J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 89–93

Stationary Vortical Structures in Electrostatic Drift Wave Turbulence

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(Received: 10 December 2003 / Accepted: 3 August 2004)

Abstract

The conditions of generation of vortical flows in fusion plasma are reviewed with particular interest for the role of the small scale vortices in the intermittency events of the internal transport barriers (ITB). Exact solutions are provided for stationary states.

Keywords:

ion drift wave, vortice, zonal flow, Integrable structure

1. Introduction

It is a well known fact, supported by experiments and numerical simulations, that the drift wave turbulence is accompanied by generation of cuasi-coherent vortical structures. The aim of this work is to examine the role they can have in the formation and in the intermittent states of the transport barriers. Three elements are particularly relevant in this respect.

First, it is recalled that the drift wave has different nonlinear dynamics on different space scales [1]. At largest space scale one finds in numerical simulations the dominance of the eddies driven by the ion temperature gradient (ITG), which have large radial extension due to the ballooning character and linear mode coupling, with a particular sensitivity to the shear. The Hasegawa-Mima (HM) equation governs the electron drift waves at the small space scales, of the order of the drift wave dispersion scale, $\rho_s = (T_e/m_i)^{1/2}/\Omega_i$, where T_e is the electron temperature, m_i is the ion mass, $\Omega_i = eB/\mathrm{m}_i$ is the ion cyclotron frequency, e is the ion charge and B is the magnetic field. At intermediate space scales the scalar (or Korteweg deVries) nonlinearity is dominant. Except for the large ITG eddies (which are well studied numerically), the two other scales can be satisfactorily described analytically in closed form and have been the subject of many studies.

Second, the flow structures at mesoscopic scale have a strong influence on the transport properties [2]. This is an essential part of the theories aiming to explain the high confinement barriers by the reduction of the radial correlation length of the turbulent fluctuations in the presence of a sheared flow, even if there is not yet an unanimous opinion on the origin of these flows (ion-orbit losses, Reynolds stress, poloidal asymmetry, etc). We have found that the nonlinear drift mode equation has, at this scales, an exact, periodic solution whose pattern is identical to the zonal flows, with good agreement with the experimental observations and numerical simulations. This flow, which naturally suppresses the radial transport, becomes structuraly unstable when the vectorial nonlinearity is significant and decays into an ensemble of vortices with scale close to ρ_s.

The third aspect is derived from the necessity to assess the role of these vortical structures, generated under the Hasegawa-Mima nonlinearity in the phase where the barrier is destroyed (an intermittent event). It is well known from numerical simulations that an ensemble of vortices (which are not solitons) evolves by collisions and merging, generating larger structures from which the ITG eddies may recover. We are concerned here with the nature of the solutions consisting of ensembles of vortices. We prove that the Hasegawa-Mima equation has stationary periodic solutions consisting of a lattice of vortices. The basic procedure in this rather technical analysis consists of the mapping of the HM dynamics to a system of point-like vortices interacting via a short range potential. This model is formalized in a field theoretical framework and we search for the states having a particular property, called self-duality (a minimal characterization is that they extremize the energy functional). The outcome of this formalism consists of nonlinear differential equations which must replace at stationarity the HM equation, precisely as the sinh-Poisson equation replaces the Euler equation for the ideal fluids. We prove that these equations are exactly integrable.

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2. The space scales of the drift instability

We consider the equations of continuity and momentum conservation for the electrons and ions in two-dimensions [1]. We assume the quasineutrality $n_i \quad n_e$ and the Boltzmann distribution of the electrons along the magnetic field line $n_e = n_0 \exp\left(-\frac{|e|\phi}{T_e}\right)$. The equation written for vorticity $\boldsymbol{\Omega} = \nabla \times \boldsymbol{v}$ is $\frac{d}{dt}(\Omega + \Omega_i) + (\Omega + \Omega_i)(\nabla_\perp \cdot \boldsymbol{v}) = 0$. Here $\nabla_\perp$ is the gradient operator transversal to the magnetic field.

