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Critical Contribution of Low-Density Background Particles in the Formation of Ordered Structures in Pure Electron Plasma

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Abstract

We study how low-density (background = BG) electrons affect the generation processes of ordered structures in the distribution of high-density electrons (clumps) in two-dimensional dynamics transverse to a homogeneous magnetic field. Here, by reviewing recently-published work [1,2], we discuss mainly on two experimental results: (1) two-clump dynamics focusing on bifurcation between merger and close-separation under critical contribution of BG electrons, and (2) their contribution to advecting three clumps to a symmetric configuration and sustaining the geometry of an equilateral triangle that represents a unit cell of vortex crystals. We regard the two phenomena are among key processes supporting the self-organization of ordered structures in many-particle systems with long-range interacting force.

Keywords:

nonneutral plasma, vortex dynamic, self-organization

1. Introduction

Vortex has become one of the key concepts in contemporary physical science. [3] Quantized vortex represents major property governing the dynamics of super fluid. [4] The accumulation and fusion-triggered abrupt release of the vortex strings in the super fluid core is called for as a possible mechanism of glitch in the spinning speed of a pulsar. [5] Most prominent features of Bose-Einstein condensates include vortex dynamics of probability distribution functions. [6] Needless to say, vortex dynamics appears in fusion-oriented plasmas in many essential ways including convective cells that enhance unfavorable radial transport and internal transport barrier that benefits the confinement. There is a possibility that one can explore the frontier of plasma physics as an active and easy-to-experiment field to investigate vortex as a concept common in the wide spectrum of physical science.

Strongly magnetized pure electron plasmas are characterized by the long-range coulomb interaction without effective shielding and by incompressible two-dimensional (2D) dynamics equivalent to the 2D Euler fluid. The particle density $n(x,y)$ is related to the vorticity $\zeta(x,y)$ by $\zeta = en/\varepsilon_0 B_0$, and the circulation Γ is defined as $\Gamma = \int dxdy\zeta(x,y) = eN/\varepsilon_0 B_0 L$. Here N is the total number of electrons trapped within the axial length of L in a homogeneous magnetic field B_0.

One of the culmination among studies of fluid dynamics with pure electron plasmas is the discovery of crystallized states of vortex strings. The first experimental study was initiated by generating many string distributions of electrons in the nonlinear stage of Kelvin-Helmholtz instability. The subsequent process of the relaxation of the vortex strings is characterized by repeated organization of highly symmetric arrays of strings whose number decreases intermittently as a function of time. [7]

To explore abundant physical significance involved in the self-organization process, we have carried out a series of experimental investigations by singling out a simplified configuration in the process. [1,2,8,9,10] Our typical configuration consists of a set of discrete distributions of electrons (clumps) initially immersed in a well-defined continuous distribution of tenuous electrons that act as a collection of background vortices (BGV).

The first topic is focused on experimental investigations on how the merger of two clumps is accelerated or blocked by different levels of BGV to a degree substantially different from the clumps' dynamics in vacuum. [1,8] Modifications in the BGV distribution generated by the clumps' motion exert critical effects on the merger or generation of a closely tied state of the clumps. The second topic is on how the unit cell of vortex crystals is established in interaction with other clumps and fragmentary vorticity distributions consisting of remnants of preceding vortical interactions. [2] Here a set of

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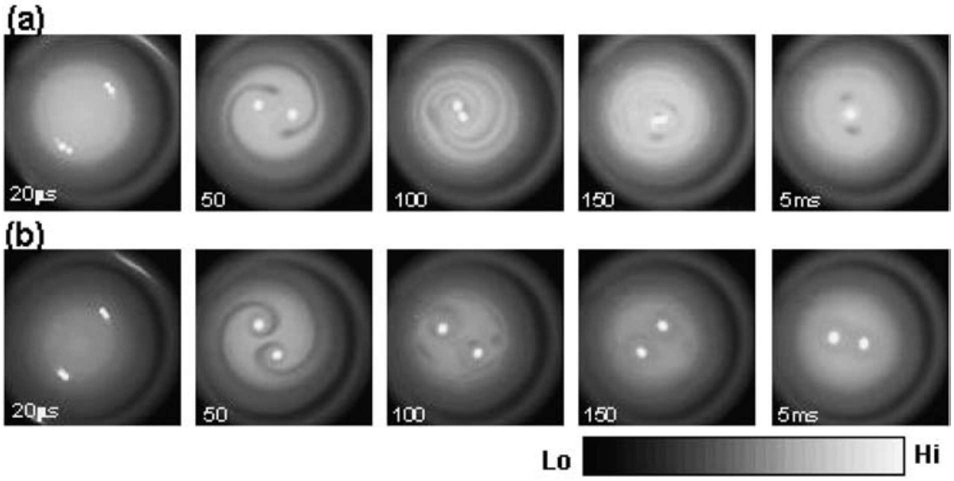

