J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 241–244

The Effect of the Aspect Ratio on the External Kink-Ballooning Instability in High-β Tokamaks

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(Received: 9 December 2003 / Accepted: 8 March 2004)

Abstract

A theory for the stability analysis of ideal external modes is developed with the property of the Newcomb equation. This theory is also useful for stabilizing resistive wall modes by feedback controls. As an application of this theory, the effect of the aspect ratio on the ideal magnetohydrodynamic (MHD) stability limits of tokamak plasmas are examined numerically. The toroidal mode number n = 1 external modes are stabilized by making the aspect ratio smaller.

Keywords:

newcomb equation, ideal MHD stability, external mode, resistive wall mode, tokamak

1. Introduction

Ideal kink-ballooning instability limits the performance of tokamaks, and resistive wall modes (RWMs) [1] should be stabilized or be controlled in order to realize stationary high-performance tokamaks. RWMs are ideal kink-ballooning modes that are stable when the plasma is surrounded by a perfect conducting wall, but become unstable due to the resistivity of a wall.

We developed a theory where the change of a potential energy δW_p for external modes in a toroidal system is expressed in terms of the boundary values by using the property of the Newcomb equation [2]. We apply this theory to analyze the stability of external kink-ballooning modes in high-β tokamaks. Emphasis is put on the aspect ratio dependence of a kink-ballooning stability, which attracts attention recently in the design research on high-performance tokamaks.

We first describe the analytical method with the property of the Newcomb equation briefly. Next, we show a benchmark test of MARG2D code [3] comparing with ERATOJ code [4]. The effect of the aspect ratio on the stability of ideal external modes is investigated in Sec. **4**.

2. External mode theory with the property of the Newcomb equation

We analyze the stability of ideal external modes according to the quadratic form;

$$\delta K + \delta W_p + \delta W_V = 0 \tag{1}$$

where δK, δW_p and δW_V are the change of a kinetic energy, a plasma potential energy and a vacuum energy, respectively.

The Newcomb equation is the inertia free linear ideal magnetohydrodynamic (MHD) equation. In the case that the plasma displacement ξ satisfies the Newcomb equation, which means that an equilibrium is marginally stable, δW_p is expressed by the boundary values of an equilibrium. On the basis of this theory, δW_p + δW_V can be expressed by the stability matrix [5], A, as

$$\begin{aligned} \delta W_p[\xi,\xi] + \delta W_V[\xi,\xi] &= \langle \boldsymbol{x} \,|\, \boldsymbol{M}_p \,|\, \boldsymbol{x} \rangle + \langle \boldsymbol{x} \,|\, \boldsymbol{M}_V \,|\, \boldsymbol{x} \rangle \\ &= \langle \boldsymbol{x} \,|\, \boldsymbol{A} \,|\, \boldsymbol{x} \rangle \\ \boldsymbol{x} &= (x_{-Lf}, \ldots, x_{Lf}) \\ \xi &= \sum_m x_m \boldsymbol{Y}^m(r) \end{aligned} \tag{2}$$

where Lf is the maximum poloidal mode number to analyze the stability, $\boldsymbol{Y}^m(r)$ are the basis functions satisfying the Newcomb equation, and $\boldsymbol{M}_p$ and $\boldsymbol{M}_V$ are the matrices for expressing δW_p and δW_V by the displacement at a plasma surface $\boldsymbol{x}$. MARG2D code constructs $\boldsymbol{Y}^m(r)$ by solving the Newcomb equation, and obtains the matrix A. We analyze the stability of ideal external modes by investigating the sign of the minimum eigenvalue of A. This theory is explained minutely in Ref. [3].

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This analysis with the stability matrix has advantages for stabilizing resistive wall modes (RWMs) by feedback controls [5]. One reason is that we can analyze even when external modes are stable. The other reason is that this analysis is executed in short time, because $\boldsymbol{M}_p$ does not change approximately when an equilibrium is stable against ideal internal modes.

3. Benchmark test of MARG2D code

We first compare the result obtained with the stability matrix, calculated by MARG2D code, with that of ERATOJ code as a benchmark test. The equilibrium has a circular cross-section and the poloidal beta β_p is very small, as shown in Fig. 1. The plasma pressure and the safety factor are expressed as p and q, and s is defined as $s = \sqrt{\psi}$, where ψ is the poloidal flux function normalized to unity at a plasma surface. In this equilibrium, Only ideal external kink modes are unstable.

Figure 2 shows the dependence of the stability of $n = 1$ external kink modes on the safety factor at a plasma surface q_a, where n is the toroidal mode number. The minimum eigenvalues obtained with MARG2D and ERATOJ are expressed as μ_0 and $\lambda_{0\text{-}ERT}$, respectively. An equilibrium with arbitrary value of q_a is obtained by σ-scaling [3]. The marginally stable point obtained with the stability matrix well agree with that calculated with ERATOJ.

