J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 119–124

Double Internal Transport Barrier Triggering Mechanism in Tokamak Plasmas

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(Received: 9 December 2003 / Accepted: 3 April 2004)

Abstract

Sheared flow layers created by energy released in magnetic reconnection processes are studied with the magnetohydrodynamics (MHD), aimed at internal transport barrier (ITB) dynamics. The double tearing mode induced by electron viscosity is investigated and proposed as a triggering mechanism for double internal transport barrier (DITB) observed in tokamak plasmas with non-monotonic safety factor profiles. The quasi-linear development of the mode is simulated and the emphasis is placed on the structure of sheared poloidal flow layers formed in the vicinity of the magnetic islands. For viscosity double tearing modes, it is shown that the sheared flows induced by the mode may reach the level required by the condition for ITB formation. Especially, the flow layers are found to form just outside the magnetic islands. The scaling of the generated velocity with plasma parameters is given. Possible explanation for the experimental observations that the preferential formation of transport barriers in the proximity of low order rational surfaces is discussed.

Keywords:

internal transport barrier, double internal transport barrier, viscosity tearing mode, double tearing mode, sheared flow, rational surface

1. Introduction

Internal transport barriers (ITBs) of ion or electron energy are the regions where corresponding temperature gradient is reduced with respect to the values in low (L) and high (H) mode tokamak discharges. The latter are governed by micro-instabilities and exhibit temperature profile stiffness.

In experiments ITBs are formed when neutral beam injection (NBI) or radio frequency (RF) heating power is higher than a minimum threshold in discharges. The minimum power required depends on plasma conditions when the NBI or RF waves are launched. Besides other advantages such as stable for a variety of instabilities, advanced tokamak (AT) operations with non-monotonic safety factor (q) profiles have lower thresholds than plasmas with monotonic q profiles. As a result, the AT mode of operation is highly desirable for a tokamak reactor.

Whereas reduced turbulent transport in the stationary phase after the formation of an ITB can be explained invoking the sheared perpendicular rotation resu1ting from the improved confinement within the lTB itself, the dynamics of the formation of ITB is not yet well understood. The delineation of the physics of ITB triggering mechanism will assist the ultimate goal of active control of the onset, duration and confinement resulting from the formation of ITB in AT plasmas. The required power may also be lowered through adjusting profiles of plasma parameters such as pressure, temperature, velocity and current. Therefore, in recent years, the conditions, especially the radial position, for triggering the formation of an ITB have been intensively investigated in theory and experiment.

There is evidence that ITBs are preferentially formed near low order rational flux surfaces (especially, q = 2, 5/2 and 3). It was first reported from JT-60U that the radial position of the ITBs for both ion temperature T_i and plasma toroidal velocity v_ϕ most likely coincides with the q = 3 flux surfaces [1]. In DIII-D too the ITB growth events are well correlated with integral q, although there are several counter examples [2]. The "type II" confinement transition in TFTR discharges requires lower NBI power than the threshold for enhanced reverse shear (ERS) transition and often occurs when q_{min}, the value of q at the surface of shear reversal, crosses rational values, q = 3 and 5/2 in particular [3]. In electron cyclotron heated L mode discharges on the RTP tokamak, the electron temperature (T_e) profile can be well simulated with a thermal diffusivity, χ_e model which has alternating layers of low and high χ_e located near and away

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from low q rational flux surfaces, respectively [4]. In ASDEX Upgrade reversed shear experiments, the onset of ITB is usually accompanied by MHD activity. In the 80 discharges studied, there is not a single one in which a clear ITB forms without the presence of MHD activity [5]. It was observed in JET experiments that the radial position of ITBs always coincided with the $q = 2$ or $q = 3$ surface in low core magnetic shear discharges. In addition, an appropriate q profile, and in particular the location of the $q = 2$ surface, is essential for triggering of ITBs. With negative central magnetic shear, the analysis of ITB triggering reveals a correlation between the formation of the ITB and q_{min} reaching an integral value (q = 2 or 3). The time of ITB emergence was also found to coincide with the time when q_{min} reaches 2 [6]. A comprehensive recent review on ITB in tokamaks, including experiment and theory, is given by Wolf [7].

