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Drift Waves and Zonal Flows

BALESCU Radu
Association Euratom - Belgian State,
Physique Statistique et Plasmas, CP 231
Université Libre de Bruxelles,
B-1050, Brussels, Belgium
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Abstract

Drift wave turbulence in a plasma is analyzed on the basis of the Wave Liouville Equation, describing the evolution of the distribution function of wave packets (quasiparticles) characterized by position $\boldsymbol{x}$ and wave vector $\boldsymbol{k}$. A closed kinetic equation is derived for the ensemble-averaged part of this function; it has the form of a non-Markovian advection-diffusion equation describing coupled diffusion processes in $\boldsymbol{x}$- and $\boldsymbol{k}$-spaces. General forms of the diffusion coefficients are obtained in terms of Lagrangian velocity correlations. The latter are calculated in the decorrelation trajectory approximation, duly accounting of the trapping of quasiparticles in the rugged electrostatic potential. The analysis of individual decorrelation trajectories provides an illustration of the fragmentation of drift wave structures in the radial direction and the generation of long-wavelength structures in the poloidal direction that are identified as zonal flows.

Keywords:

plasma turbulence, zonal flow, wave kinetic equation, non-Markovian diffusion

1. Introduction

We consider a plasma in presence of a strong magnetic field (such as a tokamak). Locally this magnetic field can be considered as constant, equal to $\boldsymbol{B}$. A Cartesian reference system is then defined by three unit vectors: $\boldsymbol{e}_z$ directed along $\boldsymbol{B}$, $\boldsymbol{e}_x$ in the direction of the density gradient, mimicking the radial direction in a tokamak (the temperature is taken constant, for simplicity), and $\boldsymbol{e}_y = \boldsymbol{e}_z \times \boldsymbol{e}_x$, representing the poloidal direction (shearless slab geometry). If $\boldsymbol{B}$ is sufficiently strong, the collisionless motion of the particles can be approximated by the quasi-two-dimensional motion of their guiding centres, characterized by their position vector $\boldsymbol{x} = (x, y)$. The plasma is in a turbulent state, produced by a fluctuating electrostatic potential denoted by $\varepsilon\phi(\boldsymbol{x}, t)$, where ϕ is a dimensionless function. It is well known that in a tokamak, under certain circumstances, a *transport barrier* may appear. At its location the turbulence level and the transport are significantly reduced.

An enormous amount of literature on transport barriers has appeared in the past twenty years; the main results are reviewed in Refs. [1-3]. An important mechanism of formation of such a barrier is produced by the action of a *shear flow* in the direction perpendicular to the magnetic field and to the gradients (i.e., the poloidal direction in a tokamak). This flow will carry along *nonuniformly* the drift wave structures, thus deforming, and finally breaking them into smaller substructures. It will be shown that under certain circumstances, the drift wave turbulence is able to generate spontaneously a large scale poloidal shear flow. This large-scale turbulent poloidal flow is called a *zonal flow*. In order to avoid confusion, let us stress that in the present work we use a slightly more general definition of the term "zonal flow" compared to other authors. We understand under this term a structure with poloidal wave vector k_y much smaller than the radial one k_x, *but not ecessarily zero*. Its effect is again, understandably, a tearing apart of the drift wave structures and their fragmentation, strikingly seen in the massively parallel numerical simulations published in Ref. [4]. The most elaborate explanation of this effect was initiated by Diamond and his collaborators [5], and is still being actively pursued (see e.g., Refs. [2,6,7]).

In the present paper we are mainly interested in the mechanism of generation of the zonal flows. The "classical" discussion of this problem is exposed in Ref. [2]. Its argument can be summarized as follows. The fragmentation produced by the complex zonal flows results in a *diffusive increase* of the mean square of the radial wave-vector $\langle k_x^2 \rangle$, which is a measure of the inverse square of the correlation length λ_x^{-2} in the radial direction (i.e., the direction of the density gradient). As a result, the drift wave frequency $\omega_d(\boldsymbol{k}) = k_y V_* / (1 + \rho_s^2 k^2)$ will *decrease*. (Here $V_* = \rho_s c_s / L_n$ is the electron diamagnetic velocity, ρ_s the ion-acoustic Larmor radius, c_s the ion-acoustic velocity and L_n the density

