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Particle Transport due to Magnetic Fluctuation in a Field-Reversed Configuration

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Abstract

Effects of fluctuating magnetic field in a Field-Reversed Configuration (FRC) on cross-field particle transport have investigated with the aid of numerical calculation of single particle orbits. The canonical angular momentum of a plasma particle ($P_\theta = mv_\theta r + q\psi$; $\psi > 0$ inside the separatrix) varies temporally in a fluctuating magnetic field used here, although it is a constant of motion in axisymmetric FRC. The calculation results show the ensemble averaged time derivative of the canonical angular momentum goes to zero, which suggests the guiding center of plasma particle does not suffer from a cross-field drift. The particle diffusion due to magnetic fluctuation, however, causes a radial flow in inhomogeneous FRC plasma

Keywords:

FRC, particle transport, drift, diffusion, fluctuation, confinement

1. Introduction

Particle transport mechanism in a Field-Reversed Configuration (FRC) is an ambiguous but important issue for the scaling of confinement time and future reactor study of FRC with advanced fuel [1]. Main mechanism of particle transport has not been determined as yet because of difficulties in diagnostic technique to measure the pulsing events. Since the obtainable data are restricted by an experimental approach, it is very important to investigate the cross-field transport in an FRC by numerical and theoretical methods. In the history of transport study of FRC, lower hybrid drift instability has believed to be the dominant process [2], however, it is disproved by Carlson's experiment [3]. Recently the combination of radial and open-field transport is also found to be possible to modify the scaling of gross confinement time [4]. One of the authors proposed the adiabaticity breaking process near X-points [5] enhances the end loss rate, which also increases the density gradient and resultant radial flow around the separatrix. Though the open-field transport is possible to affect the gross confinement time, we consider that the radial transport is still more dominant in the FRC confinement mechanisms. In the present paper, the fluctuating magnetic field is examined as the transport mechanism, and investigated its dependence on the temporal change of the canonical angular momentum, which results in the cross-field drift and diffusion of particles.

2. Model and Calculation Procedure

In order to show the cross-field transport of particles, we observe the single particle motion inside the separatrix of axisymmetric FRC whose equilibrium is written in the form:

$$\psi = \frac{1}{2} B_w r^2 \left(1 - \frac{r^2}{r_s^2} - \frac{z^2}{(l_s/2)^2} \right) \tag{1}$$

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where B_w, r_s and l_s is the magnetic field on the midplane and separatrix, the separatrix radius and length, respectively. The parameters are chosen from the conceptual D-^{3}He/FRC reactor "ARTEMIS-L". This equilibrium described by eq. (1) is known as Hill's vortex model. We have calculated the single particle motion in this equilibrium model with fluctuating magnetic field, which is given as,

$$\vec{B}_1 = \nabla \times \left(\beta \vec{B}_0\right)$$
$$\beta = \delta r_s \exp\left\{ i\left(n\theta + m\pi \frac{2z}{l_s} - \omega t \right) \right\} \tag{2}$$

where δ, n, m and ω are the amplitude control parameter, the toroidal and poloidal mode number of fluctuation, and the frequency, respectively. The resultant fluctuating magnetic field is written:

$$B_{r1} = in\frac{\beta}{r} B_{z0},$$
$$B_{\theta 1} = \beta\left(\frac{im\pi}{r} B_{r0} + \mu_0 j_{\theta 0}\right),$$
$$B_{z1} = -in\frac{\beta}{r} B_{r0}. \tag{3}$$