We also compare the eigenfunctions near the marginally stable point, $q = 2.9$. Both eigenfunctions obtained with MARG2D and ERATOJ, shown in Fig. 3, have the profiles peaking near a plasma surface. It is a well-known feature of ideal external kink modes in low-β tokamaks [6,7].

From these comparisons, the analysis with the stability matrix is effective for examining the stability of ideal external modes.

4. Effect of the aspect ratio on the stability of external modes

As an application of the analysis with the stability matrix, we analyze the effect of the aspect ratio on the stability of $n = 1$ external modes with the stability matrix. The shape parameters of equilibria are fixed as the elongation $\kappa = 1.8$, the triangulity $\delta = 0.45$. The profiles of $dp/d\psi$ are also fixed. The toroidal magnetic field at the magnetic axis, B_t, and the poloidal field current, I_p, are changed as

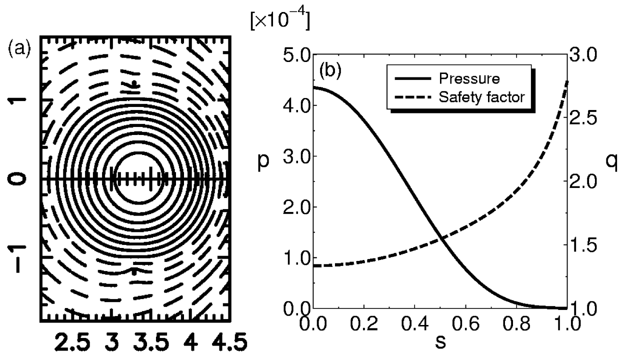

Fig. 1 Equilibrium for the benchmark test. (a) Cross-section. (b) Profiles of the pressure and the safety factor. s is defined as $\sqrt{\psi}$.

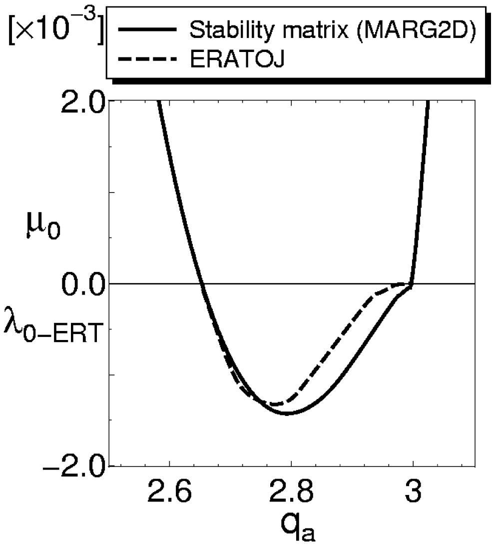

Fig. 2 Dependence of the stability of ideal external kink modes on q_a. μ_0 and $\lambda_{0\text{-}ERT}$ is the minimum eigenvalue obtained with MARG2D and ERATOJ, respectively.

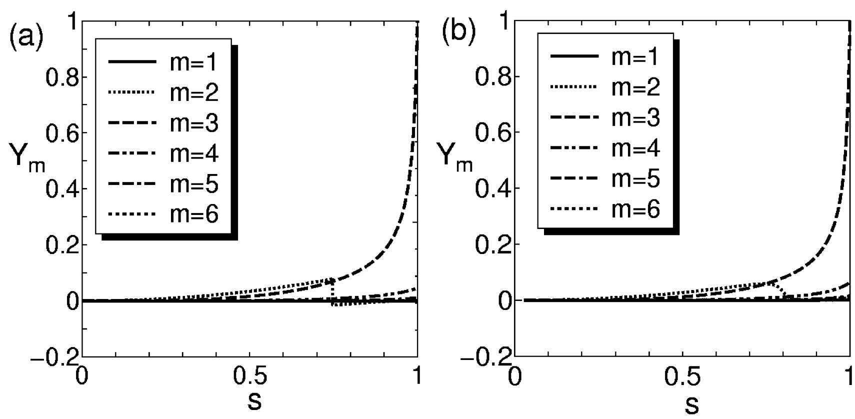

Fig. 3 Eigenfunctions of ideal external kink modes at $q_a = 2.9$. (a) MARG2D code. (b) ERATOJ code.