Corresponding author's e-mail: rbalescu@tiscalinet.be

gradient length; k_y is the y-component of the wave vector $\boldsymbol{k}$, mimicking the poloidal component k_θ in a tokamak, $k^2 = k_x^2 + k_y^2$.) It follows from the conservation of the action density $N(\boldsymbol{k}) \approx e(\boldsymbol{k})/\omega_d(\boldsymbol{k})$ that the drift wave energy $e(\boldsymbol{k})$ must also decrease, it being transferred to the large-scale zonal flow energy. We are thus in presence of an instability that produces an inverse cascade in k_y-space. After its saturation, a new state appears, in which the level of small-scale turbulence, hence the anomalous radial transport is significantly reduced: a *transport barrier* has been created.

This process is described in Ref. [2] by a diffusion coefficient in $\boldsymbol{k}$-space, estimated in the quasilinear approximation. The authors of Ref. [2] correctly note that their quasilinear treatment of $\boldsymbol{k}$-space diffusion is of limited validity. When the turbulence level is not very weak, new features become important. The rugged fluctuating potential landscape (as it appears in the numerical simulations of Ref. [4]) produces transient *trapping of the particle trajectories* in its troughs or around its peaks, as well as formation of coherent structures [7]. The difficult nonlinear treatment of these processes has been approached in various ways in the recent literature; it will not be further discussed here.

In the present paper we study the trapping effect on the global transport. We base our treatment on the *decorrelation trajectory (DCT) method*, introduced and developed in several works, starting in 1998 [8], and extended here in order to include the random walk in $\boldsymbol{k}$-space. The method and the main results obtained sofar will be outlined here; the details of the calculations and additional references will be found in a recent publication [9].

2. The wave kinetic equation.

Of particular importance for the characterization of the turbulent state is the *action density* $N(x, \boldsymbol{k}, t)$, which is a conserved quantity. It is a quadratic functional of the potential, averaged over small scales (with respect to ρ_s), and slowly varying in $\boldsymbol{x}$-space. Its precise definition, in the case of drift waves, was derived in Ref. [6], together with its equation of evolution:

$$\partial_t N(\boldsymbol{x},\boldsymbol{k},t) = \frac{\partial H(\boldsymbol{x},\boldsymbol{k},t)}{\partial \boldsymbol{x}} \cdot \frac{\partial N(\boldsymbol{x},\boldsymbol{k},t)}{\partial \boldsymbol{k}} - \frac{\partial H(\boldsymbol{x},\boldsymbol{k},t)}{\partial \boldsymbol{k}} \cdot \frac{\partial N(\boldsymbol{x},\boldsymbol{k},t)}{\partial \boldsymbol{x}}, \tag{1}$$

with:

$$H(\boldsymbol{x},\boldsymbol{k},t) = \omega_d(\boldsymbol{k}) + \boldsymbol{k} \cdot \boldsymbol{V}(\boldsymbol{x},t), \tag{2}$$

where $\boldsymbol{V} = (\varepsilon c/B)(\boldsymbol{e}_z \times \nabla\phi)$ is the (long-range part of the) electric drift velocity. The remarkable feature here is the Hamiltonian structure of the evolution of a set of "quasi-particles", i.e., wave packets, characterized by the canonically conjugate variables $(\boldsymbol{x}, \boldsymbol{k})$. The action density plays the role of the phase space distribution of the quasi-particles, and obeys the Liouville equation (1), which can be written in terms of a Liouville operator L_W, defined as in statistical mechanics by the Poisson bracket of H and N:

$$\partial_t N = L_W N \equiv [H, N]. \tag{3}$$

The action density is a fluctuating quantity in the large-scale domain; hence the Wave Liouville Equation is a *stochastic differential equation*, whose associated characteristic equations are typical *Langevin equations*.