The subscription 0 means the equilibrium field at a steady state. Fluctuating electric field is given

$$\vec{E}_1 = i\omega\beta\vec{B}_0 \tag{4}$$

Implicitly, eq. (4) results from the following assumption of the charge density ρ_1,

$$\rho_1 = -\frac{m\pi\omega\varepsilon_0}{b}\beta B_z, \tag{5}$$

The perturbed density is 180 degrees out of phase with the perturbed magnetic field. This anti-correlating feature was observed in the recent high-beta plasma experiment [6]. The fluctuating fields are shown in Figs. 1 and 2. Without the fluctuation, the particles in a steady state and an axisymmetric FRC are accessible in a limited region. We refer to this region as the *accessible region*. The particles are never lost in a case this region is closed against the wall and the mirror throat. Figure 3 shows the accessible region for a 100-keV deuteron with the canonical angular momentum P_θ ($= mv_\theta r + q\psi$) of 0.05 and 0.1. The values of P_θ are normalized by $qB_w r_s^2$. Particles with smaller P_θ are confined in the region closer to the separatrix. Thus, the mechanism that reduces P_θ contributes the cross-field particle transport. We calculate 5000 different particles with the same kinetic energy K and the same canonical angular momentum P_θ initially so as to find these dependences on the

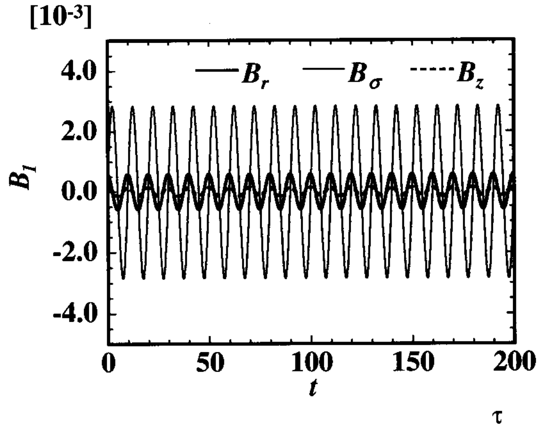

Fig. 1 The model fluctuating magnetic fields $\vec{B}_1$ with the frequency $\omega/\omega_c = 2\pi \times 0.1$, where $\omega_c = qB_w/m$. Here, the amplitude control parameter δ is 10^{-3} and time t is normalized by $1/\omega_c$. In this case, $\vec{B}_1$ of the mode numbers (m,n) = (1,2) are observed at $r/r_s = 1/\sqrt{2}$ and $z/r_s = 1.0$ in the FRC with the separatrix elongation $E = l_s/(2r_s) = 6.61$.

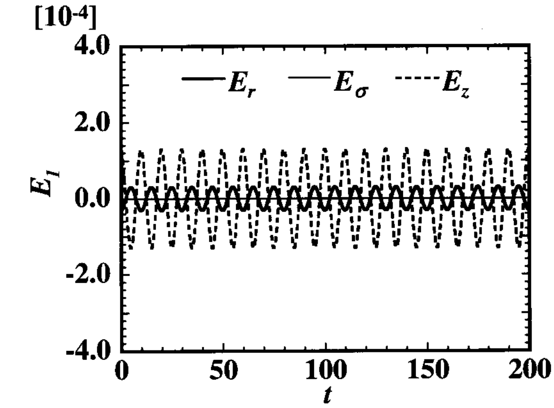

Fig. 2 The model fluctuating electric fields $\vec{E}_1$ for the same cases as in Fig. 1.

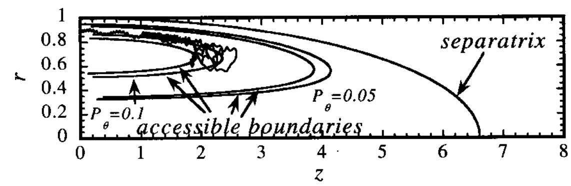

Fig. 3 Accessible region for the particles with the canonical angular momentum $P_\theta = 0.05$ and 0.1. The separatrix of Hill's vortex FRC and sample orbit of particle with $P_\theta = 0.1$ in the fluctuating magnetic field are also presented.

transport coefficients. They are injected from the guiding center surface $\psi = P_\theta / q$ uniformly. Numerical integration of equation of motion is carried out together with ensemble average of information on particle motion. The time step of the calculation is $0.01/\omega_c$, where $\omega_c = qB_w/m$. Then the calculation is valid for the fluctuation with the frequency ω less than $10\omega_c$. We observe the first and second order cumulants of P_θ for the cross-field drift motion and particle diffusion.