Fig. 1 Snapshots describing clumps' dynamics in BGVD with $\Gamma_b \propto N_b/10^8 = 3.55$ (a) and $\Gamma_b \propto N_b/10^8 = 3.39$ (b). Two clumps with equal circulation $\Gamma_c \propto N_c/10^7 = 0.6$ are placed symmetrically in the initial BGVD. The luminosity nonlinearly increases with electron density and saturate at high level. The panel size is 37.4 mm × 37.4 mm at the mid-plane of the trap.

three clumps with different number $N_c \propto \Gamma_c$ of electrons are observed to settle down to the vertices of an equilateral triangle in the presence of BGV, roughly with a speed increasing with $\Gamma_b \propto N_b$, the circulation of BGV.

2. Bifurcation: merger or close separation of two clumps [1]

Stable maintenance of relative locations of clumps is one essential feature of vortex crystals, and merger between them is another feature associated with the transition of the vortex configuration to other with decreasing number of clumps. In the relaxation processes involving the crystallization, the BGV distribution (BGVD) is generally low, and contributions from third clumps add to the BGVD's contribution. Here we examine the interaction between two clumps by smearing other clumps into BGVD.

Two clumps in vacuum are known to merge when placed within the distance of 1.6 × vortex diameter. [11] If a tenuous distribution of electrons (∝ BGVD) fills the space around the clumps well-separated in space, they approach quickly as shown in Fig. 1 in the time scale $\tau_V \approx 50$ μs of vortex orbitation. [1] If BGVD is high and peaking enough, the clumps continue to approach and merge in the time scale of τ_V. [8] However, if the BGVD is low and / or rather flat in shape, the clumps rebound at $t \approx \tau_V$ and stay apart for period longer than τ_V by orders of magnitude. The conditions relevant to the formation of crystals are positioned in a narrow parameter range between the two cases.

We examine the separation between the two clumps as a function of time for slightly different initial distributions of BGV as shown in Fig. 2(a), that are systematically varied with a precise electronical control. The ratio between the peak density of the clumps and the local density of the BGV is typically $n_c/n_b = \zeta_c/\zeta_b = 13$–$15$. We have measured the distance $D(1\text{ ms})$ between the clumps at 1ms for various initial conditions, and extensively examined its correlation with various parameters characterizing the initial BGVD. The

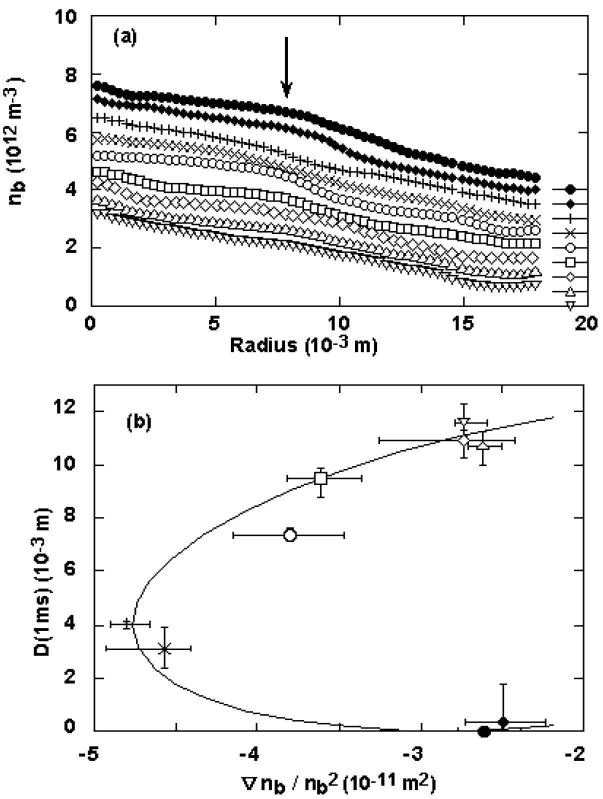

Fig. 2 (a) Radial profiles of the electron density distribution $n_b(r) = \varepsilon_0 B_0 \zeta_b(r)/e$ constituting BGVD with $n_b/10^8 =$ 3.55 (●), 3.46 (closed diamond), 3.25 (closed square), 3.14 (×), 3.39 (○), 3.08 (square), 3.13 (open diamond), 3.17 (△), and 3.16 (▽). The arrow indicates the initial radial locations of the clumps. (b) The inter-clump distance at 1 ms is plotted against $\nabla n_b/n_b^2$ as evaluated at the initial locations of the clumps.