The distribution function is separated into an average part and a fluctuating part: $N(\boldsymbol{x},\boldsymbol{k},t) = n(\boldsymbol{x},\boldsymbol{k},t) + \delta N(\boldsymbol{x},\boldsymbol{k},t)$ where $n(\boldsymbol{x},\boldsymbol{k},t) = \langle N(\boldsymbol{x},\boldsymbol{k},t)\rangle$. Our first purpose is to derive a closed equation for the average distribution function. The strategy is standard [9,10]: the equation for the fluctuation is solved by the method of characteristics, and the result is substituted into the equation for the average, which becomes:

$$\begin{aligned}
&\partial_t n(\boldsymbol{x},\boldsymbol{k},t) + \boldsymbol{V}^g(\boldsymbol{k}) \cdot \frac{\partial}{\partial \boldsymbol{x}} n(\boldsymbol{x},\boldsymbol{k},t) \\
&= \int_0^t \mathrm{d}t_1 \left\{ \frac{\partial}{\partial \boldsymbol{x}} \cdot \overleftrightarrow{L}^{XX}(t-t_1) \cdot \frac{\partial}{\partial \boldsymbol{x}} + \frac{\partial}{\partial \boldsymbol{x}} \cdot \overleftrightarrow{L}^{XK}(t-t_1) \cdot \frac{\partial}{\partial \boldsymbol{k}} \right. \\
&\left. + \frac{\partial}{\partial \boldsymbol{k}} \cdot \overleftrightarrow{L}^{KX}(t-t_1) \cdot \frac{\partial}{\partial \boldsymbol{x}} + \frac{\partial}{\partial \boldsymbol{k}} \cdot \overleftrightarrow{L}^{KK}(t-t_1) \cdot \frac{\partial}{\partial \boldsymbol{k}} \right\} n(\boldsymbol{x},\boldsymbol{k},t_1),
\end{aligned} \tag{4}$$

where $\boldsymbol{V}^g(\boldsymbol{k}) = \partial\omega_d(\boldsymbol{k})/\partial\boldsymbol{k}$ is the (deterministic) group velocity of the drift waves. Equation (4) will be called the (true) ***Wave Kinetic Equation***. It is a *non-Markovian "advection-double-diffusion" equation* for the average distribution $n(\boldsymbol{x},\boldsymbol{k},t)$, describing two coupled diffusion processes, respectively in $\boldsymbol{x}$-space and in $\boldsymbol{k}$-space, combined with propagation (advection) in $\boldsymbol{x}$-space. Equation (4) contains the precise formulation of what Diamond and his group call the "random walk in $\boldsymbol{k}$-space".

Equation (4) introduces four 2×2 Lagrangian correlation tensors of the fluctuating $\boldsymbol{x}$-velocities $\boldsymbol{V}$ and the $\boldsymbol{k}$-velocities $\boldsymbol{W} \equiv -\partial[\boldsymbol{k} \cdot \boldsymbol{V}(\boldsymbol{x},t)]/\partial\boldsymbol{x}$. Typically:

$$L^{KK}_{r\,|\,s}(t-t_1) = \Big\langle W_r[\boldsymbol{x},\boldsymbol{k},t]\, W_s[\boldsymbol{x}(t_1),\boldsymbol{k}(t_1),t_1]\Big\rangle, \tag{5}$$

and similar definitions for the three other tensors. Assuming stationary turbulence, these functions only depend on the difference of the two times. *If* Eq. (4) could be Markovianized by neglecting the memory effect, i.e., setting $n(\boldsymbol{x},\boldsymbol{k},t_1) \approx n(\boldsymbol{x},\boldsymbol{k},t)$, it would become an "ordinary" multidimensional advection-diffusion equation, with the following coefficients, which are time-dependent, running diffusion coeficients:

$$D^{KK}_{r\,|\,s}(t) = \int_0^t \mathrm{d}t_1\, L^{KK}_{r\,|\,s}(t_1), \tag{6}$$

and similar definitions for the three other tensors. The asymptotic values $D^{KK}_{r\,|\,s}(\infty) \equiv D^{KK}_{r\,|\,s}$ are the ordinary diffusion coefficients. The Markovianization process is, however, not always justified [10].

3. The DCT method for zonal flows

An essential role in the nature of strongly turbulent phenomena is played by the trapping processes in the rugged potential landscape. It is therefore natural to study these phenomena by using the *decorrelation trajectory (DCT)* *method*, developed for "ordinary" drift wave turbulence in Ref. [8]. This method requires an extension in the present case.

