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Excitation of Zonal Flows and Fluid Closure

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Abstract

The excitation of zonal flows by ion temperature gradient driven modes is studied by analytical and numerical methods using a reactive advanced fluid model. It is shown analytically that the dominant nonlinearity is the convection in the energy equation. The excitation of zonal flows is particularly strong just above linear marginal stability leading to a nonlinear upshift in the critical gradient for steady transport. Turbulence simulations confirm the overall expectations and a nonlinear upshift in agreement with the Dimits nonlinear particle simulations is obtained. The strong excitation of zonal flows just above marginal linear stability is due to the fluid resonance in the energy equation. This is exactly where the fluid closure is made so the excitation of zonal flows depends sensitively on this closure.

Keywords:

fluid closure, zonal flows, drift waves, turbulence, transport

1. Introduction

One of the most important aspects of turbulent systems is the presence of background flows [1-7]. Such flows may be due to pressure difference as in pipe flow, rotating planets or, in plasmas, neoclassical effects [8], pressure gradients or external excitation by beams. However, there can also be flow generated by the turbulence itself [1-7]. Such flows are generally parallel (streamers) or perpendicular (zonal flows) to the background pressure gradient. We will here focus on zonal flows which tend to reduce transport and thus have a regulating effect on it. The importance of flows for tokamak transport has been recognized relatively late. The beginning actually was in connection with the L to H mode transition where most models involve the effects of radial electric fields which generate rotation at the edge giving an edge transport barrier. Later flows were also used to explain internal transport barriers. Recently nonlinear gyrokinetic simulations have shown strong effects of zonal flows. In particular Dimits [4] discovered a nonlinear upshift in the critical ion temperature gradient for the onset of steady ion thermal transport and some simulations indicate that flows have a general tendency to regulate transport. For very short wavelengths ETG (electron temperature gradient) modes, streamers were found to be very important in nonlinear gyrokinetic simulations [9]. As will be argued in this paper, kinetic resonances are playing an important role for the nonlinear upshift in kinetic simulations. So how is it possible for fluid drift wave models without zonal flows to describe tokamak transport? First we need to distinguish two separate effects of zonal flows. One is the already mentioned nonlinear upshift.

Another is the damping of modes with long radial wave lengths. Since the $\boldsymbol{E} \times \boldsymbol{B}$ nonlinearity tends to isotropize turbulence, a damping of eddies with long radial size means that the zonal flow acts as a general sink for long wavelengths. This sink can be caused by sheared poloidal flows in general *i.e.* generated either externally or by the turbulence. Thus, in codes without external sources for the flow, the nonlinearly generated zonal flow will be essential for damping out long wavelengths thus generating an absorbing boundary in this limit. Since a reflecting boundary for long wavelengths increases the turbulence level strongly, this can introduce a very strong sensitivity of the transport level to the flow. Since, however, long wavelengths are more easily torn apart by the flow, having an absorbing boundary for long wavelengths is much easier for a large radial box size. Since the effective radial box size in the experiments is quite large, an absorbing boundary for long wavelenths will usually be a realistic assumption. The other aspect, *i.e.* the nonlinear upshift, is only important sufficiently close to the critical gradient. In the Cyclone case [4] the nonlinear upshift was 50 % but the experiment from which the data were taken (D-Ⅲ-D 81499) had a gradient which was 73 % above the linear critical gradient at half radius. Thus the experiment was well above the nonlinear upshift and there our fluid model, without nonlinear upshift, was in reasonably good agreement with Dimits gyrokinetic simulation. This situation can be expected

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to occur generally for collisionless plasmas with strong heating. Thus a fluid model which assumes absorbing boundary for long wavelengths should be applicable for most present day tokamak experiments in steady state.

We will in the present paper analyze the effects of zonal flows in kinetic and fluid simulations. In particular the effects of the fluid and kinetic resonances turn out to be important. This will naturally connect to the questions of the fluid closure which will be discussed in detail.

