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Plasma Turbulent Transport and Fine-Scale Structures of Phase Space Distribution Function

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Abstract

Recent results from kinetic simulations of steady and quasisteady states of the ion temperature gradient (ITG) driven turbulence are reviewed in focus on the fine-scale structures of the phase space distribution function associated with the anomalous transport. Importance of proper treatment for the fine structures in numerical simulations of the collisionless ITG turbulence is emphasized by a parameter survey for the velocity space resolution. Preliminary results of the flux tube simulation of the toroidal ITG mode are also given for the linear benchmark test and the collisionless damping in a tokamak configuration.

Keywords:

gyrokinetics, turbulence, transport, computer simulation

1. Introduction

The plasma turbulent transport has long been one of the central subjects in the magnetic confinement fusion research. Since the collision frequency is much lower than characteristic ones of the turbulence in a core region of tokamaks with high temperature, in order to study the turbulent transport, one needs to deal with a kinetic description of the magnetized plasma such as the gyrokinetic equation. Numerical simulations of the gyrokinetic plasma turbulence have rapidly developed in the last decade [1], and revealed several important aspects of the anomalous transport. For example, turbulence suppression by the self-generated zonal flow has been investigated by many authors [2], where interactions of low-order velocity space moments of the one-body distribution function f and the electromagnetic field have been discussed intensively. In the collisionless or weakly-collisional plasma, however, a velocity space profile of f should have more degrees of freedoms than those characterized by the low-order moments such as the density, the flow velocity, the temperature and so on. Even though a transport flux is defined by a correlation of the low-order moments and the electromagnetic field, fine-scale structures of f should be properly treated in numerical simulations.

The one-body velocity distribution function f in a collisionless plasma turbulence is stretched and folded by the Hamiltonian flow in the phase space while preserving its amplitude. This is known as the phase mixing which leads to continuous generation of fine-scale fluctuations of f, and is closely related to the paradoxical argument how the collisionless turbulence described by the kinetic equation with the time-reversal symmetry causes the steady transport flux with no dissipation mechanism. If the steady transport driven by constant density and/or temperature gradients is observed through coarse-graining of f with fine-scale structures, a *quasisteady state* of the collisionless turbulence should be realized, where high-order velocity-space moments of f continue to grow while keeping low-order ones constant in average [3,4].

In comparison of collisionless turbulence simulation results with actual plasma experiments, it is implicitly assumed that an asymptotic behavior of the transport coefficient in a low-collisionality limit agrees with the collisionless one in the quasisteady state. The finite collisionality is indispensable to realizing the *real statistically steady states* for all order-moments of f. The entropy variable associated with fluctuations is produced by the turbulent transport on a macro velocity scale corresponding to the low-order moments. Then, it is transfered in the phase space by the $\boldsymbol{E} \times \boldsymbol{B}$ nonlinearity and the phase mixing, and is, finally, damped by collisions acting on a micro scale associated with the high-order moments.

In order to confirm the critical issues on collisionless and weakly-collisional turbulence given in the above two paragraphs, we have investigated the slab ion temperature gradient (ITG) driven turbulence by means of the Eulerian (Vlasov) kinetic simulations with high velocity-space resolution [5-7]. Results obtained in our previous works are reviewed in the next section with the aim of introducing the

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background of the simulation studies given in the latter part of this paper. Artificial effects of coarse numerical grid on the transport coefficient are shown in Sec. **3**, which demonstrates importance of the velocity-space resolution in simulations of the kinetic plasma turbulence. Our recent progress in the toroidal ITG simulation is also reported in Sec. **4**. A summary is given in Sec. **5**.