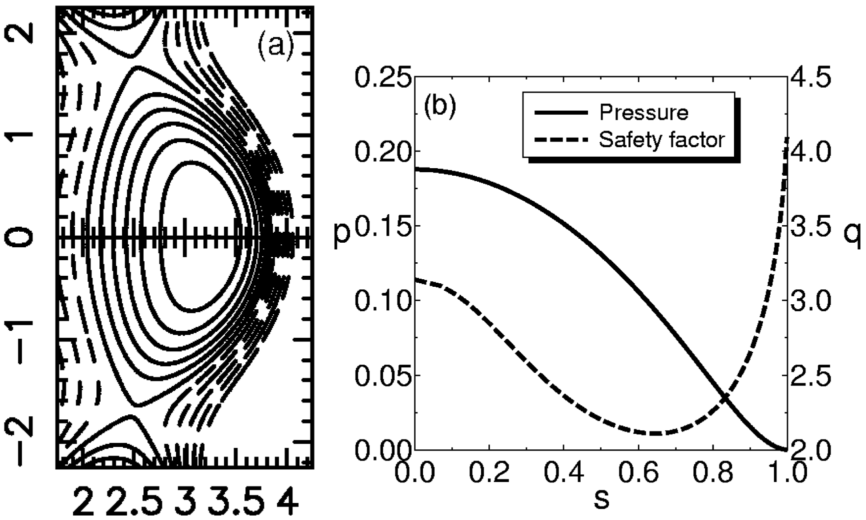

Fig. 4 Equilibrium for analyzing the effect of aspect ratio. Parameters are A = 3.26 and β_N = 6.0. (a) Cross-section. (b) Profiles of the pressure and the safety factor.

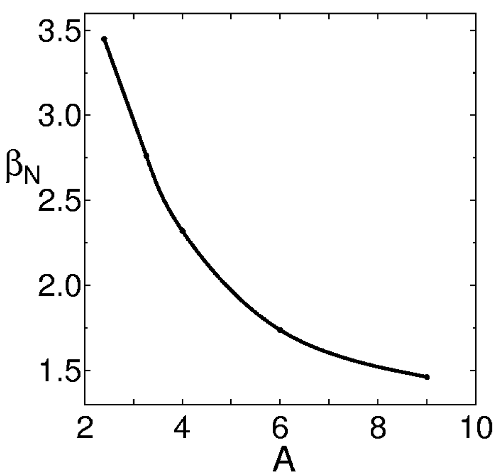

Fig. 5 Dependence of the β_N limit against n = 1 ideal external kink modes on the aspect ratio. No perfect conducting wall surrounds the equilibrium.

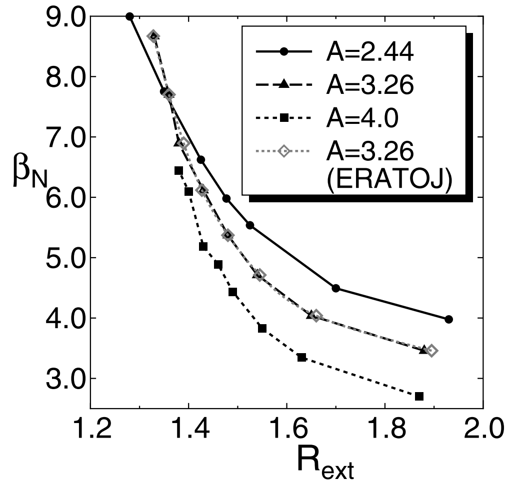

Fig. 6 Dependence of the β_N limit against n = 1 ideal external kink modes on the position of a perfect conducting wall.

$$B_t = B_{t0} \times \frac{R_c}{R_{c0}}, \tag{3}$$

$$I_p \simeq I_{p0} \times \frac{R_{c0}}{R_c}, \tag{4}$$

where R_c is the major radius and the subscript 0 means the values when the aspect ratio A = 3.26, as B_{t0} = 3.36 and R_{c0} = 2.93. I_{p0} of the normalized beta β_N = 5.0 equilibrium is set to 4.0. Though the q profile varies when A and β_N are changed, q value at s = 0.95 is fixed as q_{95} = 3.5 by adjusting I_p. The profiles of the A = 3.26 equilibrium are shown in Fig. 4.

We first investigate the dependence of the β_N limit on the aspect ratio against n = 1 ideal external modes without a perfect conducting wall. As shown in Fig. 5, a low aspect ratio equilibrium is more stable against n = 1 external modes.

Figure 6 shows the dependence of the β_N limit against n = 1 ideal external modes on the position of a perfect conducting wall. The position of a conducting wall R_{ext} is normalized with a. When β_N < 7.5, a small aspect ratio equilibrium is more stable than a large aspect ratio one as in a no wall case.

