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Relation between the Radial Electric Field and the Flow in a Tokamak Plasma with $\nabla T = 0$

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Abstract

The toroidal viscosity determines the neoclassical radial electric field in an axisymmetric tokamak plasma as in non-axisymmetric systems. This toroidal viscosity is very small and obtained only when the finite orbit width effect is considered. In the case of uniform temperature ($\nabla T = 0$), the wave equation is derived to show the oscillatory behaviour of the radial electric field. The oscillatory radial electric field converges a steady state which satisfies the satandard neoclassical relation between the parallel flow and the radial electric field.

Keywords:

neoclassical radial elelctric field, tokamak, non-axisymmetric system, finite orbit width, toroidal viscocity, parallel flow

1. Introduction

It is a very important subject to examine what mechanism determines the radial electric field in toroidal systems, because the radial electric field plays an important role for the plasma transport. The radial electric field is determined by the ambipolar condition : Γ_e (E_r) = Γ_i (E_r), where Γ_e and Γ_i are electron and ion particle fluxes, respectively, and E_r is the radial electric field. The particle fluxes arise from various physical processes such as neoclassical transport, anomalous transport, charge exchange, particle loss out of the confinement region.

In the present paper, we consider only neoclassical transport to determine the radial electric field in axisymmetric tokamaks. It is well known, in axisymmetric tokamaks, that the particle transport is intrinsically ambipolar, which means that the electron and ion particle fluxes are independent of the radial electric field, and thus it is not determined. However, this intrinsically ambipolar condition holds only in the lowest order of ε in the drift kinetic equation, where $\varepsilon = \rho_p / L \ll 1$ with ρ_p the poloidal Larmor radius and L the characteristic length of the plasma. The standard neoclassical theory [1,2] holds in this lowest order of ε and thus in the limit of the small orbit width (SOW) of the plasma particles. Therefore, to determine the neoclassical radial electric field E_r in an axisymmetric tokamak, the fourth order $O(\varepsilon^4)$ of particle flux must be calculated solving the drift kinetic equation up to the second order $O(\varepsilon^2)$. Assuming the constant termperarure ($\nabla T = 0$), Rosenbluth *et al.* [3] calculated the fourth order particle flux and obtained the radial electric field at the steady state. This fourth order particle flux is very small and attributed to the finite orbit width (FOW) effect.

In the general case of non-zero ∇T, it is difficult to obtain higher order flux without solving numerically the drift kinetic equation. To include comprehensively the FOW effect in studying neoclassical transport, the δf Monte Carlo method has been developed [4-11]. Since this method includes accurately both FOW effect and collisional effect which ensures the momentum and energy conservations, it is applicable to the neoclassical transport independent of ε ordering. Recently, Okamoto *et al.* [11] calculated the neoclassical radial electric field in a tokamak plasma with a flow by using a δf simultion code which solves simultaneouslly the drift kinetic equation and the equation for the time development of the radial electric field. The radial electric field oscilates with a frequency of the geodesic acoustic mode and converges to a steady state value.

In the present paper, we derive the wave equation of the oscilatory electric field in the case of $\nabla T = 0$, assuming that the radial particle flux is calculated by the δf simulation. It is shown that the oscilatory electric field converges and the resultant steady state radial electric field and the parallel flow has the same relation with that of the standard neoclassical theory. It is conjectured that the standard neoclassical relationship between the radial electric field and the parallel flow holds even for the plasma with FOW.

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2. Parallel and toroidal viscosities

The equation for momentum balance is given, at the steady state, by

$$n_a e_a\left(\vec{E} + \vec{u}_a \times \vec{B}\right) + \vec{F}_a - \nabla P_a - \nabla \cdot \Pi_a = 0 \tag{1}$$

Taking toroidal component ($R^2\nabla\phi$) gives the particle flux in the radial direction for species a,

$$\Gamma_{a\psi} \equiv \left\langle n_a \vec{u}_a \cdot \nabla\psi \right\rangle = \left\langle R^2\nabla\phi \cdot n_a \vec{u}_a \times \vec{B} \right\rangle \tag{2}$$

$$\Gamma_{a\psi} = -\left\langle R^2\nabla\phi \cdot \left(\frac{1}{e_a}\vec{F}_a + n_a\vec{E} \right) \right\rangle + \frac{1}{e_a}\left\langle \nabla \cdot \left(R^2\nabla\phi \cdot \Pi_a \right) \right\rangle \tag{3}$$

It is noted that the relations $\Sigma F_a = 0$ (momentum conservation during the collision), $\Sigma e_a n_a = 0$ (charge neutrality), and $\Sigma \Pi_a = 0$ (viscosity force balance) hold. Then, $\Sigma e_a \Gamma_{a\psi} = 0$, which is consistent with vanishing radial current at the steady state.