The minimum power threshold represents an external condition that must cause required changes in local (at the position of ITB formation) as well as global (profile) plasma and magnetic configuration parameters in order to form an ITB at the position. An increase in the $\boldsymbol{E} \times \boldsymbol{B}$ shear (in the region of ITB formation) has been identified as one of these changes. In addition, it has been shown that a self-organized layer with the features of an ITB may exist in collisionless plasmas. One of the necessary conditions for the formation of such a layer is a poloidal velocity corresponding to a poloidal Mach number of order unity [8]. However, we still have to establish the relevant relationship between the position where such dramatic increase of flow velocity takes place and plasma properties. In other words, the reason that a flow layer emerges and an ITB forms at a specific position and not elsewhere in a discharge has not been addressed in detail. Especially, the observations that the ITBs form preferentially in the proximity of low order rational surfaces has not been satisfactorily explained.

It is well known that low order rational flux surfaces are prone to excitation of ideal and dissipative MHD instabilities and that magnetic energy released in the development of the modes may create significant plasma flows. Therefore, besides fishbone oscillations, other MHD instabilities, forming magnetic islands are proposed as plausible triggering mechanisms for the formation of ITBs in the proximity of low order rational surfaces in ASDEX Upgrade reversed shear discharges [5]. External kink modes coupled to inner rational surfaces have been shown to be able to trigger ITBs in positive shear discharges in JET [6]. However, the double ITB (DITB) structure observed in JET reversed magnetic shear discharge has not been addressed in detail [9].

The linear double tearing mode (DTM) mediated by anomalous electron viscosity was studied in Ref. [10] where the possibility for such modes to drive DITB was also suggested. By simulating the quasi-linear development of this mode, we demonstrate in this paper the creation of sizable sheared poloidal flow layers in the vicinity of the magnetic islands. The nature and magnitude of the generated flows makes the double tearing mode a strong candidate for the triggering of ITBs in tokamak plasmas with non-monotonic q profiles.

2. Physics model and MHD equations

We consider a plasma slab of length a in the x-direction, with a current in the z-direction, and zero equilibrium flow velocity $\boldsymbol{V}_0 = 0$ embedded in the standard sheared magnetic field

$$\boldsymbol{B}_0(x) = B_{0y}(x)\hat{\boldsymbol{y}} + B_{0z}(x)\hat{\boldsymbol{z}}, \tag{1}$$

where $B_{0y}(x)$ equals zero at $x = \pm x_s$. The stability of this initial configuration will be examined with respect to two-dimentional, incompressible perturbations. The vector fields are expressible in terms of two scalar potentials: the flux function $\psi(x, y, t)$,

$$\boldsymbol{B}_\perp = \nabla\psi \times \hat{z}, \tag{2}$$

and the stream function $\phi(x, y, t)$,

$$\boldsymbol{V}_\perp = \nabla\phi \times \hat{z}. \tag{3}$$

With electron viscosity, the Ohm's law becomes

$$\boldsymbol{E} = \eta\boldsymbol{j} - \frac{1}{c}\boldsymbol{V}\times\boldsymbol{B} - \frac{m_e\mu_e}{n_e e^2}\nabla^2\boldsymbol{j}. \tag{4}$$

