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Effect of Magnetic Axis Shift on Neoclassical Transport in Helical Torus

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Abstract

Neoclassical transport for large helical device (LHD) configurations is studied by solving the bounce-averaged Fokker-Planck equation. Numerical code employed in the present paper (CHD1) is much faster and more efficient than existing transport codes. Effects of the magnetic axis shift on the mono-energetic transport coefficients are studied in detail for the LHD configurations, revealing that a strong inward shift of the magnetic axis can reduce remarkably the neoclassical ripple transport.

Keywords:

neoclassical transport, bounce averaged kinetic equation, magnetic axis shift, LHD

1. Introduction

During more than two decades, a number of analytical and numerical approaches have been developed to study the neoclassical transport in toroidal helical systems. Analytical descriptions of neoclassical transport are effective because they give clear physical insight associated with various transport and/or loss mechanisms. However, the derivation of the most of these analytical formulae are originally based on a number of assumptions such as so-called frequency ordering, which leads somewhat disconnected expressions for the transport coefficients in different collisionality regimes. On the other hand, lots of numerical methods have been discussed to avoid some of these assumptions and to determine the transport coefficients over the wide range of collisionality regime. Among these studies there are the Monte-Calro simulations [1,2], the methods based on the bounce-averaged Fokker-Planck equation, namely, FPSTEL [3], FLOCS [4], and numerical code by solving the drift kinetic equation, DKES [5]. A general solution of the ripple-averaged kinetic equation; GSRAKE is also developed [6]. The GSRAKE makes it possible to analyze the cases including the contributions due to $E \times B$ and ∇B drifts without the assumption, $\Omega_{E\times B} \gg \Omega_{\nabla B}$.

An analytic representation based on the longitudinal adiabatic invariant for the general magnetic configuration instead of a simple model magnetic field frequently employed, has been presented in a convenient form for the purpose of numerical calculation in realistic magnetic configurations [7]. In the previous studies [8,9], a neoclassical transport in a helical torus based on the bounce averaged Fokker-Planck equation has been developed by using the generalized bounce-averaged orbit theory mentioned above. The reduction of neoclassical transport, particularly in the low collisionality region is one of the most important issues for any reactor scale heliotron devices. In the previous paper [10], the effect of radial electric field on neoclassical transport has also been analyzed in detail when a boundary layer and/or resonance are present. Furthermore, great efforts have been undertaken to optimize magnetic configurations associated with the reduction of neoclassical transport [7,11]. In this paper, we study the neoclassical transport in typical configurations such as inward-shifted configuration of LHD device by solving the bounce averaged Fokker-Planck equation (CHD1). In the present study the magnetic field are calculated by using the MAGN code for fixed coil currents. Then, the transport coefficients are evaluated for a realistic magnetic field relevant to the operation parameters of the LHD experiment.

The bounce-averaged Fokker-Planck equation together with the numerical code (CHD1) is presented in Section 2. In section 3, the numerical results of the transport for the LHD configuration are presented. The last section is devoted to the summary and discussions.

2. Bounce-averaged kinetic equation

Considerable efforts have been paid on the bounce-averaged drift kinetic equation to describe the neoclassical particle and energy transport in toroidal helical systems. Since both fast and slow drift motions are involved in the toroidal helical systems, some time average is frequently employed in the formalism. The time average is usually performed over the periodic bounce motion of particle trapped within local

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ripples of helical magnetic field. Recently, the definition of time average is extended to so-called "ripple average" in which the average is taken over a characteristic time for particles to traverse a local ripple regardless of whether a particles bounces or not. To discuss this formalism, we here employ the Boozer's coordinates (ψ, ϑ, ϕ), ψ being the toroidal flux. We consider the helical torus with toroidal period N and the rotational transform per period is assumed to be small, $\iota/N \ll 1$. Namely, ϑ and all functions of this variable are considered constant with respect to the time average and the average is carried out along ϕ. The motion of charged particles in case of this assumption is described by the longitudinal adiabatic invariant

$$J(E,\mu,\psi,\vartheta)=\frac{1}{2\pi}\oint P_\phi \mathrm{d}\phi,\ P_\phi = mv_{//}\frac{B_\phi}{B}+e\psi_\phi. \quad (1)$$

