J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 199–202

The Effect of Reduced Pfirsch-Schlüter Current on the Ideal MHD Stability and Alpha Particle Confinement in 2-period Compact Configurations

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(Received: 9 December 2003 / Accepted: 1 April 2004)

Abstract

The CHS-qa 2b32 [1] configuration has been considered as a starting point for the further numerical optimisation towards small Pfirsch-Schlüter current. The configurations obtained have about 4 times smaller PS current and are stable with respect to Mercier modes for β values around 0.085. However, the α-particle confinement and the Magneto-Hydro-Dynamic (MHD) stability with respect to the ballooning modes appear to be more difficult issues to improve in the configurations obtained. In particular, the maximum value of β, stable with respect to the ballooning modes, has been obtained on the level of 0.02. Detailed calculations of the configurations computed with the 3D fixed boundary ideal equilibrium code VMEC2000 [2], with the 3D ideal MHD stability code TERPSICHORE [3], which reconstruct the equilibrium state in Boozer magnetic coordinates and with the MCT [4] code, which follows collisionless mono-energetic α-particles drift orbits, are presented.

Keywords:

stellarators, quasi-axisymmetry, 3D codes, CHS-qa

1. Introduction

Quasi-axisymmetric helical configurations have axially symmetric magnetic field in the Boozer magnetic coordinate system [5], therefore tokamak-like neoclassical transport properties can be expected. Because helical systems need no inductive current, a quasi-axisymmetric stellarator has essentially no problem with respect to steady-state operation. Such type of quasi-symmetry, quasi-axisymmetry [6], constitutes the basis of new stellarator projects, NCSX [7] and CHS-qa [8].

CHS-qa is a 2-period quasi-axisymmetric compact concept with a low aspect ratios of A = 3.2 – 3.9 which is currently under design and optimisation at the National Institute for Fusion Science (NIFS, Japan). During last few years, several candidates of magnetic configuration for CHS-qa with the different optimised features have been proposed. The first engineering design was made for the 2w39 configuration with an aspect ratio A = 3.9 and a very weakly increasing rotational transform profile approaching 0.4 at the plasma edge. The β value of this configuration is limited below 0.02 where a large Shafranov shift accompanied with a drop of central rotational transform appears.

In accordance with VMEC2000 calculations, the next candidate 2b32 with A = 3.36 has a larger initial (at the beginning of the period) elongation E_0 = 5.08, while the rotational transform is designed to be limited between 1/3 and 2/5. The Shafranov shift is reduced by about 30% compared with the 2w39 configuration, the drop of the central rotational transform is also suppressed and the β limit is above 0.05 with respect to the Mercier criterion [9]. Recently a new candidate 2b32m3 has been found with a larger elongation E_0 = 5.33, small negative shear, larger aspect ratio A = 3.41 and even further reduced Shafranov shift. This case is somewhat more stable than the 2b32 candidate.

In this paper, a further optimisation of the 2b32m3 case has been performed with respect to small Pfirsch-Schlüter (PS) current. Such optimisation should generally give a small Shafranov shift and a high equilibrium β limit, the configuration becomes non-sensitive to the plasma pressure. During this optimisation, we can impose different weight factors for other desired characteristics of the magnetic configurations such as quasi-axisymmetry, aspect ratio, Mercier criterion, maximal elongation (for the coil design restrictions), etc.

The paper is organised as follows. Section **2** presents the Pfirsch-Schlüter current minimisation issues. Section **3** shows the equilibrium, Mercier and ballooning stability as well as

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α-particle confinement properties of the configuration with reduced PS current and is followed by a summary.