Using the normalizations $t \to \Omega_i t$, $(x, y) \to (x/\rho_s, y/\rho_s)$ and $\phi \to e\phi/T_e$, the equation for the electrostatic potential ϕ is obtained

$$\frac{\partial}{\partial t}\left(1-\nabla_\perp^2\right)\phi-\left(-\nabla_\perp\phi\times\hat{\boldsymbol{n}}\right)\cdot\boldsymbol{v}_d^*+\left(-\nabla_\perp\phi\times\hat{\boldsymbol{n}}\right)\cdot\boldsymbol{v}_{dT}\phi$$
$$+\left[\left(-\nabla_\perp\phi\times\hat{\boldsymbol{n}}\right)\cdot\nabla_\perp\right]\left(-\nabla_\perp^2\phi\right)$$
$$=0 \qquad (1)$$

where $\boldsymbol{v}_d^* = -\nabla_\perp \ln n_0 - \nabla_\perp \ln T_e$ and $\boldsymbol{v}_{dT} = -\nabla_\perp \ln T_e$. The two nonlinearities that appear in the equation have different physical effect. However, they are active on distinct space scales, as shown by a multiple space-time scales analysis [3,4]. By introducing a translation velocity u, the equation is expressed in a moving frame, $\eta = y - ut$. One takes $x_i = \varepsilon^i x$ and $t_i = \varepsilon^i t$, $\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \ldots$ for $\varepsilon \ll 1$. One space scale is specified by the choice $\phi_1 = \phi_1(x_0, x_1, x_2, \ldots, \eta_0, \eta_1, \eta_2, \ldots, t_0, t_1, t_2, \ldots)$, with the density and temperature variable on the $i = 1$ scale. The characteristic space scale for the potential is then ρ_s which is the typical extension of the dipolar vortex. The equation obtained in the first order in ε, is the Hasegawa-Mima equation.

A different dynamical equation is obtained on other space-time scale $\phi = \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \ldots$ and the density, the temperature and the velocity u are assumed to vary on the second (slower and larger) scale. Assuming that $\frac{1}{T(x_2)} + \frac{\kappa_n(x_2)}{u_2} \sim O(\varepsilon^2)$, the equation on the time scale of order 5 has the form of the Flierl-Petviashvili (FP) equation.

$$\Delta\phi = \alpha\phi - \beta\phi^2 \qquad (2)$$

where $\alpha = \frac{1}{\rho_s^2}\left(1 - \frac{v_*}{u}\right)$ and $\beta = \frac{T_e}{2u^2 e B_0^2 \rho_s^2}\frac{\partial}{\partial x}\left(\frac{1}{L_n}\right)$ (v_* is the electron diamagnetic velocity and L_n is the density gradient length). This equation has been derived by Petviashvili [5] in a study of the Jupiter's Red Spot. The dominant space variation is here on the scale $x_1 = \rho_s/\varepsilon$ which is much larger than the dipolar vortex scale.

3. The intermediate-scale structures

3.1 The monopolar vortex

The one dimensional form of the equation (2) has been solved on an infinite domain, obtaining as solution the KdV soliton [6]. The two dimensional equation has vortical monopolar solutions, well studied numerically [7,8]. By defining a functional of the solution expressed in one-dimensional (radial) geometry and taking the extremum of the functional under the condition of asymptotic decay, Petviashvili and Pokhotelov have found the approximate solution $\phi(r) = \frac{4.8\alpha}{2\beta}\left[sech\left(\frac{3}{4}\sqrt{x^2+(y-ut)^2}\,r\right)\right]^{4/3}$. The condition imposed to the Flierl-Petviashvili equation is to provide an isolated, finitely extended vortical solution, obeying boundary conditions at infinity (on the $2D$) of smooth decay. There is no known analytical form for an exact solution.

We consider that the insuccess in determining the analytical vortex solution has a fundamental motivation. First of all the equation does not pass the Painlevé test, so it is very likely that it is not integrable. This, in the definition that is suggested by the Inverse Scattering Transform method, means that there is no Lax pair of operators for this equation. This equation has close ressemblance with some approximative form of other nonlinear equations, for example the differential equation $\Delta\phi = \exp(\phi) - 1$, known as the Abelian-Higgs equation (AH), governing the vortices of the superconducting media. This equation has also been derived by Jacobs and Rebbi. An expansion of the right hand side $\Delta\phi = \exp(\phi) - 1 = \phi - \frac{1}{2}\phi^2$ leads to something close to the FP equation, especially because the FP equation has actually been derived under the neglect of the third order powers of ϕ. We have proved elsewhere [9] that the AH equation is integrable on periodic domains and we have provided analytical solutions in terms of Riemann *theta* functions. It is a known fact that equations derived as deformations of exactly integrable equations preserve some of their properties: solutions are robust and localised even if they are not exactly solitons. However it is difficult to give a formal characterisation of this kind of soft-nonintegrability.