2. A review of the slab ITG simulation by the Eulerian kinetic code

2.1 Simulation model

We consider the following simulation model which is defined in a two-dimensional slab geometry with the translational symmetry in the z-direction. The periodic boundary condition is used in the x and y directions. The uniform magnetic field is set in the y-z plane such that $\boldsymbol{B} = B(\hat{z} + \theta\hat{y})$ for $\theta \ll 1$. The governing equations are derived from a $v_\perp$-integral of the electrostatic gyrokinetic equations [8] by ignoring the parallel nonlinear term and by assuming $\delta f_{\boldsymbol{k}}(v_\parallel, v_\perp) = \tilde{f}_{\boldsymbol{k}}(v_\parallel)\,F_M(v_\perp)$, where F_M is the Maxwellian velocity distribution. We also assume constant density and temperature gradients of the background ions in the x-direction with scale-lengths of $L_n^{-1} \equiv -\mathrm{d}(\ln n)/\mathrm{d}x$ and $L_T^{-1} \equiv -\mathrm{d}(\ln T_i)/\mathrm{d}x$, where L_n, $L_T \gg L$ (L denotes the perpendicular system size). Therefore, we arrive at the following equations given in the wave number space $\boldsymbol{k} = (k_x, k_y)$, such that

$$\partial_t \tilde{f}_{\boldsymbol{k}} + \mathrm{i}\Theta v_\parallel k_y \tilde{f}_{\boldsymbol{k}} + \sum_{\boldsymbol{k}=\boldsymbol{k}'+\boldsymbol{k}''} \left(k_y' k_x'' - k_x' k_y''\right)\Psi_{\boldsymbol{k}'} \tilde{f}_{\boldsymbol{k}''}$$
$$= -\mathrm{i}k_y \Psi_{\boldsymbol{k}}\left[1 + (v_\parallel^2 - 1 - k^2)\eta_i/2 + \Theta v_\parallel\right]F_M(v_\parallel) + C_i(\tilde{f}_{\boldsymbol{k}}) \qquad (1)$$

and

$$\left[1 - \Gamma_0(k^2)\right]\phi_{\boldsymbol{k}} = e^{-k^2/2}\int \tilde{f}_{\boldsymbol{k}}(v_\parallel)\mathrm{d}v_\parallel - \tilde{n}_{e,\boldsymbol{k}}, \qquad (2)$$

where the electric potential $\phi_{\boldsymbol{k}}$ is related to $\Psi_{\boldsymbol{k}}$ by $\Psi_{\boldsymbol{k}} = e^{-k^2/2}\phi_{\boldsymbol{k}}$ with $k^2 = k_x^2 + k_y^2$. Also, $\Theta = \theta L_n/\rho_i$ and $\eta_i = L_n/L_T$. The background electron temperature $T_e = T_i$ and the adiabatic electron response are also assumed. Here, we consider a limiting case with no zonal flow component of $k_y = 0$ by fixing $\tilde{f}_{k_y=0} = \phi_{k_y=0} = 0$. The instability drive is contained in the first group of terms on the right-hand side of Eq. (1). The last term on the right-hand side denotes the ion-ion collision term for which we employ the Lenard-Bernstein model collision operator,

$$C_i(\tilde{f}_{\boldsymbol{k}}) = \nu\partial_{v_\parallel}[\partial_{v_\parallel} + v_\parallel]\tilde{f}_{\boldsymbol{k}}(v_\parallel), \qquad (3)$$

with the collision frequency ν normalized by v_{ti}/L_n. More complete descriptions of the simulation model as well as the normalization are given in Refs. [5,6].

Equations (1) and (2) are numerically time-integrated by means of the nondissipative integrator for the collisionless case [9-11] or the fourth-order Runge-Kutta-Gill (RKG) method for the weakly collisional ones. The $\boldsymbol{E} \times \boldsymbol{B}$ advection term is calculated by the pseudo-spectral method with the 3/2-rule for de-aliasing.