We study the stochastic motion of a drift-wave quasiparticle in presence of the fluctuating electrostatic potential $\varepsilon\phi(\boldsymbol{x}, t)$. The fluctuations are defined statistically by a homogeneous and stationary Gaussian process, whose Eulerian correlation introduces a correlation length λ and a correlation time τ_c, such that $\langle \phi(\mathbf{0},0) \sim \phi(\boldsymbol{x},t) \rangle \approx 0$ when $|x| \gg \lambda$ *and/or* $t \gg \tau_c$. We go over to dimensionless variables defined as: $\boldsymbol{x} \Longrightarrow \lambda \boldsymbol{x}$, $\boldsymbol{k} \Longrightarrow \rho_s^{-1}\boldsymbol{k}$, $t = \tau_c \theta$. The Langevin equations of motion of the quasiparticle are written as:

$$\frac{\mathrm{d}\boldsymbol{x}(\theta)}{\mathrm{d}\theta} = K_d \boldsymbol{v}^g[\boldsymbol{k}(\theta)] + K\boldsymbol{v}[\boldsymbol{x}(\theta), \theta], \quad \boldsymbol{x}(0) = \mathbf{0}, \tag{7}$$

$$\frac{\mathrm{d}\boldsymbol{k}(\theta)}{\mathrm{d}\theta} = Kw[\boldsymbol{x}(\theta), \boldsymbol{k}(\theta), \theta], \quad \boldsymbol{k}(0) = \boldsymbol{k}^0. \tag{8}$$

(It is important to take an initial wave vector of non-zero length: an initial $|\boldsymbol{k}^0| = 0$ would remain zero for all times.) The three scaled velocities $\boldsymbol{v}^g, \boldsymbol{v}$ and $\boldsymbol{w}$ are related to the corresponding dimensional quantities as follows: $\boldsymbol{V}^g = (\lambda/\tau_c) K_d \boldsymbol{v}^g$, $\boldsymbol{V} = (\lambda/\tau_c) K \boldsymbol{v}$, $\boldsymbol{W} = (\rho_s \tau_c)^{-1} K \boldsymbol{w}$. We introduced here the two basic dimensionless parameters characterizing the turbulence: the *Kubo number* $\boldsymbol{K}$, related to the intensity ε of the fluctuations, and the *diamagnetic Kubo number*, related to the gradient length through V_*:

$$K = \frac{\tau_c}{\lambda}\frac{c\varepsilon}{B\lambda}, \quad K_d = \frac{\tau_c}{\lambda} V_*. \tag{9}$$

The transport problem requires the evaluation of the four Lagrangian correlations entering Eq. (4). We concentrate here on the last one, which introduces all the new features; the three others are treated in a completely similar fashion. Its corresponding dimensionless form, for a homogeneous and stationary turbulent state, is:

$$L^{KK}_{r\,|\,s}(\theta) = \left\langle w_r(\mathbf{0}, \boldsymbol{k}^0, 0)\, w_s[\boldsymbol{x}(\theta), \boldsymbol{k}(\theta), \theta\,] \right\rangle \tag{10}$$

We first note that all fluctuating quantities (including $\boldsymbol{v}$ and $\boldsymbol{w}$) are derived from the potential ϕ. As in all theories based on Langevin equations, the primary Eulerian potential autocorrelation has to be specified *a priori*. This quantity is assumed to be of the same form as in Ref. [8]:

$$\left\langle \phi(\mathbf{0}, 0) \phi(\boldsymbol{x}, \theta) \right\rangle = E(\boldsymbol{x}) T(\theta), \tag{11}$$

where $E(\boldsymbol{x})$ is a dimensionless function of the position, with a maximum at the origin, and $T(\theta)$ a similar function of time. All other Eulerian correlations of couples of fluctuating quantities $(\phi, \boldsymbol{v}, \boldsymbol{w})$ are deduced from (11) by appropriate differentiations [9].

The extension of the *decorrelation trajectory (DCT)* method follows the line of Ref. [9]. The basic step is the decomposition of the ensemble of realizations of the turbulent ensemble into subensembles. This decomposition is now different from Ref. [8], because not only the potential ϕ and its first derivatives (through $\boldsymbol{v}$), but also its second derivatives (through $\boldsymbol{w}$) enter the theory. The subensemble S is therefore defined as the set of realizations in which the potential ϕ, the two components of the velocity v_r, and the two components of the "$\boldsymbol{k}$-velocity" w_r have a given value at time 0:

$$S\colon \phi(\mathbf{0}, 0) = \phi^0,\ v_r(\mathbf{0}, 0) = v_r^0,\ -k_s^0 \nabla_r v_s(\mathbf{0}, 0) = w_r^0. \tag{12}$$