2. Pfirsch-Schlüter current minimisation

In this section, we present several details connected with PS current minimisation and its calculations with the VMEC, JMC and TERPSICHORE codes. By definition, the Pfirsch-Schlüter current $\boldsymbol{j}_{\|}$ is that component of the plasma secondary current $\boldsymbol{j}$ connected with the plasma pressure that is parallel to the magnetic field $\boldsymbol{B}$:

$$\boldsymbol{j} = \boldsymbol{j}_{\|} + \boldsymbol{j}_{\perp} = p'(s)\left\{\alpha\boldsymbol{B} + \frac{(\boldsymbol{B}\times\nabla s)}{B^2}\right\}$$

where the perpendicular component is $\boldsymbol{j}_{\perp}$, the plasma pressure is $p(s)$, s is a flux surface label proportional to the toroidal flux, $s = 1$ corresponds to the plasma edge and α is a periodic function. A convenient nondimensional measure of the Pfirsch-Schlüter current in different configurations is given by the ratio of the average square of the PS current component along $\boldsymbol{B}$ over the corresponding average of the perpendicular component $\langle \boldsymbol{j}_{\|}^2\rangle/\langle \boldsymbol{j}_{\perp}^2\rangle$. This ratio can be used as a target function to reduce the Pfirsch-Schlüter current and to obtain a large limit for the equilibrium β-value. Here and subsequently, β indicates the $\langle\beta\rangle$ value.

One example of PS minimisation, a 6-period quasi-helically symmetric configuration with a high $\beta \approx 0.5$ approaching the equilibrium limit, has been previously obtained with $\langle \boldsymbol{j}_{\|}^2\rangle/\langle \boldsymbol{j}_{\perp}^2\rangle \approx 0.1$ [10]. The current ratio in [10] was calculated with the JMC code. It was found by using the paraxial approximation [11] that large elongation and large rotational transform can significantly reduce Pfirsch-Schlüter currents in quasi-symmetrical configurations. This conclusion is also valid for quasi-axisymmetric configurations like CHS-qa. However, the smaller number of periods (2 instead of 5 or 6) and the additional restrictions from the coil design preclude achieving the same level of PS currents as in Ref. [10].

In our numerical optimisation with the help of the NAG Fortran library E04UCF, based on a sequential quadratic programming method, we use the 3D fixed boundary ideal equilibrium VMEC code [2]. Subsequent improvements of the VMEC code, versions VMEC2000-6.80 [12] and VMEC2002-7.10, have extended the range of applicability for the computation of plasma equilibria to low aspect ratio systems with net toroidal current such as NCSX and QOS [13].

As a test of a system with small Pfirsch-Schlüter currents and of the formulation used to calculate these currents, we have made the comparison of the PS calculations obtained for the Wendelstein-7X (W7X, Greifswald, FRG) configuration with the VMEC, JMC and TERPSICHORE codes. The JMC code [14] and the TERPSICHORE code [3] calculate the magnetic field strength B in so called magnetic or Boozer coordinates taking into account a diverging parallel current density at rational values of the rotational transform (ι). The averaging over the plasma layer $\mathrm{d}V$, $\langle f\rangle = \int(W \times f \mathrm{d}s\mathrm{d}\theta\mathrm{d}\zeta)/\mathrm{d}V$ of the PS components is performed using different weight functions W. In the JMC code this weight is $W_{JMC} = \sqrt{g}/|\nabla S|^2$, where $\sqrt{g}$ is Boozer jacobian. In the VMEC code this weight is $W_{VMEC} = \sqrt{g}$. We have performed also the TERPSICHORE calculations of currents ratio $\langle \boldsymbol{j}_{\|}^2\rangle/\langle \boldsymbol{j}_{\perp}^2\rangle$ with both W_{JMC} and W_{VMEC} weights.

The nice coincidence of the current ratio calculations obtained from the JMC code and from the TERPSICHORE codes with the W_{JMC} weights and the PS ratio calculations obtained from VMEC and from the TERPSICHORE code with the W_{VMEC} weights has been achieved for the test W-7X configuration.