The explicit adiabatic invariant forms for ripple trapped and passing particles are given in Refs. [7,10]. Hence, we can write the averaged kinetic equation in the form

$$\frac{1}{e}\frac{\partial J}{\partial \vartheta}\frac{\partial f}{\partial \psi}-\frac{1}{e}\frac{\partial J}{\partial \psi}\frac{\partial f}{\partial \vartheta}+\sigma e V_\phi \frac{\partial f}{\partial E}$$
$$=\overline{C}(f,f)\equiv\oint C(f,f)\frac{B}{v_{//}}\mathrm{d}\phi, \quad (2)$$

where $\overline{C}$ is the bounce averaged collision operator. Here, the contribution from the toroidal loop voltage V_ϕ is also included in Eq. (2) and $\sigma = \pm 1$ corresponds to the sign of the parallel velocity for passing particles.

We now put $f = f_0(K, \psi)\,[1 + h(K, \lambda, \sigma, \psi, \vartheta)]$, where K is the kinetic energy, μ is the magnetic moment and f_0 is the local Maxwellian with density $n(\psi)$ and temperature $T(\psi)$, and introduce the even part h^+ and the odd part h^- of h with respect to the sign of parallel velocity, where $h^+ \equiv [h(\sigma = 1) + h(\sigma = -1)]/2$ and $h^- \equiv [h(\sigma = 1) - h(\sigma = -1)]/2$. Then, the linearized equation can be written in the following form

$$Ch^+ + Hh^- + Dh^+ = S^+,$$
$$Ch^- + Hh^+ + Dh^- = S^-,\ (\lambda<\lambda_c) \quad (3)$$

$$Ch^+ + Dh^+ = S^+,\ \ h^- = 0,\ \ (\lambda>\lambda_c), \quad (4)$$

with

$$S^+ = \frac{1}{e}\frac{\partial J}{\partial \vartheta}\frac{\partial \ln f_0}{\partial \psi},\ S^- = \frac{eV_\phi}{T},$$

$$C=\nu(v)\frac{\partial}{\partial\lambda}\left(\lambda M\frac{\partial}{\partial\lambda}\right),\ M=\oint\frac{mv_{//}B_\phi}{B}\mathrm{d}\phi, \quad (5)$$

$$H=\iota\frac{\partial}{\partial\vartheta},\ D=\omega_E\left\langle\frac{\partial J}{\partial K}\right\rangle\frac{\partial}{\partial\vartheta},\ \omega_E=\frac{\mathrm{d}\Phi_E}{\mathrm{d}\psi}. \quad (6)$$

Here, only the pitch angle scattering term, which is dominant particularly in the low collisionality region is retained in the collision operator. Also, the ϑ-dependent part of $\partial J/\partial K$ is kept in the D operator for the sake of the conservation law. The numerical scheme to solve Eqs. (3) and (4) has been discussed in the previous paper [7]. After solving Eqs. (3) and (4), the transport coefficients are represented by the following relations,

$$D_{11}\propto(S^{-c},h^{-c}),\ D_{13}\propto(S^{+S},h^{+c}),\ (a,b)=\int ab\,\mathrm{d}\lambda, \quad (7)$$

where $h^{\pm c}(\vartheta) = (h^\pm(\vartheta) + h^\pm(-\vartheta))/2$ and $h^{\pm S}(\vartheta) = (h^\pm(\vartheta) - h^\pm(-\vartheta))/2$. We note that D_{11} and D_{13} correspond to the particle transport coefficient and the bootstrap current, respectively.