2. Fluid model

The main part of this work will be based on the reactive fluid model first applied to ITG (ion temperature gradient) transport in [10]. It uses the collisionless parts of the Braghinskii fluid model where the argument for the closure is nonlinear effects in velocity space rather than collisions. This closure has been discussed in several papers and in the book [11]. Also a collisionless derivation has been made for anisotropic temperatures [12]. For isotropic temperatures we have the collisionless energy equation:

$$\frac{3}{2}\left(\frac{\partial}{\partial t}+\vec{v}_i\cdot\nabla\right)T_i + P_i\nabla\cdot\vec{v}_i = -\nabla\cdot\vec{q}\ , \tag{1a}$$

where

$$\vec{q} = \vec{q}_{*i} \equiv \frac{5}{2}\frac{P_i}{m_i\Omega_{ci}}(\vec{e}_{\parallel}\times\nabla T_i). \tag{1b}$$

We note that thermal force effects are collisional although they appear as reactive in the Braghinskii formulas. Thus they have been excluded. It is the heat flow given by eq. (1b) that defines the closure. It means that we can express the heat flow in the lower moments n and T which means that we have a closed system. Another type of closure is developed in [13] (see also [14, Sec. **11**]) on the basis of the stochastic treatment in [15]. Using the relation:

$$\nabla\cdot\vec{q}_* = -\frac{5}{3}n\vec{v}_*\cdot\nabla T + \frac{5}{3}n\vec{v}_D\cdot\nabla T, \tag{2}$$

where V_* and V_D are the diamagnetic and magnetic drifts, we obtain the temperature perturbation:

$$\frac{\delta T_i}{T_i} = \frac{\omega}{\omega - 5\omega_{Di}/3}\left[\frac{2}{3}\frac{\delta n_i}{n_i} + \frac{\omega_{*e}}{\omega}\left(\eta_i - \frac{2}{3}\right)\frac{e\phi}{T_e}\right], \tag{3}$$

where ω_* *and* ω_D are diamagnetic and magnetic drift frequencies. Equation (3) is comparatively simple but implicit due to the presence of the density perturbation. For Boltzmann electrons, however, eq. (3) directly gives a simple relation between temperature and potential perturbations. We will here only consider this case.

Equation (3) contains the fluid resonance at $\omega = (5/3)\omega_{Di}$. Since we keep it we can describe both the adiabatic regime $\omega \gg \omega_{Di}$ and the isothermal regime $\omega \ll \omega_{Di}$. However we postulate that due to nonlinear effects in velocity space we can use eq. (3) also when $\omega \cong (5/3)\omega_{Di}$. This is actually the "advanced" feature of the present fluid model.

3. The fluid closure

For comparison with nonlinear kinetic theory in general and, in particular, for the nonlinear upshift, it is important to analyze the relations between the fluid and kinetic resonances. When used in combination with the ion continuity equation and the usual low frequency fluid drifts, the temperature perturbation eq. (3) leads to a three pole density response in the presence of parallel ion motion. If we include higher order moments, we get a higher order density response in terms of poles (fluid resonances). In general we have a density response of the type:

$$\frac{\delta n}{n} = \omega_*^{\alpha_n}\frac{\omega-\omega_*+\ldots}{(\omega-\alpha_1\omega_D)(\omega-\alpha_2\omega_D)\ldots.(\omega-\alpha_n\omega_D)(\omega-k_{\parallel}v_{th})}\frac{e\phi}{T_e} \tag{4a}$$

for a fluid response of order $n + 1$. Such a response can also be written in the form:

$$\frac{\delta n}{n} = \sum\frac{\omega-\beta_j\omega_*}{\omega-\alpha_j\omega_D}\frac{e\phi}{T_e} \tag{4b}$$