3. Optimisation

The optimised configuration called chs67 is obtained using the current ratio target, constant aspect ratio, weak quasisymmetry condition (the axisymmetric mode is 2 times larger than any non-symmetric mode, but only at the edge) and edge ι value near 0.55. Such value of $\iota(1)$ is above the dangerous rational value 1/2 which has a possibly large resonant effect. The pressure profile used in the calculations is proportional to the function $14 - 22s + 8s^2$. During the optimisation it was possible to decrease the PS current almost 4 times with respect to the initial level (see Fig. 1). The strongly elongated ($E_0 = 10.4$) configuration chs67 has a very small PS current, $\min\langle \boldsymbol{j}_{\|}^2\rangle/\langle \boldsymbol{j}_{\perp}^2\rangle = 0.36$, which leads to a very high limit for the equilibrium β-value above 0.20 for 2-period configurations. The region $s < 0.1$ should not be taken into account because of the errors of VMEC near the magnetic axis.

The Boozer Fourier spectrum of chs67 (Fig. 2) can be called weakly quasi-axisymmetric only near the edge, here the magnetic field strength $B = \sum B_{mn}\cos(m\theta - n\zeta)$, θ and ζ are poloidal and toroidal Boozer angles. There the first subscript 'm' identifies poloidal index, the second 'n' - the toroidal index per period. Near the axis, the dominant component is a mirror-type mode (0,2), while another mirror-type component (0,1) is not very large for all radial points.

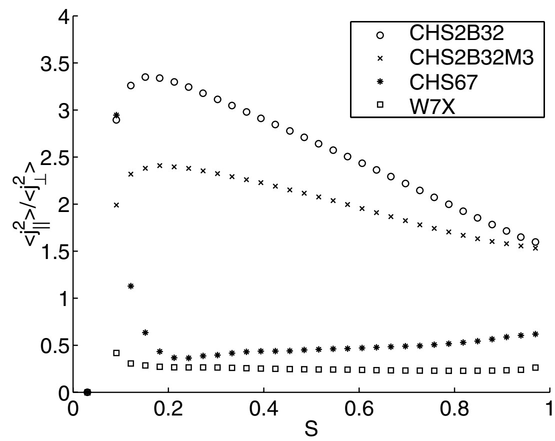

Fig. 1 The current squared ratio versus flux surface label obtained with the VMEC code for chs2b32, chs2b32m3, chs67 and W7X configurations with $\beta = 0.05$.

One can say the constraint of small PS current significantly destroy the quasi-axisymmetry.

Nonsymmetrical modes lead to the increasing of the α-particle losses during the initial 10^{-4} s period of confinement, see Fig. 3. The collisionless confinement of α-particles in chs67, $\beta = 0.085$ is calculated with the guiding-centre orbit code MCT [4] with launching surfaces $\Psi_0 = 0.0625$ (lower curve) and $\Psi_0 = 0.240$ (upper curve). Normalisation: plasma volume 10^3 m^3, magnetic field 5 T. Two thousands particles randomly distributed in the pitch angle variable and around the birth surface are followed; the dashed lines indicate the percentage of the reflected particles. Almost all trapped particles are lost during a time of 0.01 s. However, this is the case also for the starting configuration chs2b32m3, which can be attributed to the high sensitivity of α particle confinement in quasi-axisymmetrical systems to small nonsymmetrical components.

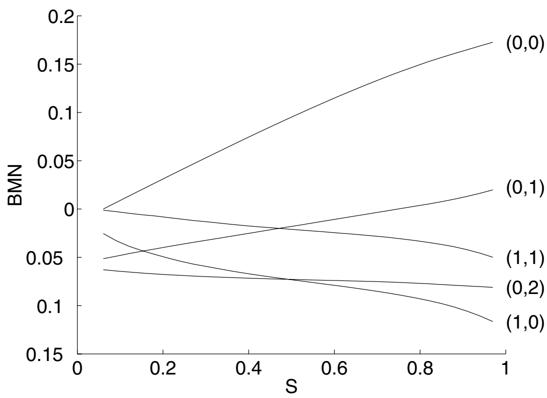

Fig. 2 Main components of the magnetic field strenght spectrum in the chs67 configuration, $\beta = 0.085$, obtained with the JMC code. Normalisation: $B_{0,0}(0) = 1.00$.