Let $P_0(\phi^0, v^0, w^0)$ be the probability distribution (pdf) of these initial values, assumed to be Gaussian. Then the $x-x$ component of the KK-Lagrangian correlation (10) is provided (exactly) by the superposition:

$$L^{KK}_{x\,|\,x}(\theta) = \int \mathrm{d}\phi^0 \mathrm{d}v^0 \mathrm{d}w^0 P_0(\phi^0, \boldsymbol{v}^0, \boldsymbol{w}^0)\, w_x^0 \left\langle w_x[\boldsymbol{x}(\theta), \boldsymbol{k}(\theta), \theta] \right\rangle^S, \tag{13}$$

where $\langle \dots \rangle^S$ denotes the average in the subensemble. Note the important fact that in the Lagrangian correlation in S: $\langle w_x[\mathbf{0}, \boldsymbol{k}^0, 0]\, w_x[\boldsymbol{x}(\theta), \boldsymbol{k}(\theta), \theta] \rangle^S$ the first factor can be taken out of the average because of Eq. (12), hence the calculation of this quantity is reduced to the simpler calculation of the average Lagrangian velocity $\langle w_x[\boldsymbol{x}(\theta), \boldsymbol{k}(\theta), \theta] \rangle^S$.

The pdf P_0 as well as the Eulerian averages of the velocities v and w in S can be calculated analytically as appropriate conditional averages. Typically, the Eulerian average k-velocity in the subensemble is of the form: $\langle \boldsymbol{w}(\boldsymbol{x}, \boldsymbol{k}, \theta) \rangle^S = \boldsymbol{w}^S(\boldsymbol{x}, \boldsymbol{k})\, \tau(\theta)$, with:

$$\boldsymbol{w}^S(\boldsymbol{x}, \boldsymbol{k}) = A\phi^0 + \boldsymbol{B}\cdot\boldsymbol{v}^0 + \boldsymbol{C}\cdot\boldsymbol{w}^0 \tag{14}$$

where $A, \boldsymbol{B}, \boldsymbol{C}$ are rather complicated functions of the Eulerian correlation E and of its derivatives [9]; v^S has a similar form. Thus, $\boldsymbol{w}^S$ and $\boldsymbol{v}^S$ depend on the parameters $\phi^0, \boldsymbol{v}^0, \boldsymbol{w}^0$ defining the subensemble S, as well as on the initial value of the wave vector $\boldsymbol{k}^0$.

The *decorrelation trajectory (DCT)* is defined as a *deterministic* trajectory $[\boldsymbol{x}^S(\theta), \boldsymbol{k}^S(\theta)]$ in the subensemble S of a fictitious quasiparticle whose motion is determined by Eqs. (7), (8) in which $\boldsymbol{v}$ and $\boldsymbol{w}$ are replaced by $\boldsymbol{v}^S$ and $\boldsymbol{w}^S$, respectively [8]. Finally, the *DCT approximation of the Lagrangian correlations* is obtained by replacing in Eq. (13) the subensemble *Lagrangian* average $\langle w_x[\boldsymbol{x}(\theta), \boldsymbol{k}(\theta), \theta] \rangle^S$ by the *Eulerian* average in S of the velocity, evaluated for each time at the deterministic point $[\boldsymbol{x}^S(\theta), \boldsymbol{k}^S(\theta)]$:

$$L^{KK}_{x\,|\,x}(\theta) = \int \mathrm{d}\phi^0 \mathrm{d}v^0 \mathrm{d}\boldsymbol{w}^0 P_0(\phi^0, \boldsymbol{v}^0, \boldsymbol{w}^0)\, w_x^0\, w_x^S[\boldsymbol{x}^S(\theta), \boldsymbol{k}^S(\theta)]\, \tau(\theta). \tag{15}$$

The calculation of the Lagrangian correlations is thus replaced by the calculation of Eulerian averages. The present problem is, however, significantly more complicated in the present case than for the simple drift wave turbulence treated in Ref. [8] because of the intimate coupling of the equations for $\boldsymbol{x}^S(\theta)$ and $\boldsymbol{k}^S(\theta)$. The Lagrangian correlations are now 5-fold integrals, which makes their numerical evaluation difficult and time-consuming. The final result depends on more parameters, viz. the Kubo number K, the diamagnetic

Kubo number $\boldsymbol{K}_d$, and also the initial wave vector $\boldsymbol{k}^0$. Some important qualitative features can be obtained from an analysis of the individual DCT trajectories.