It is straightforward to write the z-component of Eq. (4) as

$$\frac{\partial\psi}{\partial t} = -\boldsymbol{V}\cdot\nabla\psi + \frac{c^2}{4\pi}\eta\nabla^2\psi - \frac{m_e\mu_e c^2}{4\pi n_e e^2}\nabla^4\psi, \tag{5}$$

after using Eq. (2) and Faraday's law. Here, η is the plasma resistivity, μ_e is the parallel electron viscosity diffusion coefficient, m_e the electron mass, n_e the electron density, and e and c are, respectively, the electron charge and the speed of light. The z-component of the vorticity equation may be written as

$$\frac{\partial}{\partial t}(\nabla^2\phi) = -(\boldsymbol{V}\cdot\nabla)\nabla^2\phi + \frac{1}{4\pi\rho}[\nabla(\nabla^2\psi)\times\nabla\psi]\cdot\hat{z}, \tag{6}$$

where ρ is the mass density of the plasma. Normalizing all lengths to a, time to $\tau_h = a/v_A$, the poloidal Alfvén time of a plasma column of scale width a, and the magnetic field to some standard measure B_0, Eqs. (3), (5) and (6) may be transformed to:

$$\frac{\partial\psi}{\partial t} = \{\phi,\psi\} + \frac{1}{S}\nabla^2\psi - \frac{1}{R}\nabla^4\psi + E', \tag{7}$$

$$\frac{\partial}{\partial t}(\nabla^2\phi) = \{\phi,\nabla^2\phi\} - \{\psi,\nabla^2\psi\}, \tag{8}$$

where $S = \tau_r/\tau_h$ is the magnetic Reynolds number with $\tau_r = 4\pi a^2/c^2\eta$, $R = \tau_v/\tau_h$ is the fluid dynamic Reynolds number, while $\tau_v = 4\pi a^4 n_e e^2/c^2\mu_e m_e = \omega_{pe}^2 a^4/c^2\mu_e$, and

$$\{\phi,\psi\} = \frac{\partial\phi}{\partial x}\frac{\partial\psi}{\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}$$

is the Poisson bracket. Assuming the perturbation potentials

$$\phi = \sum_{m=1}^{\infty}\bar{\phi}_m(x,t)\sin(mky), \tag{9}$$

and

$$\psi = \delta\psi(x,t) + \sum_{n=1}^{\infty}\bar{\psi}_n(x,t)\cos(nky), \tag{10}$$

we obtain the following coupled quasi-linear equations from the first harmonic perturbation,

$$\frac{\partial\delta\psi}{\partial t} = -\frac{k}{2}\left(\frac{\partial\bar{\phi}_1}{\partial x}\bar{\psi}_1 + \bar{\phi}_1\frac{\partial\bar{\psi}_1}{\partial x}\right) + \frac{1}{S}\left(\frac{\mathrm{d}^2\psi_0}{\mathrm{d}x^2} + \frac{\partial^2\delta\psi}{\partial x^2}\right) - \frac{1}{R}\left(\frac{\mathrm{d}^4\psi_0}{\mathrm{d}x^4} + \frac{\partial^4\delta\psi}{\partial x^4}\right) + E' \tag{11}$$

$$\frac{\partial\bar{\psi}_1}{\partial t} = -k\bar{\phi}_1\left(\frac{\mathrm{d}\psi_0}{\mathrm{d}x} + \frac{\partial\delta\psi}{\partial x}\right) + \frac{1}{S}\left(\frac{\partial^2\bar{\psi}_1}{\partial x^2} - k^2\bar{\psi}_1\right) - \frac{1}{R}\left(\frac{\partial^4\bar{\psi}_1}{\partial x^4} - 2k^2\frac{\partial^2\bar{\psi}_1}{\partial x^2} + k^4\bar{\psi}_1\right) \tag{12}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial^2\bar{\phi}_1}{\partial x^2} - k^2\bar{\phi}_1\right) = k\left(\frac{\mathrm{d}\psi_0}{\mathrm{d}x} + \frac{\partial\delta\psi}{\partial x}\right)\left(\frac{\partial^2\bar{\psi}_1}{\partial x^2} - k^2\bar{\psi}_1\right) - k\left(\frac{\mathrm{d}^3\psi_0}{\mathrm{d}x^3} + \frac{\partial^3\delta\psi}{\partial x^3}\right)\bar{\psi}_1. \tag{13}$$

Equations (11-13) are solved as an initial value problem – E' is chosen such that the equilibrium does not dissipate due to resistivity and viscosity.