In the axisymmetric tokamak, the toroidal viscosity is small due to axisymmetry, which is on the order of $O(\varepsilon^4)$ if ε is small [3], and parallel force balance $\vec{B}\langle \vec{B} \cdot \vec{F}_a \rangle = \langle \vec{B} \cdot \nabla \cdot \Pi_a \rangle$ dominates the particle diffusion. Then, in the lowest order, the relation $\Gamma_{a\psi} = \Gamma_{i\psi}$ holds independently of the radial electric field E_ψ. This is the intrinsic ambipolarity.

The total particle flux in the radial direction is given, omitting the subscript ψ, by

$$\Gamma = -\Gamma_{ee} - \Gamma_{ei} + \Gamma_{ii} + \Gamma_{ie} \tag{4}$$

Here, Γ_{ee}, Γ_{ei}, Γ_{ii}, and Γ_{ie} are radial particle fluxes caused by electron-electron, electron-ion, ion-ion, and ion-electron collisions, respectively. In the lowest order, $\Gamma_{ei} = \Gamma_{ie}$ independent of E_ψ, and Γ_{ee} is very small compared to Γ_{ii}, since the particle fluxes due to self-collisions are attributed to FOW effects in the axisymmetric tokamak. Γ_{ii} comes from the toroidal viscosity $\Gamma_{ii} = \langle \nabla \cdot (R^2\nabla\phi \cdot \Pi_a) \rangle$, which depends on E_ψ. It is concluded that $\Gamma_{ii}(E_\psi) = 0$ determines E_ψ.

In non-axisymmetric tori (stellarator, helical system like LHD), the toroidal viscosity $\langle \vec{B}_t \cdot \nabla \cdot \Pi_a \rangle$ dominates the particle diffusion over the parallel viscosity and $\Gamma_{e\psi}(E_\psi) = \Gamma_{i\psi}(E_\psi)$ determines E_ψ.

3. Wave equation

Only ions are treated in the present paper. We consider a plasma in an axisymmetric tokamak and all the variables depend only on the radial position ψ and the poloidal angle θ. In the coordinates ($\mathcal{E}$, μ, ψ, θ), the drift kinetic equation is given by [1]

$$\frac{\partial f}{\partial t} + \frac{e}{m}\frac{\partial \Phi}{\partial t}\frac{\partial f}{\partial \mathcal{E}} + \left(\vec{v}_\parallel + \vec{v}_d \right) \cdot \nabla f = C(f, f) \tag{5}$$

where $\vec{v}_\parallel$ is the parallel velocity, $\vec{v}_d$ is the drift velocity, and C is the collision operator. This drift kinetic equation is solved numerically by the δf method with two weights [5]. However, to solve this equation, the equation for the radial electric field is required.

From Poisson equation and the continuity equation, the time development for the radial electric field is given by

$$\left[\left\langle |\nabla\psi|^2 \right\rangle + 4\pi nmc^2 \left\langle \frac{|\nabla\psi|^2}{B^2} \right\rangle \right] \frac{\partial^2 \Phi}{\partial t \partial \psi} = 4\pi e \Gamma_i \tag{6}$$

In this equation, $\Gamma_i = \Gamma_{ii}$ is the ion particle flux. (Γ_{ee} is very small and neglected.) Φ is the electric potential, ψ is the toroidal flux (the radial electric field is $E_\psi = -\partial\Phi / \partial\psi$), and the second term on the left hand side is attributed to the classical polarization current. The bracket $\langle \cdots \rangle$ means the flux average. The ion particle flux Γ_i is defined by

$$\Gamma_i = \left\langle \int d\vec{v} \left(\vec{v}_d \cdot \nabla\psi \right) f \right\rangle \tag{7}$$

Equation (6) with (7) holds independently of the magnitude of pressure gradients. That is, eqs.(6) and (7) are unrelated to the ordering of $\varepsilon = \rho_p / L$. In the standard neoclassical theory, like-particle collisions produce no particle flux in the SOW limit, and hence Γ_i vanishes. However, in the δf simulation, Γ_i remains finite even if the pressure gradient is very small, because the effect of FOW is rigorously taken into account by solving guiding center equations. If the FOW effect is small, Γ_i is small and a small E_ψ is generated. If the FOW effect is large, Γ_i is large and a large E_ψ is generated. The radial electric field evolves so as to vanish Γ_i to maintain the charge neutrality.