3. Effect of magnetic axis shift

In the previous paper [7], the dependence of the particle confinement on the magnetic axis shift, the shaping of magnetic surface and the pitch modulation of helical windings has been discussed for a LHD geometry. It is recently shown in [2] that the diffusion coefficients of the inward shifted configuration are about three to ten times smaller than those of standard configuration of LHD. In this paper, we used a magnetic field calculated by the MAGN code for the LHD parameters. We here consider three configurations with inwardly shifted magnetic axis from the center characterized by Δ(= 0 cm, –10 cm, –20 cm). The radial profiles of the Fourier spectrum of magnetic field in the magnetic coordinate are shown in Fig. 1 for typical three configurations (0 cm, –10 cm, –20 cm). As was shown in Fig. 1, the magnetic field components such as $\varepsilon_h^{\,l}(\rho)$, $\varepsilon_t(\rho)$, $\varepsilon_h^{\,l-1}(\rho)$, $\varepsilon_h^{\,l+1}(\rho)$ and $\varepsilon_h^{\,l-2}(\rho)$ change the radial profiles by shifting the magnetic axis inwardly. In particular, the components with (1,10) and (3,10) change both its strength and polarity, and it may reduce effectively the helical ripple strength, leading a significant reduction of neoclassical transport. Typical results for the collisionality dependence of the diffusion coefficient (D_{11}) and the bootstrap current (D_{13}) in the case of the LHD three configurations with inwardly shifted magnetic axis are plotted in Fig. 2 (a) and Fig. 2 (b) for fixed value of normalized electric field, $R\omega_E/v$. The parameters used in the calculations are $\rho(= r/a) = 0.5$, $B_0 = 3$ T, $\iota = 0.48$, $\varepsilon_t = 0.074$, and $\varepsilon_h = 0.059$, and $R\omega_E/v = 1.0 \times 10^{-3}$. Here, we note that the results for the case of $\Delta = -20$ cm as shown in Fig. 2 corresponds to the results for $R\omega_E/v = 1.0 \times 10^{-3}$ in Fig. 1 of Ref. [10]. It is found from Fig. 2 (a) that the diffusion coefficient decreases as the magnetic axis is shifted inwardly. The result for $\Delta = -20$ cm is smaller about 4 times than the result for $\Delta = 0$ cm in the $1/\upsilon$ region but the difference between these three cases becomes smaller in the further low collisionality regime, so called υ-regime. As was discussed in Refs. [2] and [10], the effect of radial electric field dominates the transport more than the effect of magnetic axis shift provided the electric field is large enough. Also, the local maximum of the diffusion coefficient is shifted toward lower collisionality regime as Δ increased. Figure 2 (b) indicates that the bootstrap current does not monotonically increases as the collision frequency decreases and it saturates with the local maximum at some collision frequency (at the transition). This transition point sensitively depends on the electric field strength and it is also shifted toward low collisionality region as Δ increased. The bootstrap current increases in the plateau and $1/\nu$ regions but decreases in the low collisionality regime so called ν-regime as Δ increased. It should be noted that the peaked profile of