by means of splitting into partial fractions. Now eq. (4b) is of the form of the response due to a sum of cold beams. It was shown by Dawson [16] that the usual Landau damping is recovered when such a sum is extended to infinity when the cold beams correspond to a discretization of a Maxwellian velocity distribution. Thus we conclude that the kinetic resonance will emerge if we go to infinite order n in eq. (4a). The kinetic resonance will give an imaginary part corresponding to linear drift resonances and Landau-damping. The imaginary part is, thus, expected to be the main addition obtained by going to infinite order. Gyro-Landau models [17,18] thus include a complex heat flow. The simplest generalization of eq. (1b) was obtained in [18]. There the closure was made by adding a complex heat flux to eq. (1b) and matching this to linear kinetic theory. Basically a gyrofluid model is obtained by the replacement

$$q \rightarrow q_* + iq_{gL}\ , \tag{5}$$

where q_{gL} is the Gyro-Landau resonance representing the influence of infinitely many higher order moments. There are also higher order gyro-Landau models that add the gyro-Landau resonance in the equation for a higher moment. It is obvious that this gyro-Landau resonance will differ from that added directly to the energy equation. This means that *The fluid resonances contribute a part of the full linear kinetic resonance.* Of course, once the system has been closed we can, by eliminating higher order moments algebraically, calculate a new q_{gl} to be used in eq. (5). Thus the formulation in eq. (5) is quite general.

We will now briefly recall the collisionless fluid derivation [12] of eq. (1). The derivation actually leads to anisotropic temperatures with one energy equation for the parallel temperature perturbation and one for the perpendicular. The nonlinear parts of these equations will contain curvature terms that tend to isotropize the temperature perturbations. Because of this we will assume isotropic temperature perturbation. However, the main ingredient in this derivation is to ignore the irreducible part of the four velocity correlation. We will here formally include it in the form:

$$< v_i v_j v_k v_l > = < v_i v_j >< v_k v_l > + < v_i v_k >< v_j v_l > + ... + G \,,$$

where

$$G = < v_i v_j v_k v_l >_{irr} \,. \tag{6}$$

This means that G is the part of the four velocity correlation that can not be reduced to products of two velocity correlations. G is, in fact, the first higher order fluid moment that has been omitted from our model. Just as for any other fluid moment there must be a transport equation for it. We write it in the form:

$$\frac{\partial G}{\partial t} = \frac{\partial}{\partial r}(\chi_G \frac{\partial}{\partial r})G + S_G \,, \tag{7}$$

where χ_G is the effective transport coefficient for G and S_G is the source.

Our argument for closure is that *there will be no source* ($S_G = 0$) in eq. (7) if the heat source is not resonant with the turbulent fluctuations we consider. The velocity space picture is that nonlinear effects will continuously take particles out of resonance. For comparison we note that for modes driven externally by velocity space resonances, the source will be resonant with the fluctuation (*e.g.* for Fishbone or TAE modes). In this case there will be a *source in velocity space* which will continuously fill in resonant particles.

We can write the drift kinetic equation in the form:

$$\frac{\partial f}{\partial t} + (\vec{e}_{\parallel} v_{\parallel} + \vec{v}_D + \vec{v}_E) \cdot \nabla f - \frac{q_c}{m} \nabla_{\parallel} \phi \frac{\partial f}{\partial v_{\parallel}} - \frac{q_c}{m} \nabla \phi \cdot (\frac{\vec{v}_{\nabla B}}{v_{\perp}} \frac{\partial f}{\partial v_{\perp}} + \frac{\vec{v}_{\kappa}}{v_{\parallel}} \frac{\partial f}{\partial v_{\parallel}}) = S_v \,. \tag{8}$$