For the magnetic field, we employ the configuration used in Ref. [10,11],

$$B_{0y}(x) = 1 - (1 + B_c)\mathrm{sech}(\zeta x), \tag{14}$$

where

$$\zeta x_s = \mathrm{sech}^{-1}[1/(1+B_c)]. \tag{15}$$

The constant B_c is chosen such that $B'_{0y}(x_s) = \pi/2$. We do not need to specify $B_{0z}(x)$ and plasma pressure $P_0(x)$ since incompressible equations are used. The resistivity and the viscosity are both assumed to be constant. The initial conditions for $\bar{\psi}_1$ and $\bar{\phi}_1$ are the linear eigen-functions multiplied with a small number and $\delta\psi(t = 0) = 0$ [10]. The boundary conditions are $\delta\psi(x) = \partial\delta\psi/\partial x = 0$, and the values provided by the initial conditions such as $\bar{\psi}_1(x) = 0$, $\bar{\phi}_1(x) = \partial\bar{\phi}_1/\partial x = 0$ for $x = \pm x_w$, the position of wall. The chosen parameters are $k = 0.25$, $R = 10^5$, $S = 9.4 \times 10^5$, $B_c = 0.233509$, $\zeta = 2.68298$ corresponding to two rational surfaces at $x = x_s = \pm 0.25$. The results are checked to be independent of x_w, the grid size and the time-step.

3. Numerical results

The effective growth rate $\gamma = \partial\ln B_x(0)/\partial t$ versus time is shown in Fig. 1. The run was stopped at $\gamma = 0$. For initial times, $t \lesssim 30$, $\gamma \sim 0.04$ approximates the linear growth rate [10]. After that, the quasilinear effects begin to suppress the effective growth rate-γ decreases with time and reaches zero at $t \cong 78$. Here, γ is the growth rate of x-component of the magnetic field at the origin ($x = y = 0$) point. Therefore, the mode keeps growing after γ reaches zero as shown in Figure 2 by the kinetic and magnetic energy.

The magnetic energy

$$E_m = \frac{1}{8\pi}\int\left(B_x^2 + B_y^2\right)\mathrm{d}x\mathrm{d}y = \frac{1}{8\pi}\int\left[\left(k\bar{\psi}_1\sin ky\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2\right]\mathrm{d}x\mathrm{d}y, \tag{16}$$

and the kinetic energy

$$E_k = \frac{1}{2}\rho\int(\upsilon_x^2 + \upsilon_y^2)\mathrm{d}x\mathrm{d}y = \frac{1}{8\pi}\int\left[\left(k\bar{\phi}_1\cos ky\right)^2 + \left(\frac{\partial\bar{\phi}_1}{\partial x}\sin ky\right)^2\right]\mathrm{d}x\mathrm{d}y, \tag{17}$$

as functions of time are given in Fig. 2. Here, the magnetic field is normalized to B_{0y} ($\pm\infty$), while in the final expression of E_k the velocities are measured in units of the poloidal Alfvén velocity. It is clearly shown that the magnetic energy released in the reconnection process, following the development of the DTM, converts to kinetic energy and can drive large flows.

The flux contours at the end of the run are given in Fig. 3. The flux surfaces here resemble those for resistivity DTM.

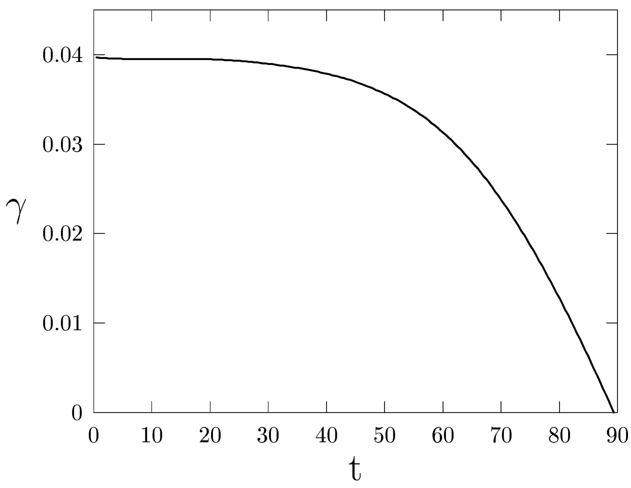

Fig. 1 Effective growth rate $\gamma = \partial\ln B_x(0)/\partial t$ versus time.

