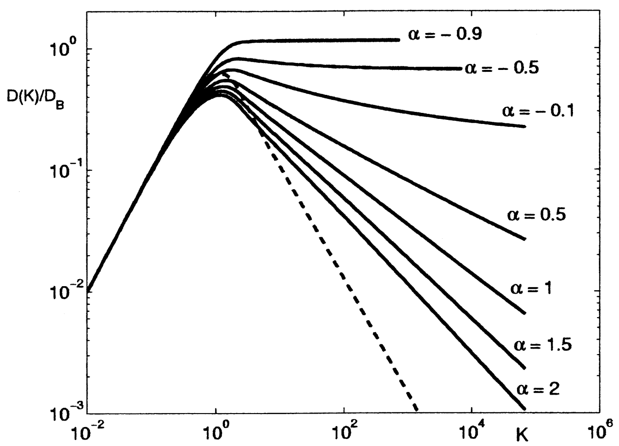

Fig. 2 The normalized diffusion coefficient obtained for an algebraic EC [$c_1 = 0$ in Eq. (10)] with different values of α. Dashed line corresponds to Gaussian EC.

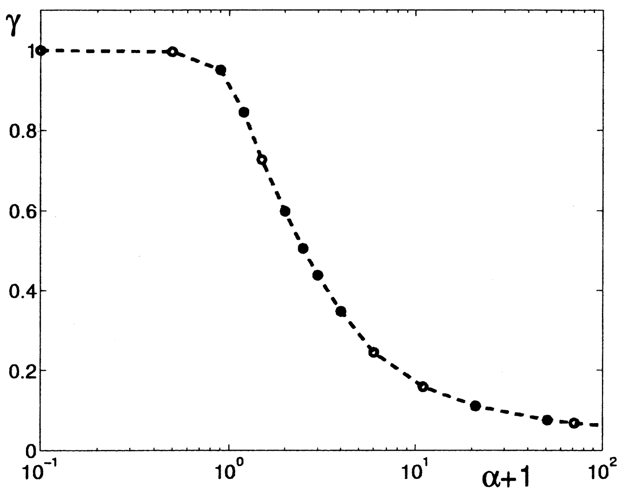

Fig. 3 The exponent γ in the scaling law of the diffusion coefficient obtained for an algebraic EC [$c_1 = 0$ in Eq. (10)] as a function of α.

algebraic function with a perturbation around zero, which decays and disappear at $c_1 = 0$. The parameter α determines the rate of decay of the EC. Its domain is $\alpha > -1$ and it defines two types of EC: short-range correlations for $\alpha > 0$ and long-range correlations for $-1 < \alpha < 0$. In the first case the EC decays to zero at large distances and in the second case the EC goes to minus infinity. This is possible because actually the Eulerian correlation of the velocity has physical significance and this correlation decays to zero in both cases. In the first case the EC of the velocity is positive around zero but at larger distances it has a region with negative values. This shows that the stochastic potential is characterized by finite size contour lines. In the second case the EC of the velocity is everywhere positive and decays monotonously to zero. This corresponds to a potential with statistically important open contour lines.

The effect of trapping is represented by the function $F(K) = D(K)/D_B$, which appears in the solution obtained with the decorrelation trajectory method [1] and is the asymptotic diffusion coefficient normalized with the Bohm diffusion coefficient. This function obtained for EC's (10) with different values of c_1 is plotted in Fig. 1 as a function of K. One can see that the scaling of this function is of power law type, $F \sim 1/K^{\beta}$, when the EC contains only one of the two terms. The exponents obtained for these two cases are different: $\beta = 0.96$ for the Gaussian and $\beta = 0.4$ for the algebraic EC with $\alpha = 1$. When the two terms in the EC (10) are present the scaling is more complicated and depends on the value of c_1. At very large K an exponent close to $\beta = 0.4$ can be observed while at K of the order 10 the exponent is closer to $\beta = 0.96$. A large transition appears between these values. The shape of the EC also influences the constant that multiplies the K dependent factor (significative distances appears between the curves). In the quasilinear regime $K < 1$, the same result is obtained for all cases which shows that the shape of the EC does not influence the diffusion coefficient.

The function $F(K)$ is represented in Fig. 2 for different values of the parameter α in the EC (10) and for $c_1 = 0$ (without the Gaussian term). One can see that the decay rate of the EC with the distance strongly influences the diffusion coefficient. The function scales as $F(K) \sim 1/K^{\beta}$ and β increases when α increases. The trapping is stronger when the EC decays rapidly (for large α) and as α increases the function $F(K)$ tends to the curve obtained for the Gaussian EC (dashed line). The diffusion coefficient obtained for the Gaussian EC is $D \sim \lambda_c^2/\tau_c K^{0.04}$, thus it practically represents the limit of total trapping $D_t \sim \lambda_c^2/\tau_c$. For negative values of α, β is small and it goes to zero when $\alpha \to -1$. In this limit $F(K)$ saturates at $K > 1$ which shows that the trapping is not present. Thus the Bohm scaling of the diffusion coefficient appears for stochastic potentials with long-range correlations.

The exponent γ of the scaling of the diffusion coefficient ($D \sim \lambda_c^2/\tau_c K^{\gamma}$) is obtained from β as $\gamma = 1 - \beta$. Its dependence on α, the decay rate of the EC, is presented in Fig. 3. We have thus obtained a continuous variation of γ with α, from the value $\gamma \cong 1$ for $-1 < \alpha < 0$ to an asymptotic value which is $\gamma = 0$. At negative value of α the trapping is practically absent and the transport is of Bohm type and when the EC is localized ($\alpha \to \infty$) the trapping is total.

4. Conclusions

We have examined the effect of the shape of the Eulerian correlation on the scaling law of the diffusion coefficient. We have obtained analytical expressions for the time dependent diffusion coefficient corresponding to given Eulerian correlation of the stochastic potential, using the decorrelation trajectory method. We have shown that in the nonlinear regime $K > 1$, the scaling law of the diffusion coefficient depends on the shape of the EC and not only on the global parameters of the EC that define the Kubo number. A strong influence has the space-dependence of the EC at large distances, i.e. the small k components of the spectrum. This effect is determined by the complicated process of dynamic trajectory trapping in the structure of the stochastic field. We have obtained in the case of Gaussian and algebraic EC's power law scalings in K for the diffusion coefficients. For the latter case, we have determined the exponent γ of the diffusion coefficient as function of the exponent α which describes the EC of the potential, Eq. (10). It is not a fixed value as in the estimation based on percolation theory [3] but a continuous function that decays from 1 to 0. For more complicated EC of the stochastic potential, the scaling of the diffusion coefficient is not of power law type.

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