In conclusion we claim that the monopolar vortex of the FP equation is only a manifestation of the close proximity (in function space) of an exactly integrable structure, related, most probably, to the Abelian Higgs model. This implies that we have to look for vortices at smaller scales, corresponding to the Hasegawa-Mima equation.

3.2 The exact periodic solution of the FP equation

In a previous work [10] we have determined an exact solution of the FP equation. The method consisted in looking for the trajectories of the one-dimensional solution singularities, in the complex plane of the spatial variables. The exact solution to the Petviashvilli equation is

$$\phi_s(x,y) = \frac{\alpha}{2\beta} + s\wp\left(iay + ibx + \omega \mid g_2 = \frac{3\alpha^2}{(s\beta)^2}\right) \qquad (3)$$

where a, b and s are parameters related by the condition $a^2 + b^2 = s\beta/6$ and $\wp$ is the doubly periodic elliptic Weierstrass function. Here ω is half of the period on the real axis of $\wp$.

Experimental measurements of the characteristics of the zonal flow have been performed on Doublet III-D tokamak [11]. In Ohmic and L-mode plasmas it has been found a perturbed potential $\tilde{\phi}_{rms} \gtrsim 10V$ and a flow shear $\omega_{E\times B} \sim 2 \times 10^5\ s^{-1}$. The value for the radial wavelength is in the range

$k_r\rho_s \in [0.1, 0.6]$ which means $\lambda_r \in (15, 30)\rho_s$. Due to the different sensitivity of the solution ϕ_s to the parameters, this set is sufficiently restrictive to determine its form. We have to take v_*/u very close to unity and $g_3 \sim -1500$ which gives: $\tilde{\phi}_{rms} \gtrsim 17V$, $\lambda_r \cong 17.4\rho_s$, $\omega_{E\times B} \sim 2.2 \times 10^5\ s^{-1}$. The relative amplitude of the perturbation results ~ 4%. An important experimental result is the radial spectrum of the perturbation. We have calculated $S(k_r)$ from the Fourier transform of the correlation of $\phi_s(x)$. The result is very close to Fig. 3 of Ref. [11], with two symmetric peaks at $k_{0r} \sim 4\ cm^{-1}$. The same sharp decay for $|k_r| < k_{0r}$ is observed, as described in Ref. [11].

3.3 Linear stability of the stationary periodic solution

The time variation of the stationary periodic solution is described by the equation from which the FP equation is derived at stationarity. We start from the equation with scalar nonlinearity, with the presence of a temperature gradient

$$\frac{\partial}{\partial t}\left(1-\nabla_\perp^2\right)\phi = \frac{\partial}{\partial y}\left[\left(-\nabla_\perp^2 + 4\eta^2\right)\phi\right] - \phi\frac{\partial\phi}{\partial y}$$

where $4\eta^2 \equiv 1 - \frac{v_*}{u} > 0$. To study the stability we take $\phi \to \phi_s(x, y) + \varepsilon(x, y, t)$, where $\phi_s(x, y)$ is the periodic flow solution (3). Taking into account the periodicity of the solution we obtain the Mathieu equation from which one of the dispersion relations is

$$\frac{1}{k_y} = \frac{\omega \pm Q^{1/2}}{P+1}\left\{1 \mp \left[1 + \frac{2(P+1)}{\left(-\omega + Q^{1/2}\right)^2}\right]^{1/2}\right\} \qquad (4)$$

where $P = -4\eta^2 + \frac{1}{2}\phi_s|_{min} + \frac{1}{2}\phi_s|_{max}$ and $Q = (\delta x)^2\ (\phi_s|_{max} - \phi_s|_{min})^2$. Evaluating the terms, the linear dispersion relation becomes

$$\frac{1}{k_y} \simeq \frac{\omega \mp \sqrt{Q}}{P+1}\left(1 \mp \frac{\sqrt{2}}{\sqrt{Q}}\right)$$

which at the limit provides an estimation for the typical length of fragmentation along the poloidal direction, $\lambda_\perp \sim 2.6\rho_s$.