3. Results and Discussion

We trace the particle orbit numerically and present the evolution of the ensemble averaged P_θ in Fig. 4, where the results for five different toroidal modes are drawn. Initial value of P_θ is set to 0.01 for all cases. The ensemble averaged P_θ is found oscillating and decaying, and appears no radial drift because of any deviation from the initial value at the end of calculation. The large amplitude of oscillation is observed for larger toroidal mode due to severe breaking of axisymmetry. The diffusion coefficients of canonical angular momentum versus canonical angular momentum itself are presented in Fig. 5. The results for the toroidal mode number $n = 1$ and 2, and poloidal mode number $m = 1$ are presented. The frequencies of magnetic fluctuation ω/ω_c are $2\pi \times 0.01$, $2\pi \times 0.1$, and $2\pi \times 1.0$. It is found that the fluctuation with the larger toroidal mode and the lower frequency diffuses particles deleteriously because of the larger cross-field diffusion coefficient. The diffusion coefficient dependent on the amplitude control parameter δ is shown in Fig. 6. The particle diffusion process saturates when the amplitude of fluctuation exceeds 10% of equilibrium field. Finally, the dependence of the diffusion coefficient on the kinetic energy is presented in Fig. 7. In this figure, the kinetic energy K is transformed to the stability parameter S by the form $S \equiv (r_s - R)/r_{Li}$, where $r_{Li} = \sqrt{2mK}/(qB_w)$. Although the conventional stability parameter is $\bar{s} \equiv \int_R^{r_s} r\mathrm{d}r/(r_s \rho_{Li})$ and is different from the above S, typical characteristics of the diffusion process can be described also by S. In the form of $\bar{s}$, ρ_{Li} is the local ion gyroradius. Higher energy ions diffuse out faster from the separatrix due to the large diffusion coefficient than lower energy ions.

We can estimate roughly the particle confinement time by using $\tau_N \propto \psi_{max}^2/D_\psi$, where ψ_{max} is flux function at magnetic axis. The diffusion coefficient we calculated $D_{p\theta}$ is easily converted to D_ψ by using $D_{p\theta} \propto q^2 D_\psi$. The solid curve in Fig. 7 represents $D_{p\theta} \propto S^{-3.156}$, thus the scaling of τ_N with respect to S becomes $\tau_N \propto S^{-3.156}$. About two times larger power with respect to S is found than the one experimentally observed; the scaling analyzed with several experimental results is reviewed in ref. [7].

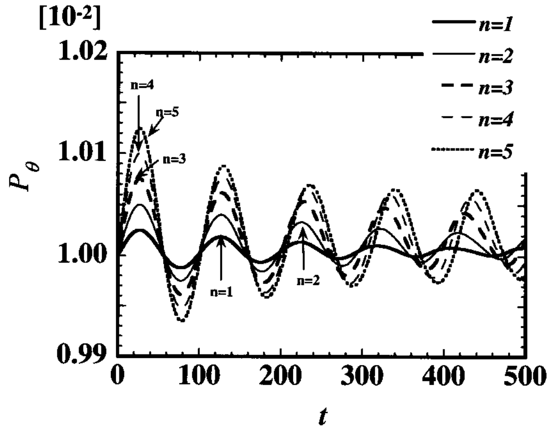

Fig. 4 The time evolutions of the ensemble averaged canonical angular momentum P_θ. The initial values of P_θ for 5000 particles are set to 0.01. The thick solid line is for $n = 1$ fluctuation with the frequency ω/ω_c of $2\pi \times 0.01$ and the amplitude control parameter δ of 10^{-3}, the thin solid line is for $n = 2$, the thick dashed line is for $n = 3$, the thin dashed line is for $n = 4$, and the thick dotted line is for $n = 5$. The poloidal mode m is 1 in all cases.