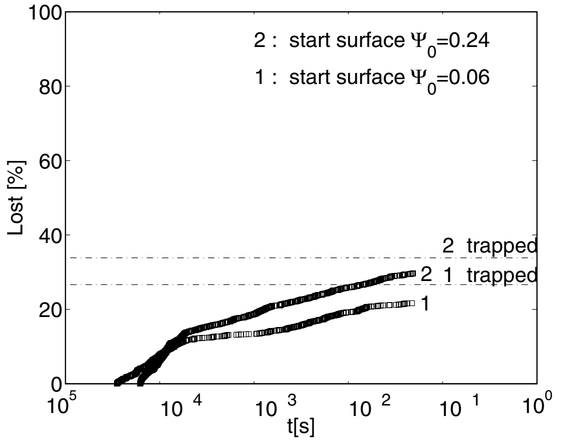

Fig. 3 Collisionless confinement in the chs67 configuration, MCT code.

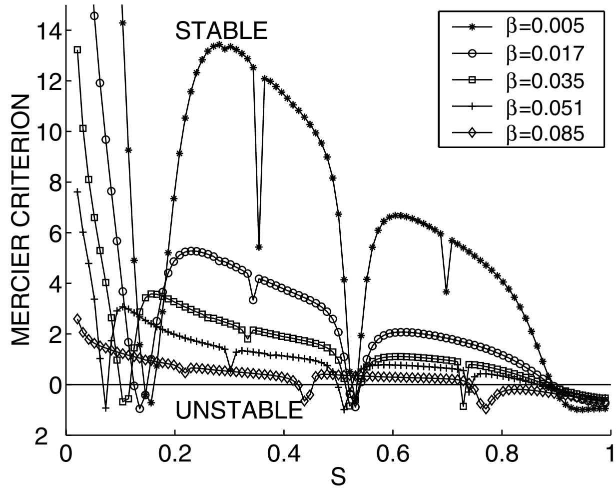

Fig. 4 The Mercier criterion obtained with the TERPSICHORE code for the chs67 configurations, $\beta = 0.005$, $\beta = 0.017$, $\beta = 0.035$, $\beta = 0.051$, and $\beta = 0.085$.