3.4 Relevance for the high confinement phenomenology

The stability properties change when the polarisation drift nonlinearity is included, together with the scalar one. Some perturbations (like, for example, the monopolar vortices embedded inside a layer) have a very long stability time since they accomodate with the background flow by reshaping the distribution of local velocities. However, perturbations that have a high amplitude relative to the background flow and/or do not conform to the flow geometry lead to the destabilisation and eventually destruction of the flow pattern. It is important however to note that the numerical simulations (presently with limitted precision) and analytical considerations suggest that the destruction of the flow pattern does not immediately lead to an arbitrary random field: a fundamental process is the generation of small (~ ρ_s) space scale vortices. We can show that the regular flow structure is replaced by a lattice of vortices which is reminescent of the exact solution of a closely related nonlinear equation. This solution evolves, due to the weak interaction between the vortices, to an ensemble of quasi-independent vortices that collide inelastically and become of various amplitudes plus a surrounding drift wave radiation. From this random field the ITG instability may regenerate the structure of eddies and reinstate the transport.

From the numerical simulations showing structural instability of the FP equation we draw the conclusion that a more careful study must be done of the small scale vortical structures arising in the dynamics of the Hasegawa-Mima equation.

4. The Hasegawa-Mima vortices

The known stable dipolar vortex of the Hasegawa-Mima equation is the Larichev-Resnik solution consisting of two monopolar lobes with positive and negative signs. However this is not an exact solution. We are looking for other possible vortex solutions. In particular we are interested in the solutions at stationarity, since the numerical simulations clearly indicate that large scale vortical structures are created at late time. There is however an important difficulty in this approach. The naive stationary form of the HM equation is extremely general and can be solved by a huge class of functions. This means that the physics is hardly controlled and suggests a reformulation of the problem. In the similar case of the Euler equation, this was possible because the ideal fluid model is known to be equivalently described as a collection of point-like vortices moving in plane under the action of a field whose propagator is the inverse of the $2D$ Laplacean [12]. This model leads to the sinh-Poisson equation for the stationary states.

There is a similar situation for the Hasegawa-Mima equation. In meteorology it has been developed a model consisting of the motion in plane of point-like vortices with a short range interaction potential. We start our analysis from the basic assumption that the HM equation is equivalent to this model. In a similar approach as we have developed for the sinh-Poisson equation, we will formalize this discrete model as a field theory. In this framework it is possible to identify an energy functional and in certain situation the functional takes a particular form, a sum of squares. Then the extremum is easy to find and results in differential equations which are simpler than the original equations of motion. This is a well known physical property in field theory, known as self-duality.

In developing this approach we face certain multitude of possible choices, which probably will be better clarified in future. We are constantly led by the condition to attain the self-dual states of the system.

The model used by Stewart and Morikawa consists of an ensemble of point like vortices moving in a velocity field. The local value of the velocity is derived from a vectorial potential. The later is constructed from contributions of all the vortices, each contribution being expressed in terms of the modified Bessel function of the second kind: $\varphi = \Sigma_j \omega_j K_0$ $(|\boldsymbol{r} - \boldsymbol{r}_j|)$. The vortex motion in plane is given by

$$\frac{\mathrm{d}\boldsymbol{r}_j}{\mathrm{d}t} = -\nabla\varphi \times \hat{\boldsymbol{n}} \tag{5}$$

This means that the vortices are shielded since the modified Bessel function of the second kind K_0 decays exponentially at large argument.

The interacting N vortices of the Eq. (5) can be described by a Hamiltonian $H = \Sigma_{i>j}\Sigma_j \omega_i \omega_j K_0$ $(|\boldsymbol{r} - \boldsymbol{r}_j|)$. We will construct a Lagrangean density for this system, keeping in mind that the major properties must be: (1) short range potential; and (2) topological nature of the elementary vortices.