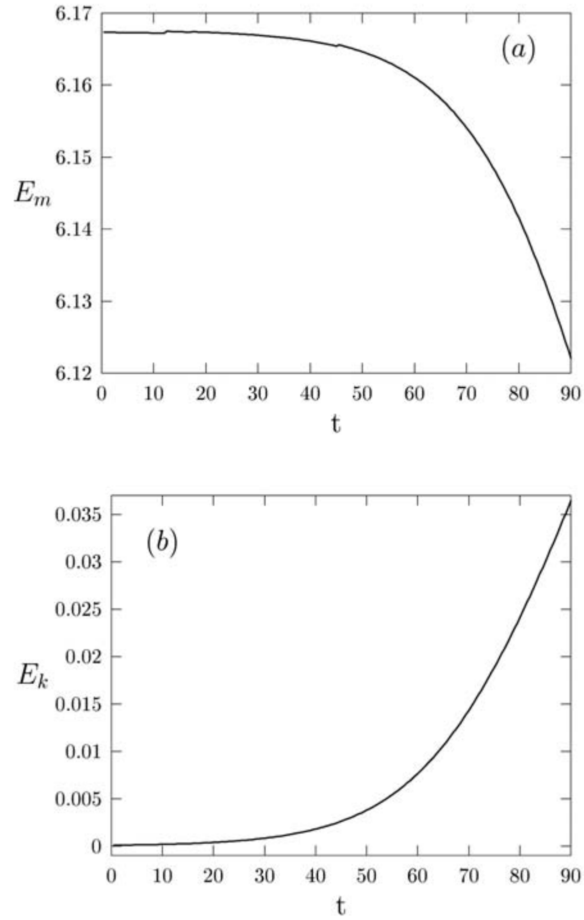

Fig. 2 Magnetic energy E_m (See Eq. (16)) and kinetic energy E_k (See Eq. (17)) as functions of time.

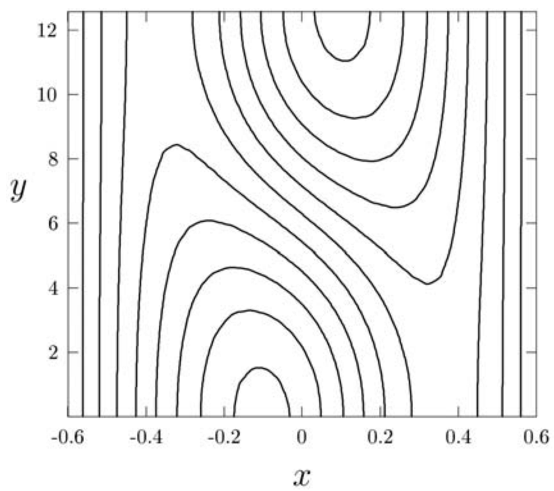

Fig. 3 Flux contours at the end of the run.