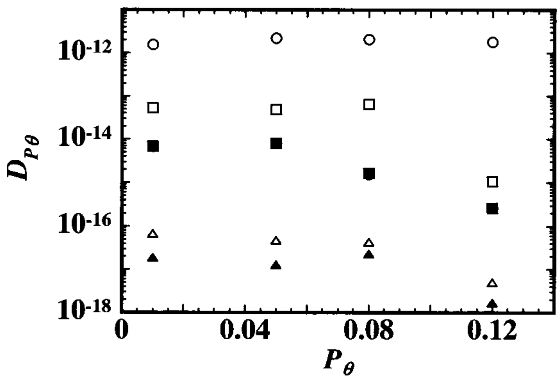

Fig. 5 The relation between the diffusion coefficients of the canonical angular momentum and the canonical angular momentum itself. The results for the poloidal mode number m of 1 and the toroidal mode number n of 1 (closed symbols) and 2 (open symbols) are presented. The ratios of the frequency of fluctuating fields to the ion cyclotron frequency of the midplane and separatrix, i.e. ω/ω_c are $2\pi \times 0.01$ (circle), $2\pi \times 0.1$ (square), and $2\pi \times 1.0$ (triangle). The amplitude control parameter δ of 10^{-3}. Note that the closed circles are behind the closed squares.

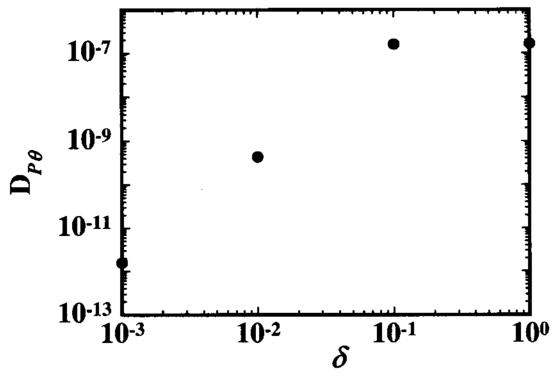

Fig. 6 The dependence of the diffusion coefficient on the amplitude control parameter of fluctuation.

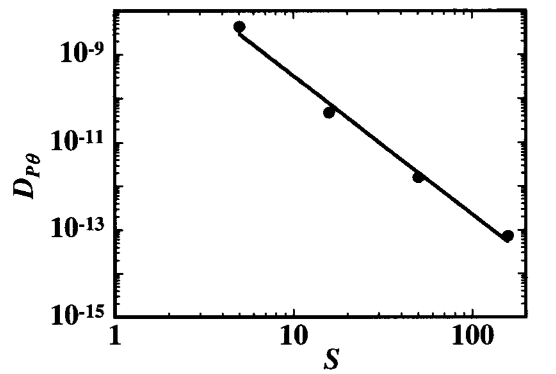

Fig. 7 Kinetic energy dependence of diffusion coefficient of canonical angular momentum. The kinetic energy K is transformed to S in the form $S \equiv (r_s - R)/r_{Li}$, where $r_{Li} = \sqrt{2mK}/(qB_w)$.

4. Conclusions

Investigation on the cross-field particle transport in a Field-Reversed Configuration (FRC) due to the electromagnetic fluctuation has been carried out. The fluctuation is described by the toroidal and poloidal mode numbers. We have traced numerically various orbits of many ions and observed the temporal change of the canonical angular momentum for the ions in the fluctuating field; this quantity is a constant of motion in axisymmetric and non-fluctuating FRC and is a good measure for the location of the guiding center. In order to estimate the cross-field transport coefficients, the first and second cumulants of the canonical angular momentum have been calculated. From the evolutions of the ensemble averaged canonical angular momentum, it is found that the cross-field drift of plasma ion can be neglected safely. The fluctuation, however, causes the diffusive flow near the separatrix, where the density gradient is large. The dependences of the diffusion coefficients on the various parameters have been studied. We have examined the fluctuation effects in the lower frequency range than ion cyclotron frequency. It is found that the fluctuation with higher toroidal mode and lower frequency affect the cross-field particle diffusion deleteriously. The scaling of the particle confinement time has been estimated with respect to the stability parameter S, and is found to be proportional to $S^{3.156}$. It appears that about two times stronger dependence on S is obtained compared with the experimental results.

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