time 1 ms, evaluated as the geometric mean of $2\tau_V$ and the coulomb collision time ≈ 10 ms, stands for the time domain characterizing the asymptotic state of the vortex dynamics before collisional dissipations set in. Clear correlations are not obtained with most of the initial parameters or their combinations except between D(1 ms) and $\nabla\zeta_b/\zeta_b^2 \propto \nabla n_b/n_b^2$ defined at the initial position of the clumps. The results of the analyses are summarized in Fig. 2(b). The smooth fitting curve, however, gives two values of D(1 ms) for a given value of $\nabla n_b/n_b^2$.

The absence of other relationship of D(1 ms) with initial parameters that resolves the degeneracy have led us to examining noticeable structures in the vorticity distributions during the evolutionary phase of the clumps' dynamics. The clumps are observed to stagnate at 50–100 μs before merger or rebounding. The BGVD's at this moment are found to exhibit systematically varying pattern of belts of depleted vorticity around the clumps. Compare the panels at 50 μs in Fig. 1. Systematic comparison of the location and length of the belts with the asymptotic separation of the clumps has led us to a model that evolution of this structure in BGVD prevents the merger.

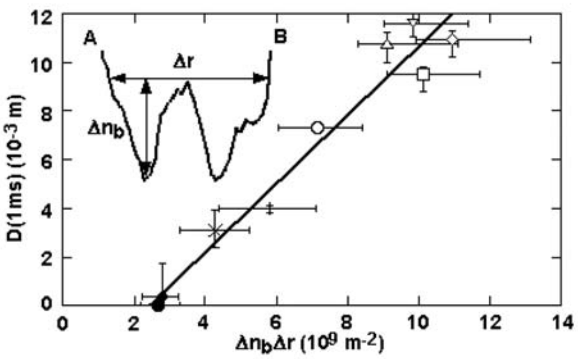

Fig. 3 The inter-clump distance D(1 ms) is plotted against $\Delta n_b \Delta r$ at 50 μs starting from different distributions of initial BGV. The inset illustrates the definition of Δn_b and Δr.

For quantitative description of this model, we evaluate the depth Δn_b of the depletion and the sum Δr of the width of the belt along the chord connecting the peaks of the clumps. Figure 3 plots D(1 ms) as a function of $\Delta n_b \Delta r$. We see that all the data points in Fig. 2(b) are continuously displayed along the linear fitting line in Fig. 3 The resolution of the degeneracy indicates that the formation of ordered structures of clumps critically depends on the fluctuating structures of the ambient BGV that are generated in mutual interaction with the clumps and with other parts of BGVD.

3. Formation of unit cell of vortex crystal [2]

We add one more clump to the previous study in order to examine the process of the formation of a unit cell of 2D symmetric arrays of vortex strings. Fourth clumps and BGV are smeared into a broad profile of the average BGV distribution. Figure 4 shows the dynamics of three clumps without BGVD (a) and with BGVD (b) starting with equal circulation $\Gamma_c \propto N_c$ from a linearly aligned configuration. The BGVD curbs the excursion of the clumps and adjusts their relative positions to the vertices of a triangle with equal side-lengths in apparent contrast with the motion of the clumps without BGVD that do not settle down to any stationary configuration.

This positive contribution of BGVD is also observed for three clumps with different circulations that finally form an equilateral triangle configuration. An appreciable difference is observed in the pattern of BGVD around each clump: The stronger clump (with larger circulation) has wider range of a depressed distribution of BGVD around itself. Our quantitative analysis has shown that the negative circulation $-\Gamma_h$ of the depression, as measured with respect to the unperturbed height of BGVD, increases with the circulation