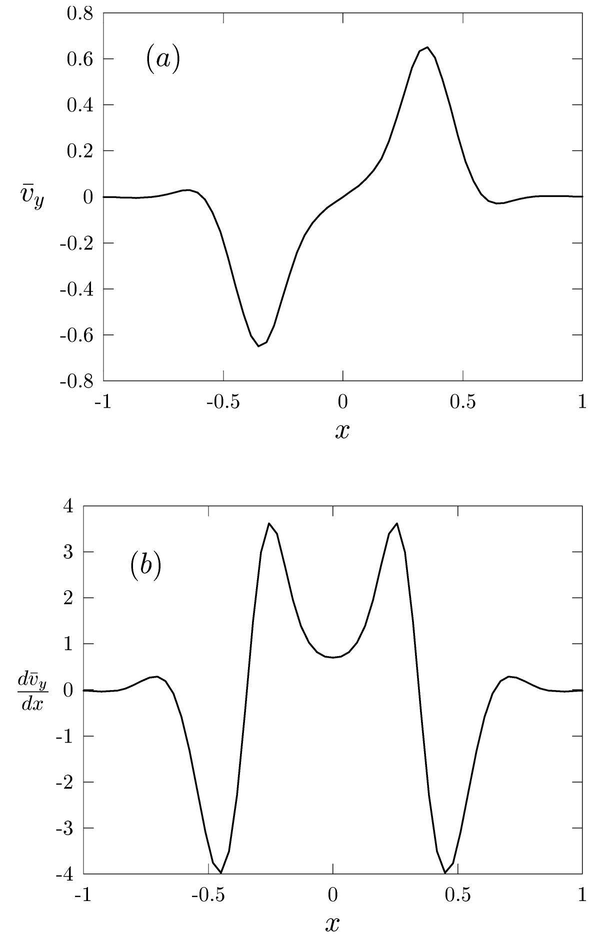

Fig. 4 Profiles of (*a*) $\bar{v}_y$ and (*b*) $\partial \bar{v}_y / \partial x$ at the end of the run.

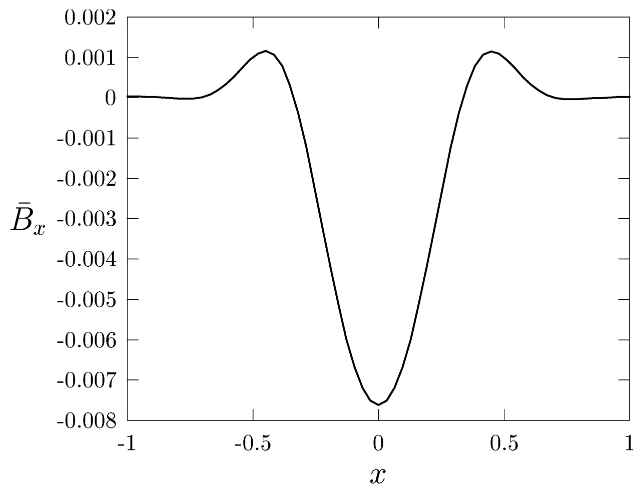

Fig. 5 Profile of the perturbed radial magnetic field $\bar{B}_x$ at the end of the run.

The profiles of (*a*) $\overline{v}_y$ and (*b*) $\partial \overline{v}_y / \partial x$ contemporaneous with Fig. 3 are presented in Fig. 4 while $\overline{B}_x$ for the same time is given in Fig. 5. Two very important points emerge: 1) the amplitude of the poloidal velocity $\overline{v}_y$ reaches the level required by the condition for ITB formation [8], and 2) the flow $\overline{v}_y$ and flow shear $\partial \overline{v}_y / \partial x$ remain at noticeable levels for $x \gtrsim 0.5$ where $\overline{B}_x$ is negligibly small. One cannot overemphasize the significance of the latter finding; because the velocity shear layers are formed outside the magnetic islands on both sides, the (magnetic) turbulence may be suppressed and DITB may form in these layers. Here, shown are the magnitudes of the perturbations. The flow patterns have to be formed from Eqs. (3) and (9) with the magnitudes given here.