2.2 Steady and quasisteady states of ITG turbulence

From Eqs. (1) and (2), one finds a balance equation of entropy variable defined by a functional, $\delta S = \Sigma_{\boldsymbol{k}} \int \mathrm{d}v_\parallel |\tilde{f}_{\boldsymbol{k}}|^2/2F_M$, that is,

$$\frac{\mathrm{d}}{\mathrm{d}t}(\delta S + W) = \eta_i Q_i + D, \qquad (4)$$

where, Q_i, W, and D mean the perpendicular ion heat flux, the potential energy, and the collisional dissipation, respectively. Their explicit definitions are as follows;

$$Q_i = \sum_{\boldsymbol{k}}\int \mathrm{d}v_\parallel(-\mathrm{i}k_y e^{-k^2/2}\phi_{\boldsymbol{k}})v_\parallel^2 \tilde{f}_{-\boldsymbol{k}}/2,$$
$$W = \sum_{\boldsymbol{k}}(2 - \Gamma_0)|\phi_{\boldsymbol{k}}|^2/2,$$

and

$$D = \sum_{\boldsymbol{k}}\int \mathrm{d}v_\parallel \tilde{f}_{-\boldsymbol{k}} C_i(\tilde{f}_{\boldsymbol{k}})/F_M$$
$$= -\nu\sum_{\boldsymbol{k}}\int \mathrm{d}v_\parallel |\partial_{v_\parallel}\tilde{f}_{\boldsymbol{k}} + v_\parallel \tilde{f}_{\boldsymbol{k}}|^2/F_M < 0.$$

Since δS is rewritten as $\delta S = S_M - S_m$ within the second order for $\delta f = f - F_M$ (where $S_M = -\int \mathrm{d}^3v\, F_M \ln F_M$ and $S_m = -\langle \int \mathrm{d}^3v\, f \ln f\rangle$ represent macroscopic and microscopic entropies, respectively), we call it the entropy variable, hereafter. Here, $\langle\ldots\rangle$ means the spatial average.

Existence of the quasisteady state in the collisionless ITG driven turbulence is confirmed by our Eulerian kinetic simulation with high resolution for the velocity space [5], where monotonic increase of δS balances with the transport in a saturated turbulent flow, such that

$$\mathrm{d}(\delta S)/\mathrm{d}t \approx \eta_i Q_i. \qquad (5)$$

It means that δf never reaches to the steady state in the collisionless case, even if a steady transport flux defined by the low-order moments is observed. Continuous generation of fine-scale structures of δf in the velocity space, due to the phase mixing caused by the parallel advection term on the left-hand-side of Eq. (1), is responsible for the increase of δS.

In the weakly-collisional case, not only low-order but also all-order moments of the distribution function can be statistically steady. This is because the fine-scale structures of δf in the velocity space are smoothed out by the collision operator with the second-order derivative. It is also numerically confirmed that $\mathrm{d}(\delta S)/\mathrm{d}t \approx 0$ and

$$\eta_i Q_i \approx -D > 0 \qquad (6)$$

in the real statistically steady state of the slab ITG turbulence [6].

2.3 Collision frequency dependence

We have also investigated collision frequency dependence of the ion thermal transport coefficient χ_i defined as $\chi_i \equiv Q_i / \eta_i$. A parameter survey for a wide range of ν shows a logarithmic dependence of the transport coefficient on relatively large values of ν while χ_i approaches a value in the collisionless case for sufficiently low collision frequency. This conclusion agrees with a concept that the quasisteady state is an idealization of the real steady state in the weak-collisionality limit [12].

A spectral analysis of δf in the velocity space has also been made by means of the Hermite-polynomial expansion [6,7]. The obtained spectrum has almost the same values at n = 1, 2, and 3 even for different values of ν (where n denotes the order of the Hermite-polynomials), which agrees with χ_i being independent of ν in the low collisionality cases. This is also consistent with the conjecture by Krommes and Hu that '*flux determines dissipation*' [3]. The entropy variable produced in the macro velocity scale on the low-n side is transfered toward the micro scale with high-n values and the large wave numbers ($\boldsymbol{k}$) by the phase mixing and the $\boldsymbol{E} \times \boldsymbol{B}$ nonlinearity. Then, it is dissipated by the collision term on the high-n side.