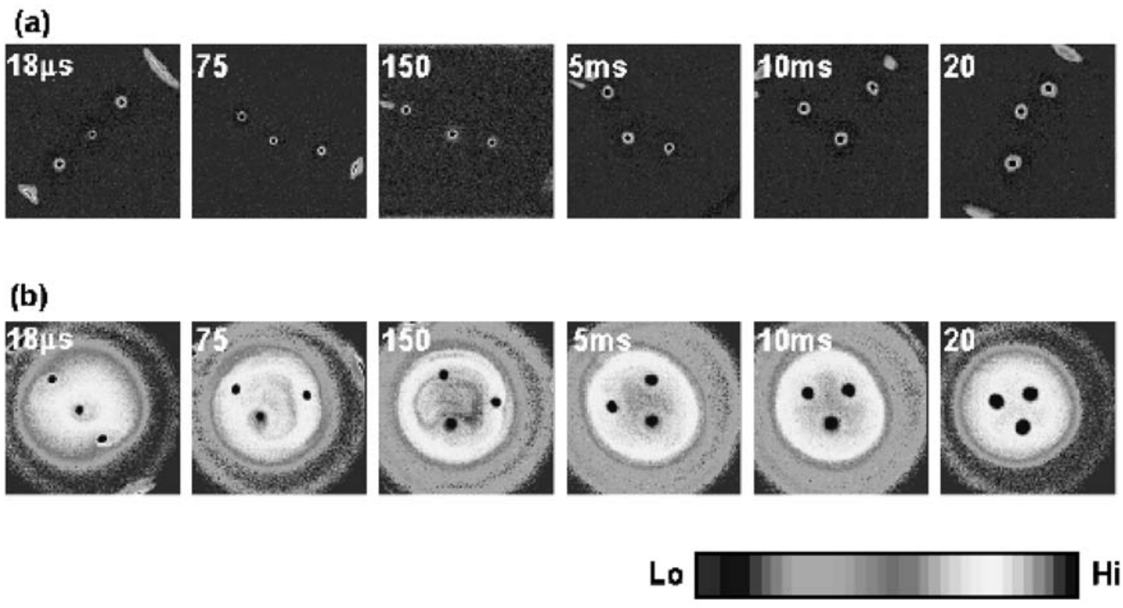

Fig. 4 Snapshots of the clumps' dynamics in vacuum (a) and in BGVD with $\Gamma_b \propto N_b/10^7 = 9.4$ (b). Three clumps with equal strength of $\Gamma_c \propto N_c/10^7 = 0.74$ are placed in straight line as the initial condition.

Γ_c of the surrounded clump but does not exceed 20%, *i.e.* $\Gamma_h < 0.2\ \Gamma_c$.

The degree of the contribution of BGVD in the formation of an equilateral triangle is most apparently observed in terms of the total circulation $\Gamma_b \propto N_b$ of BGVD. Figure 5 illustrates this feature. Here three equal clumps are placed symmetrically in BGVD at the locations indicated with arrows as illustrated in the inset. The height and profile of the initial distribution of BGV are varied systematically. The abscissa scales the radius from the axis to 2 cm, while the ordinate scales the BG density up to 2×10^{12} m^{-3}.

The main body of Fig. 5 plots the degree of proximity to the equilateral triangle of the three-clump configuration in terms of the symmetry parameter $S = 12\sqrt{3}A/l^2$, where A is the area of the triangle with total side-length of l. S is maximized at 1 when the triangle is equilateral. Figure 5 indicates that, after repeated variations in shape between thin and fat triangles in the time-scale of ≈ ms, the cell of the clumps asymptotically approaches to the symmetric form with S = 1. The asymptotic dependence is observed to be approximately $1 - S \propto t^{-1}$.

The time τ_S required to settle closely down to the equilateral cell is most strongly correlated to the circulation Γ_b of BGVD, and τ_S decreases with Γ_b. Because the clumps and BGVD rotates 10–100 times before settling, differences in the profile of initial BGVD may not dominate the evolution path of the system. On the other hand the rugged distribution in BGVD generated by the interaction with clumps may be more influential as we have found in the previous section.

4. Discussion and conclusion

We briefly compare the results of the two-clump and three-clump experiments. As read in the caption of Figs. 1-2, 4-5, the two-clump dynamics has been examined in the BGV with a narrow density range above the upper bound of the three-clump experiment. The assisting contribution of BGVD in the symmetrization of the three-clump cell is effective at densities roughly as low as 1/60 of the case where the bifurcation between merger and binary vortex is observed. We also have noticed the tendency that the size of the equilateral triangle of the clumps decreases as ζ_b or Γ_b increases and the probability of clumps' merger goes up to a statistically observable level. Our current understanding is briefly summarized that the merger process between clumps is associated with a large gradient in the local vorticity distribution that can be generated either in BGVD or by other clumps and that the merger is hindered if the sign of the local gradient is reversed.