The fact that the potential φ for the HM equation is a statistical potential does not clarify definitively from what source it might be derived, except for the fact that we now know that the Chern-Simons term must be present in the Lagrangean. We note however that there are at least two ways to obtain a short range for the interaction field (gauge field): (1) include the Maxwell contribution to the Lagrangean density; the Maxwell term and the Chern-Simons term both have numerical coefficients and a combination of these coefficients appears as a mass term in the differential equation which gives the gauge field. This is a known effect and the fact that the gauge boson aquires mass by simply considering Maxwell term in addition to Chern-Simons term is independent of the presence of interaction with other (matter) fields. The latter can simply be represented by a current. (2) By the Higgs mechanism, a gauge boson aquires mass when the symmetry is spontaneously broken for a matter field with self-interaction, after fixing the origin in a vacuum given by a degenerate minimum of the self-interaction potential. Then the mass of the boson is dependent on the form of the nonlinear potential.

4.1 From the discrete system of vortices to continuum

The Lagrangean density must have the standard structure: gauge field Lagrangean, matter field (sometimes called in the following the Higgs field) and interaction (between the matter field ϕ and the gauge field A_μ. The scalar nonlinearity is not arbitrary although it leaves certain freedom: we have to obtain the short range potential and the self-duality. The second of the method mentioned above for the photon to get massive and the interaction become short range, can be realised with only the Maxwell gauge field Lagrangean density plus the Lagrangean density of the self-interacting complex scalar (matter) field ϕ

$$L_{AH} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left|D_\mu\phi\right|^2 - V\left(|\phi|^2\right)$$

where the nonlinear potential is $V(|\phi|^2) \equiv \frac{\lambda}{4}(|\phi|^2 - v^2)^2$ (v being a constant) and $D_\mu\phi \equiv (\partial_\mu + ieA_\mu)\phi$. The Lagrangean density gives the equations $\partial_\mu F^{\mu\nu} = J^\nu$ where the current source is $J_\nu \equiv -e^2 |\phi|^2 [A_\nu - \frac{1}{e}\frac{\partial}{\partial x^\nu}(N\theta)]$ (N is the integer winding number of the phase of ϕ). At very large distance, the scalar function ϕ is close to its asymptotic value, $|\phi|^2 = v^2$, and the integral of this current around the boundary contour is $\oint J_\nu \mathrm{d}x^\nu = -e^2v^2\oint A_\nu \mathrm{d}x^\nu + ev^2\oint \partial_\nu(N\theta)\,\mathrm{d}x^\nu$. The gauge potential A_ν allow to define a magnetic field $B\hat{\boldsymbol{e}}_z = \nabla\times\boldsymbol{A}$ perpendicular on the plane. For a single vortex we can replace the core by a δ function. Then $\nabla\times\boldsymbol{J} + e^2v^2\boldsymbol{B} = 2\pi N\,ev^2\delta^{(2)}(\boldsymbol{r})\hat{\boldsymbol{e}}_z$. Using the Maxwell equation $\nabla\times\boldsymbol{B} = \boldsymbol{J}$ one arrives at the equation

$$\nabla\times\nabla\times\boldsymbol{B} + e^2v^2\boldsymbol{B} = 2\pi Nev^2\delta^{(2)}(\mathbf{r})\hat{\boldsymbol{e}}_z$$

This equation has the solution (Nielsen-Olesen)

$$\boldsymbol{B} = ev^2 N\, K_0(m_V r)\hat{e}_z$$

where the mass of the gauge boson is identified $m_V \equiv ev = e|\phi|_\infty$. Generalizing for a number of parallel vortices, with vorticities of signs $n_a = \pm 1$ the equation becomes

$$\nabla\times\nabla\times\boldsymbol{B} + e^2v^2\boldsymbol{B} = 2\pi ev^2\sum_a n_a\delta^{(2)}(\boldsymbol{r} - \boldsymbol{r}_a)\hat{\boldsymbol{e}}_z$$

and the solution is

$$\boldsymbol{B} = ev^2\sum_a n_a K_0(m_V|\boldsymbol{r} - \mathrm{r}_a|)\hat{\boldsymbol{e}}_z$$

From this derivation we learn that the Higgs mechanism leads to a finite mass for the photon, m_V which must be $m_V \equiv e|\phi|_\infty = 1/\rho_s$ and $e = c_s$.