4. Results and conclusions

We now consider the result of the numerical integration of the decorrelation trajectories. For definiteness, we assume the following form for the Eulerian potential autocorrelation (11):

$$E(x,y) = \exp(-\frac{x^2+y^2}{2}), \quad \tau(\theta) = \exp(-\theta) \qquad (16)$$

We define a subensemble S by the following values of the parameters: $\phi^0 = 2$, $v_x^0 = v_y^0 = 0$, $w_x^0 = w_y^0 = 1$. We also choose the fixed value $K_d = 1$. The choice $\boldsymbol{v}^0 = 0$ implies that the (fictitious) quasiparticle starts at time zero with the initial group velocity $\boldsymbol{v}^g(\boldsymbol{k}^0)$ and ends after a sufficiently long time ($\theta \gg \theta_{tr}$), when $v_n^S = 0, w_n^S = 0$, with the final group velocity $\boldsymbol{v}^g(\boldsymbol{k}^\infty)$. Note that, because of the factorization property $\langle \boldsymbol{w}(\boldsymbol{0},0)\boldsymbol{w}(\boldsymbol{x},\theta)t \rangle^S = \boldsymbol{w}^0 \langle \boldsymbol{w}(x,\theta) \rangle^S$ (and similarly for $\boldsymbol{v}$) [see Eq. (13)], the vanishing of $\boldsymbol{v}^S$, $\boldsymbol{w}^S$ implies the vanishing of the Lagrangian velocity correlation in the subensemble S. Thus the *trapping time* θ_{tr} is the time after which the fictitious quasiparticle is no longer correlated along its trajectory with its initial value. The trapping time θ_{tr} is determined numerically from the shape of the trajectories. (It should be clear that θ_{tr} relates to a single DCT.) Because of the limited amount of space available, we cannot reproduce here the figures representing the characteristic features of the DCT: they are found in Ref. [9]. We shall therefore summarize the main facts in words.

The initial wave vector is chosen as: $k_x^0 = 1$, $k_y^0 = 0$. As a result, the (unperturbed) group velocity at the initial time is: $\boldsymbol{v}^g(\boldsymbol{k}^0) = (1/2)\boldsymbol{e}_y$. Thus, in absence of turbulence ($K = 0$), the fictitious quasiparticle moves in a straight line in the y -direction, with a group velocity that is constant, because the wave vector $\boldsymbol{k}$ remains constant, Eq. (8).

We now consider a rather small value of the Kubo number, $K = 0.5$. Even in this relatively weak turbulence, the picture departs radically from the unperturbed motion. The beginning of a *trapping process in* $\boldsymbol{x}$*-space* is evident: the (fictitious) quasiparticle starts with the initial group velocity $\boldsymbol{v}^g(\boldsymbol{k}^0)$, but the turbulent velocity quickly overcomes the latter and deflects the particle from its rectilinear motion. The turbulent component $\boldsymbol{v}$ of the velocity $\boldsymbol{v}^S$ vanishes after a time that will be called the *trapping time* θ_{tr}, of order of the correlation time; in the present case its value is seen to be: $\theta_{tr} \approx 3$, after which the quasiparticle moves again uniformly, with its group velocity. Meanwhile, the latter has changed because the wave vector has changed. Thus the quasiparticle is deflected from its unperturbed motion in the y-direction and moves now in an oblique direction. We thus witness here a *refraction phenomenon* due to the passage through the turbulent medium. This refraction effect is also found (in a different context) in Ref. [7].