Next we extend our discussion to microscopic structures in BGVD that induce fluctuations in the velocity of the clumps and try to relate them to the geometrically determined parameter S. The speed of random motion δv of each clump can be evaluated from density-weighted average of the $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity of electrons constituting the clump. [12,13] The open circles in Fig. 6 are obtained as the deviations from the rigid-rotating velocity of the triangle. Schecter *et al.* take this scheme in the analysis of many-clump system. [14] When applied to the three-clump system, δv is larger than expected from the almost stationary shape of the triangle over 10 ms as observed in the experiments.

Further analysis indicates that a differential rotation persists in our experimental configuration with fewer clumps. If we admit, based on our experimental observations, that three clumps need not be stationary in the frame of the whole vortex system and that they can roll in the BGVD with a rotational shear, the random velocities are evaluated low as plotted with solid triangles in Fig. 6. This analysis suggests that the clumps move coherently among themselves keeping

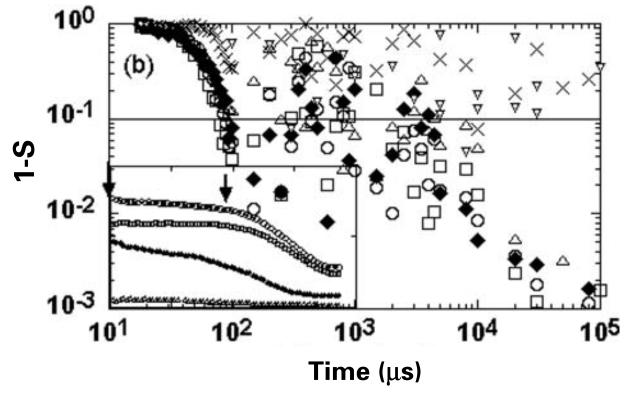

Fig. 5 Deviation 1 – S from the symmetric configuration is plotted as a function of time. Each symbol corresponds to different initial distribution of BGV as shown in the inset. The electron number constituting each profile is $N_b/10^7$ = 0 (×), 0.25 (▽), 0.52 (△), 9.4 (closed diamond), 21.0 (square), 24.8 (○). Three clumps with equal strength of $\Gamma_c \propto N_c/10^7$ = 0.74 are placed in straight line as the initial condition. Their radial locations are on the axis and at symmetric points as indicated by arrows.

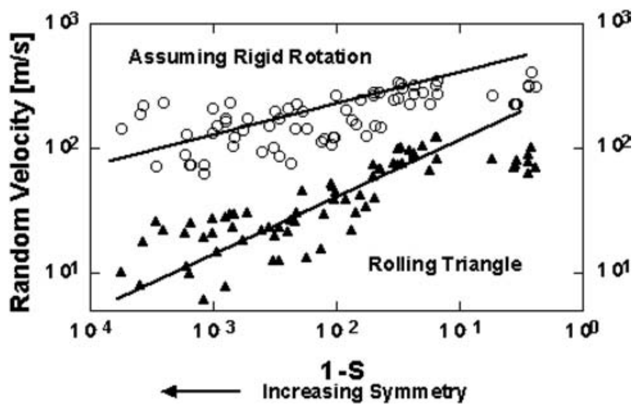

Fig. 6 Clumps' random (thermal) velocities are plotted against 1 – S. Solid triangles represent mean random velocities causing deformation from the instantaneous triangle (not necessarily equilateral) that rolls around in BGVD. Open circles stand for the mean velocity causing deviations from the instantaneous clump configuration in the global coordinates that rotate rigidly.

the configuration of an equilateral triangle. As far as we confine ourselves to the observed data, the numerical analyses of the flow field provide self-consistent view of the clumps' dynamics. However, the physical mechanism that supports the preservation of the symmetry of the triangle moving in the BGVD has not been definitely identified.

It may be of worth to take Aref's theoretical model into consideration. [15] The Aref model tells that three clumps, once placed at the vertices of an equilateral triangle, remains there in interaction with other clumps even when the circulations are unequal. Experimentally we have observed that this configuration is unstable most probably due to interaction with image charges induced on the boundary wall. Destabilizing contribution of the wall has been theoretically pointed out for clumps with equal circulations. [16] Experimentally observed dynamics appears to be more unstable than theoretically predicted. Our observations reported here indicate that, with all these destabilizing factors, the BGVD works positively to generate symmetric configurations by transporting the clumps to symmetric positions, by shielding destabilizing wall effects and by removing random velocities.

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