Here S_v is the source in velocity space and q_c is the charge. Such a source will always be present in connection with heating and/or current drive. For perturbations, we are usually considering eq. (8) in (ω, k) space. The velocity distribution would then relax towards a state where the source exactly balances the nonlinear velocity space terms for a resonant external source. If, however, S_v does not have Fourier components in the (ω, k) regime of the considered turbulence, it will not contribute. Thus we can introduce the term "correlated source" for the case when fluctuations are driven by the source in velocity space and "uncorrelated source" for the case when S_v does not have Fourier components in the regime of our turbulence. For the background distribution function we furthermore assume that there will not be a source filling in a Maxwellian distribution at the typical phase velocity of our turbulence. We note that *e.g.* ion cyclotron heating will have a frequency at least two orders of magnitude above the typical frequency of drift wave turbulence and will not be resonant even with the drift waves with the shortest frequencies. These will furthermore be strongly damped by viscosity. Now deriving fluid equations from eq. (8) we will obtain a transport equation of the type eq. (7) where the external sources will not contribute to S_G. We have one more case where S_G will not contribute. This is if S_G averages out on the transport time scale. This will be the case in a rather coherent situation with particle trapping. The energy exchange between wave and particle will then clearly average to zero in a bounce period. As we know, the nonlinear effects will all the time work in the direction of taking particles out of resonance and as we can see here, there are several possible ways in which this can happen. This leads us to the following closure strategy: We include all moments that have sources in the experiment [11]. This means that we include also $\vec{q}_*$ since it depends only on n and T. Now the fluid resonances associated with the moments that have sources in the experiment will be maintained due to these sources. Since these resonances are a part of the full kinetic resonance, this means that the velocity distribution does not have to become completely flat even in the quasilinear limit when G relaxes to zero. Another relevant aspect is that lower order moments sometimes can give sources for higher order moments. An example is Ohmic heating where a current gives a heat source. We note, however, that this is a completely random process. Such a process would not give a source for G. A further support for this point is also that the closure $G = 0$ was obtained for a semiconductor plasma by using the entropy principle for an isolated system [19]. Furthermore collisions gave a source only for the reducible part of G. This again focuses interest on the external sources. The general closure problem for driven systems is, of course, quite complicated. It is, however, possible to study idealized models of such systems. The most simple case is actually to replace the sources by fixed gradients. In steady state the sources of density and temperature will exactly balance the transport in such a way that the gradients became stationary. The real sources, may in general, also have other effects. Thus fixing the gradients is equivalent to applying ideal sources which have no other influence than to maintain the gradients. An example of such a study was the Cyclone project [4].

4. Zonal flows

As it turns out, the description of the nonlinear (Dimits) upshift, which is an effect of zonal flows, puts exceptional requirements on the fluid closure. The reason for this is that the Dimits upshift is caused by the convective nonlinearity in the energy equation which is sensitive to the fluid resonance. Actually the resonance in the energy equation is exactly at marginal linear stability in the strong ballooning limit. It is given by [20]:

$$\omega = \frac{5}{3}\omega_{Di}\,, \tag{9a}$$

$$\eta_{ith} = \frac{2}{3} + \frac{10}{3}\varepsilon_n\ ;\ \ \varepsilon_n = 2L_n / R. \tag{9b}$$

Since there is a pole in the temperature perturbation at marginal stability, it will be large just above marginal stability. Then also the convective nonlinearity $\vec{v}_E \cdot \nabla T$ will be large and there will be a strong nonlinear excitation of zonal flow.

We have calculated the nonlinear coupling factor for the zonal flow, using two different orderings in the Taniuti reductive perturbation method [21]. We consider only self-interaction of a mode at the correlation length. Nonlinear simulations have shown that this is also the linearly fastest growing mode [22]. With a standing wave structure in the radial *(x)* direction with mode number k_m and propagating wave in the poloidal *(y)* direction we can write the dimensionless (normalized by *T/e*) potential $\Phi_0^{(2)}$ of the flow as [6,7]:

$$\Phi_0^{(2)} = k_m L k_y^2 \rho_s{}^2 T \left|\phi_1^{(1)}\right|^2 \sin 2k_m x\ . \tag{10}$$