4. Conclusions and discussion

ITBs are generally accompanied by a strong $\boldsymbol{E} \times \boldsymbol{B}$ shear flow, low or negative magnetic shear, and turbulence suppression. It has been pointed out that $\boldsymbol{E} \times \boldsymbol{B}$ shear flow may be generated by a variety of mechanisms [3]. In positive magnetic shear discharges, for instance, it is the coupling of an internal mode with an external MHD mode that leads to the local braking of the toroidal and poloidal rotations and then the $\boldsymbol{E} \times \boldsymbol{B}$ sheared flow results. This mechanism does not seem to work in discharges with central negative shear. The very stimulating observation that may provide supporting evidence for the mechanism proposed in this work is that two radially separated ITBs (DITB) simultaneously exist and follow the two $q = 2$ surfaces in a section of JET discharge pulse 51573 [9]. Moreover, it is confirmed that the DITB is terminated by an $m = 2$ MHD mode which extends from the inner to the outer foot point location of the DITB. This is precisely the defining theoretical characteristic of the proposed DTM.

ITB emergence is sensitive to local conditions and, in particular, to the properties of the integral q surfaces with regard to MHD instabilities and other phenomena. Our initial results, though by no means a final explanation for experimental observations, are highly encouraging as they stand up to the test of a quantitative comparison with experiment. It is very likely that additional experimental and theoretical investigations in this direction will help to expose the ITB triggering mechanism.

For the viscosity DTMs, it is easy to estimate that the saturated poloidal shearing velocity

$$V_y \sim \frac{2^{4/15} B_{0y}'^{4/5} \upsilon_A}{(Rk)^{1/5}} \sim 0.1\upsilon_A \sim 0.1\upsilon_{ti},$$

for $R \sim 5 \times 10^7$ and $k = 0.1$ [10], here, υ_{ti} is ion thermal velocity. The scaling of the generated velocity with plasma parameters may be written as,

$$V_y \propto \frac{\mu_e^{4/5} B_{0y}'^{4/5}}{k^{1/5} n_e^{3/5} A^{2/5}}$$

where A is mass number of plasma ions. The fully nonlinear development of the mode is under investigation. Although the scaling with A and k seems in line with the experimental observations [3], the relation between the mode behavior and the minimum input NBI and RF power is a challenge for the model. Possible directions to explore are: (1) macroscopic and microscopic electromagnetic perturbations that develop and cause anomalous electron viscosity do so only when the auxiliary heating power exceeds the threshold; (2) existence of a "proper" magnitude of viscosity that does allow DTMs to develop (and the velocity shear layer to form) but does not allow the DTMs to develop too fast creating violent MHD activity. The threshold depends on the competition between $\boldsymbol{E} \times \boldsymbol{B}$ stabilization and the force that drives turbulence through instabilities like the ion and electron temperature gradient (ITG and ETG) modes, and the trapped electron (TE) modes. The fact that the required threshold power in plasmas with reversed magnetic shear is lower than that with positive shear is attributed to the stabilization of the ballooning instability, larger Shafranov shift and lower safety factor near the magnetic axis [7]. In addition, the $\boldsymbol{E} \times \boldsymbol{B}$ velocity shear required to completely suppress the (ITG) modes in the former is lower than that in the latter [12].

There is a common refrain in the literature that ITBs occur under various conditions depending on the interplay between the mechanisms that drive and suppress plasma turbulence. It is quite natural to expect that the mechanism proposed in this work is relevant only in plasmas with non-monotonic q profiles. Other mechanisms such as fishbone instability, mode coupling etc. could be relevant elsewhere [7]. It is even possible that the resonance mechanism does not play a major role under certain conditions. At this stage there is no unique or universal mechanism.

The electron viscosity induced DTM is shown to generate localized shear flows in plasma configurations with non-monotonic safety factor. The proposed mechanism emerges as a major contender for a DITB trigger because the generated shear flows bear such striking qualitative and quantitative similarity to the flows that accompany DITB formation (located in the proximity of low order rational surfaces, and just outside the magnetic islands).

Acknowledgements

One of the authors (JQD) acknowledges academician Li, Zheng Wu for continuous encouragements and Prof. W. Horton for very helpful discussions. This work is supported by the National Natural Science Foundation of China, Grant No. 10135020 and by US Department of Energy, Grant No. DE-FG03-96ER-54346.

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