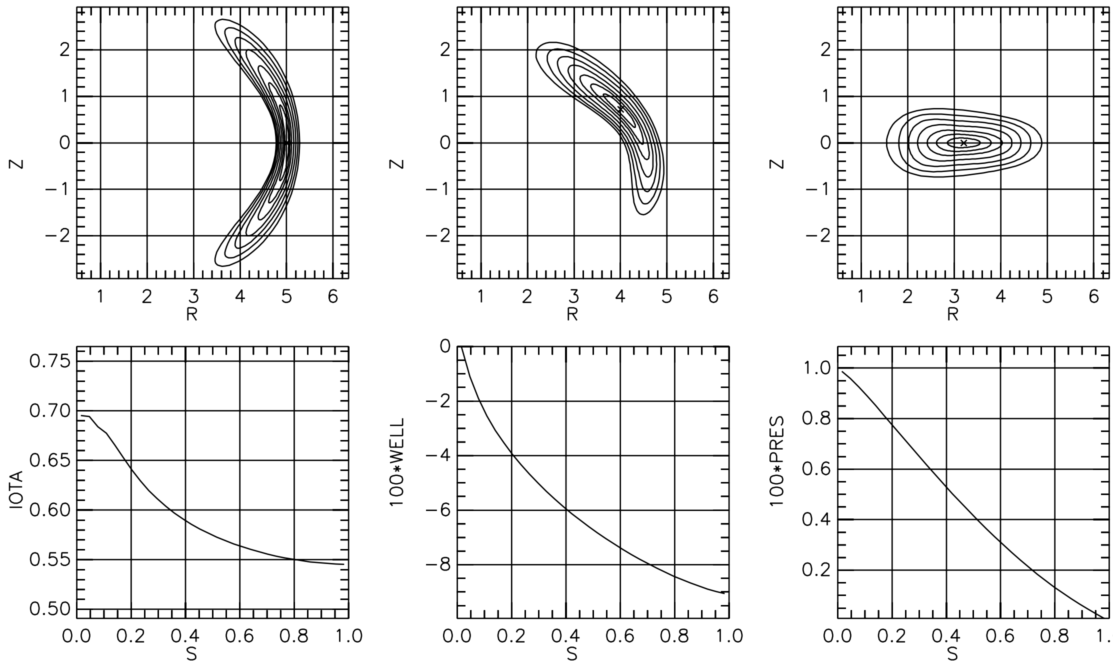

Fig. 5 Plasma surface cross-sections at the beginning, at one quarter and at the middle of the period in a chs67 configuration, $\beta = 0.093$ (top row); equilibrium profiles of the rotational transform, magnetic well and plasma pressure (bottom row).

The TERPSICHORE calculations of the Mercier criterion for chs67 configuration are shown in Fig. 4. This case is stable with respect to the Mercier modes almost for all plasma surfaces up to the $\beta = 0.085$, which is a little bit higher than the $\beta = 0.051$ limit with respect to these modes in the chs2b32m3 configuration. However, the stability limit for chs67 case with respect to the ballooning modes is $\beta = 0.017$ and is lower than the ballooning stability limit of the chs2b32 configuration ($\beta = 0.02$). Here we take into account only the symmetrical ballooning modes.

Figure 5 presents the main equilibrium profiles of the rotational transform, magnetic well, pressure and view of magnetic surface cross-sections in the chs67 configuration at the beginning, at one quarter and at the middle of a period for a value of $\beta = 0.093$.

4. Summary

The CHS-qa compact helical quasi-axisymmetric concept (NIFS, Japan) is currently in the process of the further optimisation and configuration design so it is important to consider all possible limits and extreme configurations to increase the field of future selection for the device candidates. Pfirsch-Schlüter current minimisation has been performed in several CHS-qa configurations. Such optimisation significantly improved the equilibrium β-limit and the ideal MHD stability properties with respect to Mercier modes. However, the strong stability towards ballooning modes becomes a more difficult issue to achieve. In the configurations obtained, an equilibrium β-limit is above 0.20, the stability limits with respect to the Mercier and ballooning modes are at the $\beta \approx 0.085$ and $\beta \approx 0.02$ levels, respectively.

The optimised configurations with small PS current have strong elongation of the plasma boundary cross-section. Large elongation can impose difficulties for the modular coil design, which is currently underway for the configurations obtained. We also almost did not impose quasi-axisymmetry constraints, so the configurations obtained have a large mirror-type field component (0,2). It is not clear yet how strong this component will influence the plasma confinement. These problems together with other possible restrictions, for example, due to the bootstrap current, magnetic islands or free-boundary equilibria issues, will be considered in future optimisations.

Acknowledgements

The authors are thankful to Drs. S. Hirshman and R. Zille for their help and providing us with the VMEC and JMC codes. The work was supported by INTAS Grant 99-00592, by Russian-German Agreements WTZ RUS 01-581, by the Russian Foundation of Basic Research, Grant N 00-02-17105, by the Russian Federal Programme on Support of the Leading Scientific School, Grant N 00-15-96526 and by the Fonds National Suisse pour la Recherche Scientifique. A working visit to the National Institute for Fusion Science (NIFS, Japan) of M. Isaev was initiated and supported by the Director of NIFS Prof. M. Fujiwara and Prof. C. Namba (Grant-in-aid from the Ministry of Education, Science, Culture, Sports and Technology, Japan). The computational results were performed on the NEC-SX5 platforms at the Centro Svizzero di Calcolo Scientifico in Manno, Switzerland and at the Rechenzentrum Garching, Max-Planck-Institut für Plasmaphysik, Germany.

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