2.4 Methodological issues in numerical simulations of the collisionless plasma turbulence

In order to simulate the collisionless plasma dynamics with the time-reversibility, we have developed an Eulerian simulation method based on the symplectic integrator [9-11]. The nondissipative simulation scheme for solving Eq. (1) is designed so as to preserve the time-reversibility and the phase-space integral of f^2 that is an invariant of the Vlasov and the collisionless drift (gyro-) kinetic equations. The numerical accuracy in dealing with the time-reversible plasma dynamics including the phase mixing process has been verified by the plasma echo simulation [11] as well as the 3-mode ITG problem [9,10]. The sufficient velocity-space resolution to the fine-scale structures of δf is indispensable to keep soundness of the numerical simulation, which is demonstrated in the next section by comparing the results with different grid spacings in the velocity space.

The kinetic-fluid closure model is based on existence of the quasisteady state of the collisionless plasma turbulence. The kinetic simulation result for the collisionless ITG turbulence agrees well with the collisionless fluid one with the nondissipative closure model (NCM) [12,13]. By employing the NCM in the fluid equations, the time-reversibility of unstable linear kinetic eigenmodes can be preserved, which is essential to fluid simulations not only of the 3-mode ITG problem [12] but also of the turbulent transport [13].

3. Effect of velocity space resolution

As summarized in the above, it is important in kinetic simulations of collisionless plasma turbulence to keep sufficient resolution to the fine-scale structures of δf generated by the phase mixing. We have observed artificially enhanced transport flux in cases with a coarser grid in the velocity space as shown below. For comparison, we have carried out the collisionless slab ITG turbulence simulations for $\Delta v / v_{ti}$ = 5/16, 5/32, 5/64, and 5/256 with the same physical and other numerical parameters as given in Ref. [5]. Time-evolutions of the transport coefficient χ_i are plotted in Fig. 1 (top and middle) for different Δv. As typically seen in cases for $\Delta v / v_{ti}$ = 5/16 and 5/32, χ_i gradually increases in the turbulence after t ~ 300 or 400. The higher values of χ_i averaged from t = 800 to 1000 are observed for larger Δv in the bottom of Fig. 1. For $\Delta v / v_{ti}$ = 5/16 and 5/32, χ_i is twice larger than that reported in Ref. [5] where the sufficient resolution was kept during the simulation ($\Delta v / v_{ti}$ = 5/4096). As Δv becomes smaller, the time-averaged χ_i approaches to its proper value (shown by the horizontal dashed line) obtained in the quasisteady state of the collisionless turbulence. This is because the finest scale of δf in the velocity space reaches to the grid scale at earlier time for larger Δv. Thus, the result from the simulation with a coarser grid suffers from the larger aliasing error. The obtained result demonstrates the importance of keeping enough velocity-space resolution in the collisionless ITG turbulence simulation. In order to avoid the aliasing error, one should stop running the collisionless simulation before the finest scale-length of δf becomes comparable to the grid size. Then, if the collisionless turbulence had not reached to the quasisteady state yet, it is necessary to employ the finer resolution.

4. Development of a toroidal ITG simulation code for a flux tube geometry

We are developing the gyrokinetic-Vlasov simulation code for the toroidal ITG mode in a flux tube geometry for a tokamak configuration. The coordinate system and the boundary condition are based on the work by Beer [14], where concentric circular magnetic surfaces with a large aspect ratio are assumed. We consider the gyrokinetic equation for ions [15,16], the adiabatic electron response, and the quasi-neutrality. As the first trial, we deal with passing ions only, while neglecting the mirror force term. Spatial coordinates in radial (x) and field-line-label (y) directions are discretized by the Fourier expansion, while the parallel (z) derivatives are approximated by the fifth-order upwind finite difference. The parallel velocity ($v_{\|}$) and the magnetic moment (μ) are chosen for the velocity space coordinates which are discretized by grid points. The RKG method is used for the time-integration. The simulation code is well optimized in order to achieve high efficiency for vector and parallel operations.