The eigenfunctions, when β_N = 6.0 and the wall position is slightly more far from a plasma surface than the marginally stable position, are shown in Fig. 7. In A = 3.26 case, the growth rate calculated with ERATOJ is $\gamma^2 = -1.6 \times 1.0^{-5}$; that is normalized with the toroidal Alfvén transit time at the magnetic axis. These eigenfunctions have a global mode structures unlike in a low-β case shown in Fig. 3.

The m = 3 Fourier components obtained with MARG2D in each aspect ratio cases are pointed near the outer q = 3.0 rational surface. However, since the widths of these points are nearly 0.001 measured in s, a general structure of the

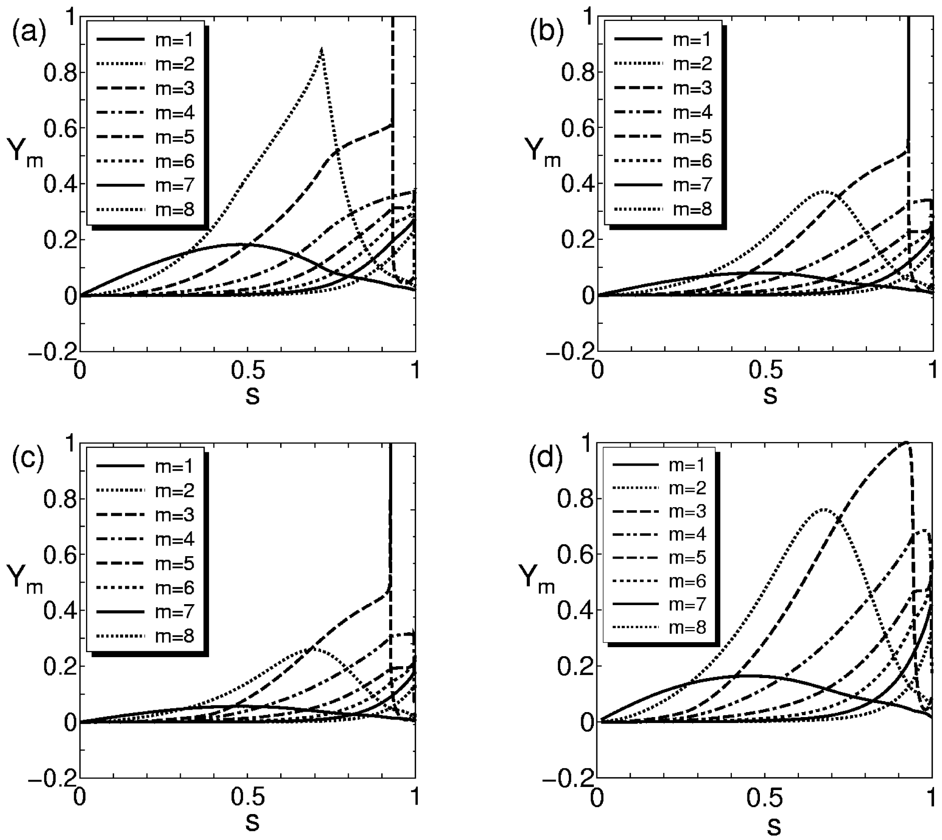

Fig. 7 Eigenfunctions when β_N = 6.0. (a) A = 2.44 obtained with MARG2D code. (b) A = 3.26 with MARG2D. (c) A = 4.0 with MARG2D. (d) A = 3.26 obtained with ERATOJ code.

eigenfunction is almost the same as that obtained with ERATOJ.

5. Summary

We developed a theory that the change of a potential energy of a plasma is expressed in terms of the boundary values when the plasma displacement satisfies the Newcomb equation. On the basis of this theory, we are able to analyze the stability of ideal external modes numerically with the stability matrix. This analysis has advantages for stabilizing resistive wall modes by feedback controls, because this analysis can be executed even when external modes are stable; and since the matrix expressing the change of a plasma potential energy does not change approximately in the case that ideal internal modes are stable, we can analyze the stability of external modes in short time.

We executed a benchmark test of the analysis with the stability matrix, computed with MARG2D code, by comparing that with ERATOJ code, and confirmed that the result obtained with MARG2D well agree with that examined by ERATOJ.

As an application of MARG2D, we analyzed the effect of the aspect ratio on the stability of external modes. From this result, external modes are stabilized when the aspect ratio is made small, and the eigenfunction of the most unstable external mode in high-β tokamaks has a global mode structure. MARG2D is effective for analyzing the stability of ideal external modes, and for the design research on high-performance tokamaks.

Acknowledgements

The authors express their gratitude to Dr. H. Ninomiya, Dr. Y. Kishimoto for supporting our research. The authors also thank Dr. Y. Kamada and Dr. G. Kurita for helpful discussions and suggestions.

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