Here $\phi_1^{(1)}$ is the dimensionless drift wave potential, here assumed to be of order ε, L is the system size in x and T is the nonlinear coupling factor. It contains the denominator of eq. (3) in the denominator but also a factor $\omega(1 + k_y^2\rho_s{}^2) - \omega_L$ where $\omega_L = \omega_r - \mathrm{i}\gamma_d$, ω_r is the real part of the local eigenfrequency and $\gamma_d = (1 + \frac{5}{3\tau})\frac{\varepsilon_n|s|}{4q_s}\omega_*$ is the magnetic shear damping *(s is the usual shear parameter, q_s the safety factor and $\tau = T_e/T_i$)*. Thus we have a resonance which can be detuned both by linear instability and by magnetic shear damping. Both these features show up in nonlinear turbulence simulations [7].

The resonant ordering gives a Zakharov like system where, however, the low frequency equation has only first derivatives. It is [23]: (compare also the resonant ordering in [21])

$$\frac{\partial \Phi_0^{(2)}}{\partial t} - c_f \frac{\partial \Phi_0^{(2)}}{\partial \xi} = c_n \frac{\partial \left|\phi_1^{(1)}\right|^2}{\partial \xi}\,. \tag{11}$$

We note that eq. (11) becomes of the same form as eq. (10) in the quasistationary case when the time derivative can be ignored. We can then integrate eq. (11) with respect to ξ. A resonance of the same type as in eq. (10) is thus also present in eq. (11). The behaviour of the couplingfactor c_n at marginal linear stability was studied in [23] where also a gyrofluid resonance was included in one case for comparison. As expected, the gyrofluid term had a significant detuning effect on the resonance giving a much smaller $|c_n|$ than with our reactive model. This seems to be the reason why the IFS-PPPL model got only half the kinetic nonlinear upshift in the Cyclone simulations [4] while recent simulations [7] with our model gave the same nonlinear upshift as Dimits nonlinear kinetic simulations [4]. This is shown in Fig. 1 where the dotted line is the quasilinear result [4] and the full line is the nonlinear simulation from [7]. Our simulations also confirmed the detuning of zonal flows by magnetic shear predicted by our analytic calculations [6]. This trend, *i.e.* an increase of transport with magnetic shear, is also in agreement with simulations of ETG turbulence [9] as it seems for similar reasons.

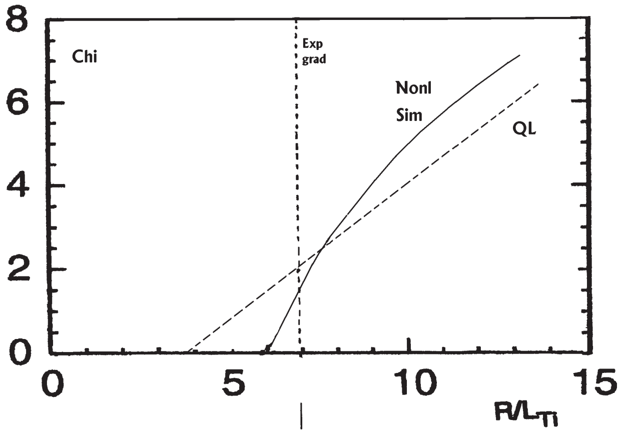

Fig. 1 The nonlinear upshift shown as the difference between threshold for transport in a quasilinear model and a nonlinear simulation including zonal flow. Parameters from ref. [7] corresponding to data from ref. [4].