This model is able to support the self-dual states. For the explicit form one has to write the squared differential operator at stationarity as shown by Bogomolnyi

$$|\boldsymbol{D}\phi|^2 = |(D_1 \pm iD_2)\phi|^2 \mp eB|\phi|^2 \pm \varepsilon^{ij}\partial_i J_j$$

where the spatial part of the current is, as above $J_j = \frac{1}{2i}[\phi^*(D_j\phi) - \phi(D_j\phi)^*]$. The static energy functional $\mathcal{E} = \int \mathrm{d}^2x\,[\frac{1}{2}B^2 + |\boldsymbol{D}\phi|^2 + \frac{\lambda}{4}(|\phi|^2 - v^2)^2]$ is for $\lambda = 2e^2$, $\mathcal{E} = \int \mathrm{d}^2x\,[E_1^2 + E_2^2] \mp ev^2B$, where $E_1 \equiv B \mp e(|\phi|^2 - v^2)$, $E_2 \equiv |D_\pm\phi|$, and $D_\pm \equiv D_1 \pm iD_2$. The self-duality means here the vanishing of the squared terms plus the condition between the constants of the theory (implicitely between the masses)

$$D_\pm\phi = 0\,,\ B = \pm e\left(|\phi|^2 - v^2\right)$$

At this point the energy is bounded from below by the magnetic flux $\mathcal{E} \gg v^2 2\pi N$. We write the complex scalar field $\phi = \sqrt{\rho}\exp(i\chi)$ and then the vanishing of the covariant derivatives $D_\pm$ yields $eA_j = -\partial_j\chi \mp \frac{1}{2}\varepsilon_{ij}\partial_j \ln\rho$. This must be substituted in the equation of B

$$\nabla^2\ln\rho = 2e^2\left(\rho - v^2\right)$$

With the substitution $\rho = \exp(\psi)$ (and rescalings) the self-duality equation becomes

$$\Delta\psi = \exp(\psi) - 1 \tag{6}$$

The physical vorticity is obtained as $\omega = |\phi|^2 = \rho$ and, in physical units, $\rho = \nabla^2\psi + \Omega_i$. The Eq. (6) can be solved exactly on periodic domains and gives a lattice of vortices in plane, which is what the numerical simulations show.

The most general model of a field mediating the interaction of vortices (represented by a density) and which is able to generate fields with short range and to present self-dual stationary states is the nonabelian Maxwell Charn-Simons Higgs theory. The Lagrangean density is

$$L = \left(\begin{array}{l} -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa}{4} \varepsilon^{\mu\nu\rho} F_{\mu\nu} A_\rho \\ -\mathrm{tr}\left[\left(D_\mu \phi \right)^\dagger \left(D_\mu \phi \right) \right] - V\left(\phi, \phi^\dagger \right) \end{array} \right.$$

It has been found that the only possiblity this model has to reach self-dual states is to choose a scalar field nonlinear self-interaction given by a sixth order potential $V(\phi, \phi^\dagger) = \frac{1}{4\kappa^2}$ tr $Y^\dagger Y)$ where $Y = [[\phi, \phi^\dagger], \phi] - v^2\phi$ and the trace is defined over the finite dimensional representation of the Lie algebra to whom A and ϕ belong.

The self-duality equations lead to $\partial_+\partial_- \ln|\phi|^2 = \frac{2}{\kappa^2} |\phi|^2 (2|\phi|^2 - v^2)$. After several transformations, with the substitution $\ln|\phi|^2 = \psi$ the equation becomes

$$\Delta\psi = \exp(3\psi)\sinh(\psi)$$

This equation is also exactly integrable and its solutions consists of lattices of vortices In Ref. [13] a graphical representation of the solution of Eq. (6) is provided.

5. Conclusion

Robust vortical structures appear in a transition from zonal flows to the random field, from which the ITG eddies may grow again. The possibility of such solution is nontrivial since they are different of the only known dipolar vortex (Larchev-Resnik) of the HM equation. We have proved that such solutions exist at stationarity and we have provided the particular forms the HM equation takes in this case. The equations are exactly integrable and exhibit lattices of vortices.

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