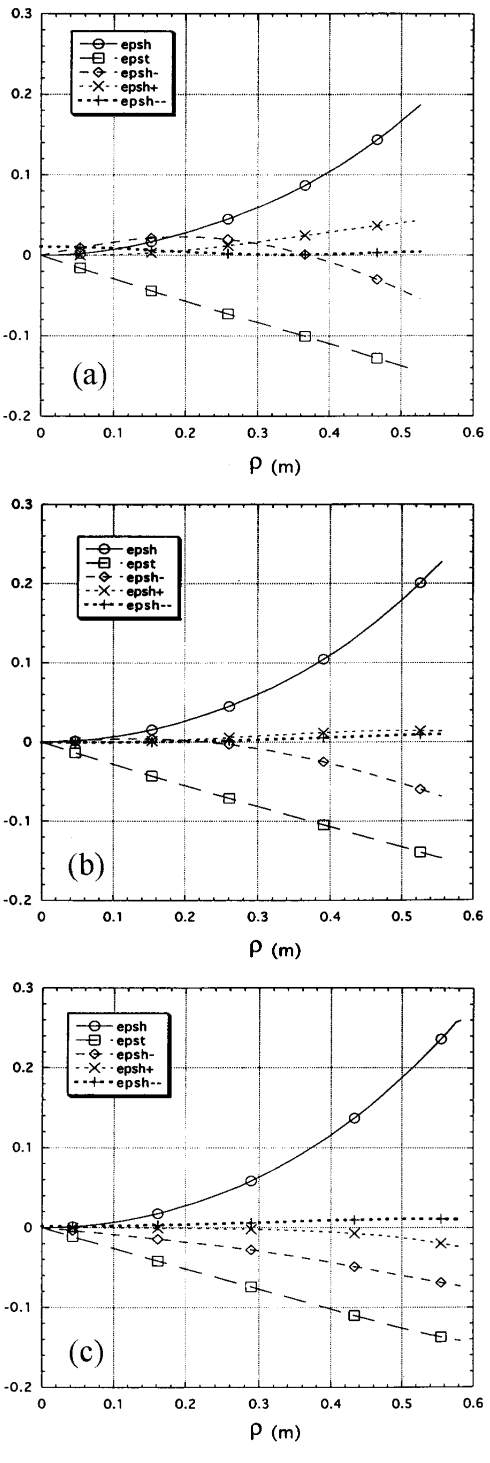

Fig. 1 Radial profiles of magnetic field spectra in vacuum configurations. (a) Δ = 0 cm , (b) –10 cm, and (c) –20 cm. Here, epsh, epst, epsh-, epsh+ and epsh-- indicate $\varepsilon_h(\rho)$, $\varepsilon_t(\rho)$, $\varepsilon_h^{l-1}(\rho)$, $\varepsilon_h^{l+1}(\rho)$ and $\varepsilon_h^{l-2}(\rho)$, respectively.

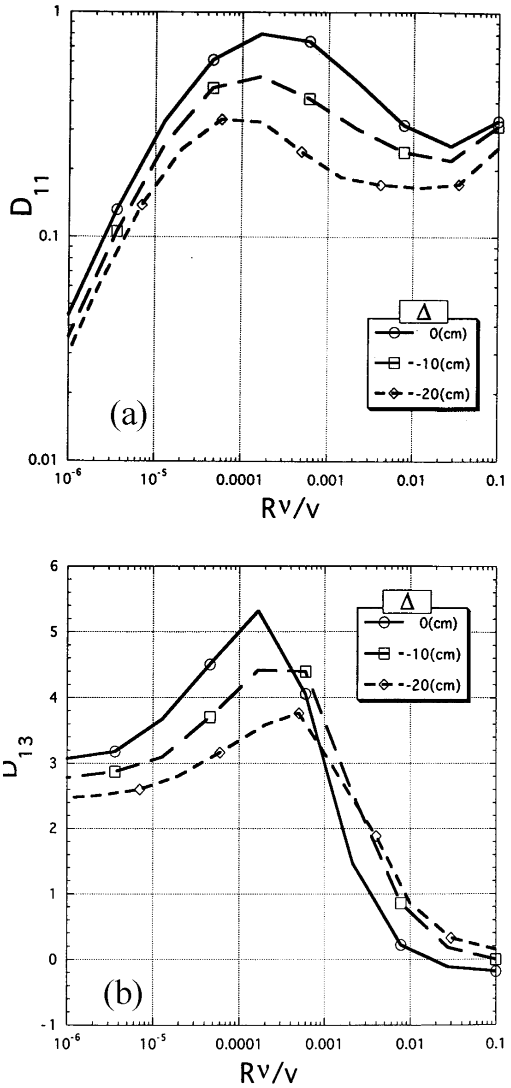

Fig. 2 Collisionality dependence of the diffusion coefficient (D_{11}) [2(a)] and the bootstrap current (D_{13}), [2(b)] for Δ = 0 cm, –10 cm and –20 cm.

D_{13} around the maximum as indicated by the solid line in Fig. 2(b) contains somewhat ambiguity because there are no explicit data around the transition point in this figure.

4. Summary and discussions

The theory and numerical code solving the bounce averaged Fokker-Planck equation has been developed in two dimensional space (λ, ϑ). As for the numerical scheme, the function in the ϑ-direction is expressed in terms of Fourier expansion, and the meshes with variable distances are concentrated in the boundary layer. So, the structure of the solution near the boundary layer and the case of helical and/or toroidal resonances can be well analyzed as shown in Ref. [10]. A symmetric band matrix solver is used. Then, the computation time is dramatically reduced, particularly, in the low collisionality region compared with the case of DKES code. On the basis of this improvement of the numerical scheme, the calculation can be extendable over wide range of collision frequency, especially in the very low collisionality region as shown in Fig. 2, compared with the previous studies [2,11]. Although the relation $\Omega_{E\times B} \gg \Omega_{\nabla B}$ is assumed in the DKES code, there is no such limitation in CHD1 code, which is discussed, in the present study. Detailed and realistic discussions associated with the effects of the magnetic axis shift and the electric field on the transport of the LHD configuration in low collisionality region will be discussed in a separate paper. In finite beta plasmas, the deformation of the magnetic surface due to the Pfirsch-Schluter current may degradate the particle confinement. Discussions on this problem await further investigations.

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