In this paper, only the case of $\nabla T = 0$ is considered. For simplicity, it is assumed that the magnetic surfaces are concentric and circular. Then, equation (7) becomes

$$\left(1 + \frac{c^2}{v_A^2} \right) \frac{\partial E_r}{\partial t} = -4\pi e \left\langle \int d^3v f\, v_{dr} \right\rangle \tag{8}$$

where $v_A = B/(\sqrt{4\pi nm})$ is the Alfven speed. The radial and poloidal drift velocities are given by

$$v_{dr} = -\frac{v_\parallel^2 + v^2}{2\Omega_0 R_0}\sin\theta \tag{9}$$

$$v_{d\theta} = -\frac{E_r}{B} - \frac{v_\parallel^2 + v^2}{2\Omega_0 R_0}\sin\theta \tag{10}$$

where R_0 is the major radius and $\Omega_0 = eB_0/mc$ with B_0 the magnetic field at the axis. The parallel flow velocity has a form of

$$v_{\parallel}(r, \theta) = \langle v_{\parallel} \rangle \frac{B_0}{B} \tag{11}$$

with

$$B = B_0 \left(1 - \frac{r}{R} \cos\theta \right). \tag{12}$$

Taking the time derivative of Eq.(8) yields

$$\varepsilon_{\perp} \frac{\partial^2 E_r}{\partial t^2} = -4\pi e \left\langle \int d^3v \frac{\partial f}{\partial t} v_{dr} \right\rangle \tag{13}$$

with

$$\varepsilon_{\perp} \equiv 1 + \frac{c^2}{v_A^2} \simeq \frac{c^2}{v_A^2}. \tag{14}$$

From the drift kinetic equation (5)

$$\frac{\partial f}{\partial t} = -\frac{e}{m} \frac{\partial \Phi}{\partial t} \frac{\partial f}{\partial \mathcal{E}} - \left(\vec{v}_{\parallel} + \vec{v}_d \right) \cdot \nabla f + C. \tag{15}$$

The standard neoclassical theory gives an exact solution to the drift kinetic equation for the case of $\nabla T = 0$ [1]. The solution is a shifted-Maxwellian distribution function f_{SM} expressed as,

$$f_{SM} = \left(1 + 2 \frac{v_{\parallel} v_z}{v_{th}^2} \right) f_M \tag{16}$$

where f_M is a Maxwellian with a thermal speed v_{th} given by

$$f_M = \frac{n}{\left(\pi v_{th}^2 \right)^{3/2}} \exp\left(\frac{e\Phi}{T} - \frac{2\mathcal{E}}{m v_{th}^2} \right) \tag{17}$$

Here, T is the ion temperature. It is noted that the shifted Maxwellian annihilates off the collision term ; $C(f_{SM}, f_{SM}) = 0$.

Insering Eq.(16) together with Eq.(17) into Eq.(13) yields, up to the order of $O(\varepsilon_t = r/R_0)$ and $O(\langle v_{\parallel} \rangle / v_{th})$,

$$\frac{\partial^2 E_r}{\partial^2 t} + \omega_{GAM}^2 E_r = H \tag{18}$$

where ω_{GAM} is the GAM (Geodesic Acoustic Mode) frequency [12] given by

$$\omega_{GAM}^2 = \frac{7}{4} \frac{v_{th}^2}{R_0^2} \tag{19}$$

and

$$H = \omega_{GAM}^2 \left(\frac{T}{e} \frac{1}{n} \frac{dn}{dr} + \frac{rm\Omega}{eq R_0} \langle v_{\parallel} \rangle \right) \tag{20}$$

where q is the safety factor, m the ion mass, e the ionic charge, and Ω is the ion cycrotron frquency. The wave equation is the same as that in ref. [11] in the small $\langle v_{\parallel} \rangle$ limit.

Equation (18) suggests that the radial electric field interacts with moving particles to generate an oscillation with a frequency ω_{GAM}. The solution to eq.(18) is

$$E_r = \bar{E}_r + A e^{-i\omega_{GAM} t} \tag{21}$$

The amplitude A is Landau damped and E_r is changed slowly by magnetic pumping [13]. It is known that the damping rate depends strongly on the value of the safety factor q. In the long time limit after the GAM oscillation damps out or after time averaging, the relation between the radial electric field and the flux averaged parallel flow becomes

$$\langle v_{\parallel} \rangle = -\frac{q R_0 v_{th}^2}{2r\Omega} \left(\frac{1}{n} \frac{dn}{dr} - \frac{e}{T} E_r \right). \tag{22}$$

This is just the relation given by the standard neoclaassical theory [1]. Note that the equation (18) or (21) has been derived without SOW assumption.

4. Summary

The analysis has been restricted to the plasma with no temperature gradients ($\nabla T = 0$). The purpose of this restriction is to compare the relation of the radial electric field and the parallel flow between the standard neoclassical theory in the SOW limit and the present analysis assuming no SOW limit. Both relations agree completely. This suggests that the FOW effect may not affect the standard relation between the radial electric field and the parallel flow velocity.

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