It is very interesting to compare the above results for excitation of zonal flows in a fluid model with the corresponding kinetic results. The main paper to compare with is here the Cyclone paper [4] where particle collisions were not included. As pointed out above, there is complete agreement with the nonlinear upshift between our fluid simulation [7] (Fig. 1) and Dimits simulation. The upshift in the kinetic simulation was, however, sensitive to the convergence with regard to number of particles. Thus only half the upshift was obtained with 4 million particles while all simulations with 8 million particles or more got the published upshift. The maximum number of particles was 134 million particles. It has recently been pointed out [24,25] that it is the resonant regimes in phase space that are most sensitive to the convergence with regard to number of particles and that a particle loading with more particles in the regimes of wave-particle resonances reduces the total number of particles needed. Since convergence with regard to number of particles was obtained in [4] above R/Lt = 7 (experimental gradient) also with 4 million particles we conclude that wave particle resonances are important in the nonlinear upshift regime and more specifically: the wave particle resonance is important for obtaining the correct nonlinear upshift. Reducing the number of particles in the resonant regime decreases the nonlinear upshift. Thus we may conclude that the nonlinear upshift is due to the kinetic resonance in kinetic simulations. As we have already seen, the nonlinear upshift is due to the fluid resonance in an advanced fluid simulation. We then conclude that obtaining the correct nonlinear upshift in a fluid simulation is a sensitive test on the closure.

5. Collisions

As pointed out above, the Cyclone simulations had ideal sources and no collisions. This means ideal conditions to apply our fluid closure (the present closure would work also in a collision dominated case but this is not realistic for application to tokamak cores). A remaining question is that turbulence which cascades to smaller and smaller scales can cause an apparent effective dissipation. We can describe this as an additional diffusion acting in a similar way as viscosity. Such an effect was recently added to the Mattor Parker system. Mattor and Parker [26] introduced a nonlinear closure into a system of three interacting slab ITG modes. The nonlinear closure essentially meant including a nonlinear frequency shift into the plasma dispersion function. The solution was very similar to the usual three wave interaction but the closure gave a slight reduction in the nonlinear oscillations. The addition of diffusion to this system gave a slow decrease in time of the oscillations indicating relaxation to a nonlinear equilibrium [27]. Qualitatively the time development became similar to that in the Cyclone simulations. The final transport level can be considerably lower than for the gyrofluid, Hammet-Perkins closure [18] just as the Cyclone simulations gave an asymptotic transport level that was considerably lower than that obtained in the gyrofluid simulations.

In real systems we will also have effects of electron-ion collisions. Here an important aspect is the result in [19] that collisions only give a source for the reducible part of G. It is also very instructive to read ref. [28] where the beam-plasma instability was simulated using a particle code. There the only observable effect of collisions was to damp the wave. Since drift waves are maintained by gradients in real space this would be of no consequence as long as the turbulence level is not strongly modified. We recall that ITG modes are fairly insensitive to collisions. The weak nature of the turbulence in [28] is not an important limitation here. In fact nonlinear detuning of wave particle resonances are likely to be stronger in strong turbulence [29,30]. A direct comparison between collision frequency and trapping frequency or inverse time of quasilinear flattening also shows that effects of collisions would be subdominant for typical drift wave saturation amplitudes in core tokamak plasmas [31]. Another aspect of collisions is that they can damp zonal flows in the nonlinear upshift regime [5].

6. Discussion

We have here discussed recent developments in fluid modelling and compared to nonlinear kinetic results. In particular the fact that fluid resonances form part of the kinetic resonance is very important in understanding how a reactive fluid model can be in agreement with a nonlinear kinetic code in a regime where both kinetic and fluid resonances are very important. We have distinguished a simple limit where the reasons for discrepancy between fluid and kinetic models can be narrowed down. This is the case of ideal sources without collisions that was studied in the Cyclone work. This is the most fundamental limit in which the closure can be studied and should be understood first. The nonlinear upshift due to zonal flows was found to be due to the kinetic resonance in nonlinear kinetic simulations and due to the fluid resonance in a fluid simulation. The comparison of these simulations is thus a sensitive test of the fluid closure.

Concerning general influence of zonal flows in experiments, it seems that a situation with absorbing boundaries in k-space and gradients above the nonlinear upshift region is most likely. In transient experiments [32] with time varying heating, however, the nonlinear upshift may become more important, in particular for stiffness, since the profiles may transiently enter the nonlinear upshift regime.

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