The linear growth rate of the toroidal ITG modes for the Cyclone DIII-D base case parameters [1] are shown in Fig. 2, where the solid and dashed lines represent the real and imaginary parts (denoted by ω_r and γ) of the eigenfrequency obtained by the linear gyrokinetic code [17], respectively.

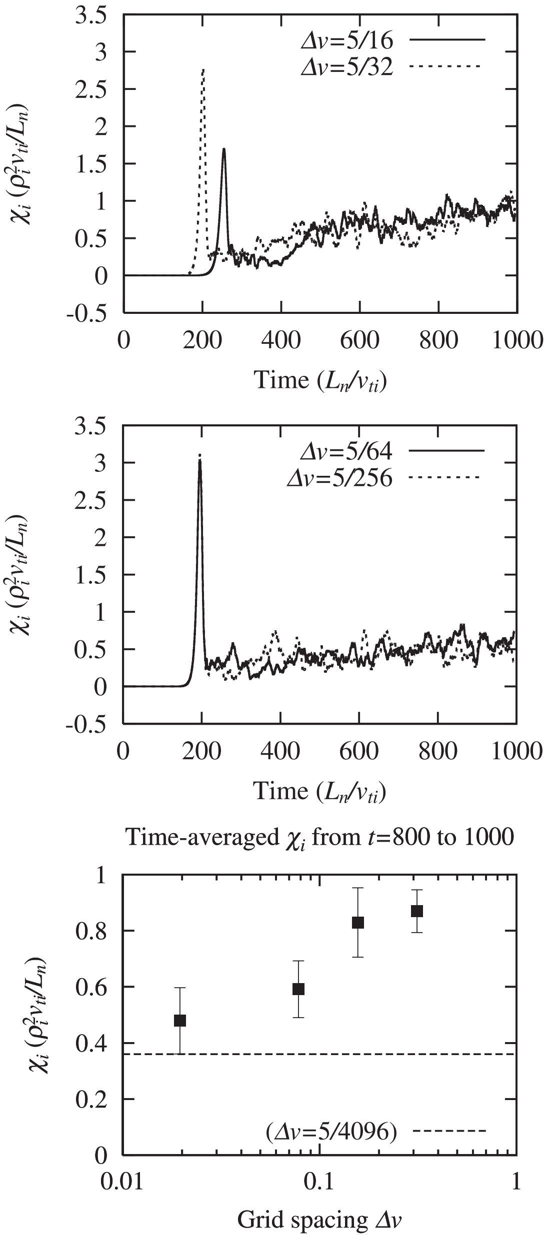

Fig. 1 (top and middle) Time-evolutions of the ion thermal transport coefficient (χ_i) obtained by the collisionless ITG turbulence simulations for $\Delta v / v_{ti}$ = 5/16, 5/32, 5/64, and 5/256. (bottom) Time-averaged χ_i from t = 800 to 1000 for different Δv in comparison to the case with the high velocity-space resolution of $\Delta v / v_{ti}$ = 5/4096 (horizontal dashed line).

Here, we have employed (84, ±20, ±32, ±64, 32) modes/grid points in the five-dimensional (k_x, k_y, z, $v_{\|}$, μ)-space, where k_x and k_y denote the wave numbers in the x- and y-directions, respectively. Solid squares and open circles indicate ω_r and γ given by the gyrokinetic-Vlasov simulation results which agree well with the linear code prediction.