The wave vector starts from $\boldsymbol{k}^0$ and initially increases in both directions, the growth being largest in the x- ("radial") direction. Note that the increase of k_x^S is monotonous, until it reaches a saturation value after $\theta \approx \theta_{tr}$. The y- ("poloidal") component k_y^S quickly reaches a maximum: its monotonous growth is then stopped and reversed after a certain time ($\theta \approx 1$). It then changes sign, and finally ($\theta \approx \theta_{tr}$) reaches a negative saturation value. This is the manifestation of the *trapping process in* $\boldsymbol{k}$*-space*, which only affects (in the present situation) the y-component. This process ends after $\theta \approx \theta_{tr}$, after which k^S remains constant: $\boldsymbol{k}^S(\theta) \to \boldsymbol{k}^\infty$ [and $\boldsymbol{w}^S(\theta) \to 0$]. It is important to note that (in the present case) $|k_y^\infty| < k_x^\infty$ ($k_x^\infty = 1.54, k_y^\infty = -0.047$). As a result, in the asymptotic state ($\theta > \theta_{tr}$), the average length scale of the turbulence in the radial direction is much smaller than in the poloidal direction. This obviously explains the fragmentation process described qualitatively in the first section, and implies the *generation of a (small-k_y) zonal flow (in the subensemble S).*

To sum up: *during the trapping time θ_{tr}, the wave vector changes from its initial value to a constant asymptotic value with $|k_y^\infty| < |k_x^\infty|$; the fictitious quasiparticle is trapped during that time, and ends up moving with a new, deflected group velocity (**refraction**). In the final state, the length scale of the turbulence in the y-(poloidal) direction is much greater than in the x-(radial) direction.*

This analysis shows that the decorrelation process is now richer than in the case studied in Ref. [8]. In particular, the position and the wave vector move in a strictly coupled way and cannot be considered separately from each other. There is weak trapping in $\boldsymbol{x}$-space, but the trapping of the k_y wave vector component is already quite significant at this relatively modest value of the Kubo number ($K = 0.5$).

We consider next a situation of strong turbulence, $K = 10$, with the same subensemble parameters. The features which were merely sketched in the case $K = 0.5$ are now greatly enhanced. The (fictitious) quasiparticle is *strongly trapped in* $\boldsymbol{x}$*-space*: its motion is a "broken rotation" during the time θ_{tr} (which has barely changed, $\theta_{tr} \approx 3$). During the trapping time it is more and more strongly deflected. Its trajectory makes two turns, thus confining the quasiparticle to a finite region of space. But after θ_{tr}, the turbulent component of the velocity vanishes, and the fictitious quasiparticle moves away with the final group velocity $\boldsymbol{v}^g(\boldsymbol{k}^\infty)$. Meanwhile, both the k_x^S- and the k_y^S-components of the wave vector undergo a broken oscillation which stops at $\theta \approx \theta_{tr}$, when they reach the values: $k_x^\infty = 0.879$, $k_y^\infty = 0.965$. We thus witness *strong trapping in both* $\boldsymbol{k}$. In the present subensemble we can no longer speak of zonal flow generation: k_y^∞ is now of order one, and moreover it surpasses k_x^∞ in absolute value. The strong turbulence produces in this case a fragmentation both in the radial and in the poloidal directions.

Clearly, the present discussion refers to a single subensemble. Recently we obtained, in collaboration with I. Petrisor, some preliminary results of a global nature (these results are not contained in Ref. [9]: they will be the object of a forthcoming publication). Thus, the Lagrangian correlation of the radial $\boldsymbol{k}$-velocities $L^{KK}_{x\,|\,x}(\theta)$, Eq. (10) was calculated for two values of the Kubo number. For very weak turbulence, $K = 0.1$, it is a function decreasing monotonously in time from a positive value towards zero. This is a typical behaviour in the quasilinear regime. The corresponding running diffusion coefficient $D^{KK}_{x\,|\,x}(\theta)$, Eq. (6) is a positive monotonously increasing function of time, reaching a finite asymptotic value. For strong turbulence, $K = 7$, the behavior is quite different. The Lagrangian correlation decreases rapidly from an initial positive value, becomes negative, reaches a minimum at $\theta \approx 0.4$, after which it performs a broad oscillation, and ends up with a long negative power-law tail reaching zero. The corresponding running diffusion coefficient rises quickly to a maximum, and then decreases to a much smaller asymptotic value (remaining always positive, as it should). These features are typical signatures of the trapping effect.

In conclusion, we showed that the quasilinear approximation, valid in the weak turbulence regime, leads to seriously overestimated values of the diffusion coefficients in the regimes of large Kubo number. The decorrelation trajectories indicate a fragmentation of the radial structures and a spontaneous generation of zonal flows in particular subensembles. A global study of the coupled advection-diffusion process of drift waves requires a detailed scanning of the parameter space. This task will be the subject of forthcoming work.

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