In the absence of the electric field, the initial density perturbation $\tilde{n}$ with the ballooning type mode structure is damped due to the phase mixing associated with the toroidal particle drift. Its asymptotic behavior is proportional to t^{-2}

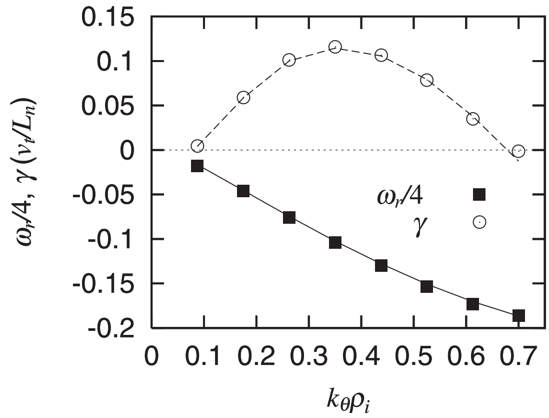

Fig. 2 Real frequency (ω_r) and linear growth rate (γ) of the toroidal ITG modes for the Cyclone DIII-D base case obtained by the gyrokinetic-Vlasov simulation code. Solid and dashes lines indicate ω_r and γ obtained by the linear gyrokinetic code [17].

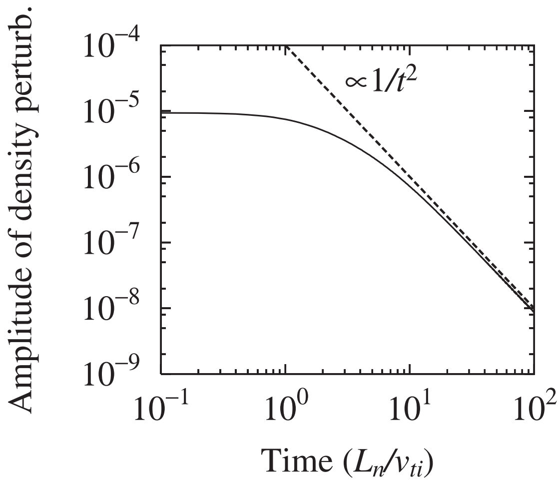

Fig. 3 Collisionless damping of the density perturbation in a tokamak configuration in absence of the electric field (solid). Dashed line represents the asymptotic behavior ($\propto t^{-2}$) predicted in Ref. [17].

[17], since not only the parallel advection term but also the toroidal magnetic drift terms contribute to generation of fine-scale structures of the distribution function in the phase space. The collisionless damping process can be successfully reproduced by our simulation as shown in Fig. 3, where a finer numerical grid for the ($v_{\|}$, μ)-space is employed, such as (±192, 64) grid points, in order to continue the run up to t = $100\,L_n/v_{ti}$. In lack of the resolution, otherwise, $\tilde{n}$ unphysically grows at earlier time. The result demonstrates that, also in a tokamak configuration, treatment of the fine-scale structures of the distribution function is one of the key issues for simulating the collisionless damping.

5. Conclusions

In this paper, we have reviewed our recent progress in kinetic simulations of the steady and quasisteady states of the slab ITG turbulent transport. Even though the transport flux is directly related to the correlation between the low-order moments and the electric field, high-order moments associated with the fine-scale structures of the distribution function should also be taken into account accurately. This is because they are related with each other through the transfer of the entropy variable in the phase space. In numerical simulations of the kinetic plasma turbulence, thus, it is essential to keep enough resolution for the fine structures which are spontaneously generated by the phase mixing in the velocity space. Lack of the resolution may lead to artificially enhanced transport flux through the aliasing error as demonstrated in this paper. In addition, taking into account of the time-reversibility of the collisionless kinetic equation is also a key issue in numerical studies of the collisionless turbulence, which was a fundamental motivation in development of the nondissipative closure model [12,13]. In a toroidal configuration, the phase mixing by the toroidal magnetic drift and the parallel advection terms generates the fine-scale structures. A toroidal version of our Eulerian kinetic simulation code is successfully applied to the collisionless damping in a tokamak configuration. Simulations of the toroidal ITG turbulent transport is currently in progress and will be reported elsewhere.

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