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One Possible Method of Mathematical Modeling of Turbulent Transport Processes in Plasma

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Abstract

It is proposed to use the mathematical modeling of the increments of fluctuating plasma variables to analyzing the probability characteristics of turbulent transport processes in plasma. It is shown that, in plasma of the L-2M stellarator and the TAU-1 linear device, the increments of the process of local fluctuating particle flux are stochastic in nature and their distribution is a scale mixture of Gaussians.

Keywords:

plasma turbulence, stable distribution, stellarator, probability analysis, Laplacian and Cox distributions.

1. Introduction

In the last years, attention is drawn to studies of the probability characteristics of turbulent transport in plasma, in particular, to the analysis of local fluctuating particle fluxes in a magnetoactive plasma produced both in toroidal and in linear devices [1-3]. However, the authors usually restrict themselves to the calculation and description of characteristics of the probability density function (PDF) of the turbulent particle flux, with a final conclusion that the distribution deviates from a Gaussian (normal) distribution. Until the present time, there have been no attempts to discuss the mechanisms responsible for the characteristic features of the PDF of fluctuating particle flux in plasma. Only in the recent papers [4-6], attempts were made to relate the statistical-probability characteristics of fluctuating fluxes to the nonlinear wave processes in plasma.

2. Experimental Results

In this paper, we will consider the problem of the correct (from the standpoint of mathematical statistics) processing of experimental data on turbulent plasma fluctuations using as an example the measured local turbulent fluxes in the L-2M stellarator and the TAU-1 linear device and will discuss in more detail the resultant statistical characteristics of the turbulent processes under study. The parameters of these devices and plasma are presented in refs. [3,7]. The main distinctions are the magnetic field topology (the toroidal field in the L-2M, the homogeneous field in the TAU-1) and essentially different values of the electron temperature (T_e = 0.6–1.0 keV in the L-2M and T_e = 5–7 eV in the TAU-1). On the other hand, as was shown in the previous experiments [5], the spectral and statistical plasma fluctuation characteristics are very similar in both devices. Let us consider the probability characteristics of the local fluctuating particle flux obtained in the course of data processing. Before proceeding to a consideration, we define the local fluctuating particle flux as $\tilde{\Gamma} = (\delta n_e \cdot \delta v_r)$, where variables are measured by a 3-tip probe [3]. δn_e denotes

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the plasma density fluctuations (measured by one tip), $\delta v_r = \delta E_\theta / B$ is expressed through the fluctuation of the poloidal electric field $\delta E_\theta = (\delta\varphi_1 - \delta\varphi_2)/r\Delta\Theta$, where $\delta\varphi_{1,2}$ are the fluctuations of the plasma floating potential (measured by two tips spaced by 4 mm in poloidal direction), and Θ is the poloidal angular coordinate, and r is the mean radius of the magnetic surface. The signal sampling rate was 1 MHz and was high enough for fluctuations with frequencies up to 200 kHz to be identified with high accuracy (5 points per period). This frequency range corresponds to MHD fluctuations due to the interchange resistive ballooning instability in L-2M [3] and to the drift-dissipative gradient instability in TAU-1 [7].

Figure 1 shows the autocorrelation functions (ACF) of the flux $\tilde{\Gamma}$ and its increment $\Delta\tilde{\Gamma} = \tilde{\Gamma}_j(t_j) - \tilde{\Gamma}_{j-1}(t_{j-1})$, respectively. It is seen that, within the time window, the ACF for $\tilde{\Gamma}$ does not attain the noise level, whereas the ACF for $\Delta\tilde{\Gamma}$ attains this level in several microseconds. The slow decrease of the ACF of $\tilde{\Gamma}$ testifies that the flux amplitudes do not represent a homogeneous independent sample, whereas the rapid drop in the ACF of $\Delta\tilde{\Gamma}$ indicates a random character of the increments and their independence. It follows from here that, when the conventional techniques of probability analysis are used to study plasma fluctuations, one should use the increments of the fluctuating flux instead of the flux amplitudes. Figure 2 shows the PDFs of the process of the local fluctuating particle flux and its incremental values in the L-2M (Figs. 2(a) and 2(c)) and TAU-1 (Figs. 2(b) and 2(d)). Also shown in the figures are the asymmetry coefficient M_3 and excess M_4[1] for the corresponding PDFs. The PDF of the process $\tilde{\Gamma}$ (Figs. 2(a) and 2(c)) demonstrates a noticeable deviation from the Gaussian law. What is even more interesting is that both distributions of $\Delta\tilde{\Gamma}$ deviate from the normal Gaussian PDF. The characteristic feature of the process $\Delta\tilde{\Gamma}$ is that its PDF is symmetric, which indicates a dynamic symmetry of the flux increments. The excess values for the PDF of the process $\Delta\tilde{\Gamma}$ are equal to $M_4 = 9$ in L-2M and $M_4 = 7$ in TAU-1. Note, the histogram in Fig. 2(d) shows the best correlation with the Laplacian probability density distribution which has $M_3 = 0$ and $M_4 = 6$.

In Fig. 3(a), this histogram of the amplitudes of the increments of the local flux in TAU-1 (see Fig. 2(d)) is approximated by a Laplacian distribution. The two curves differ by only several percent, and the peakedness of the PDF is described better by the Laplacian process than by the Gaussian distribution which is also shown in the figure. From Fig. 3(b), it is obvious that the Gaussian distribution is inappropriate for describing the heavy tail in the histogram of the amplitudes of the local flux increments.

The Laplacian probability density distribution (correct to the scale length parameter) can be represented as a scale mixture of Gaussians with the exponential mixing distribution. Thus, if $L(x)$ is a Laplacian distribution function

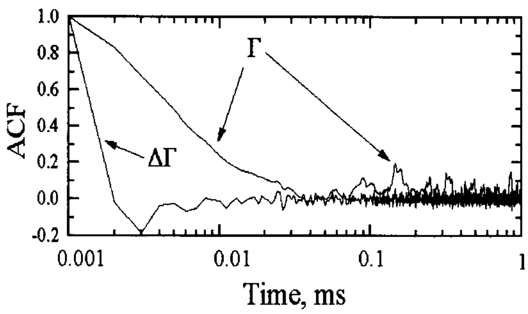

Fig. 1 Autocorrelation functions (ACF) of the local fluctuating particle flux $\tilde{\Gamma}$ and its increments $\Delta\tilde{\Gamma}$ for the L-2M device.

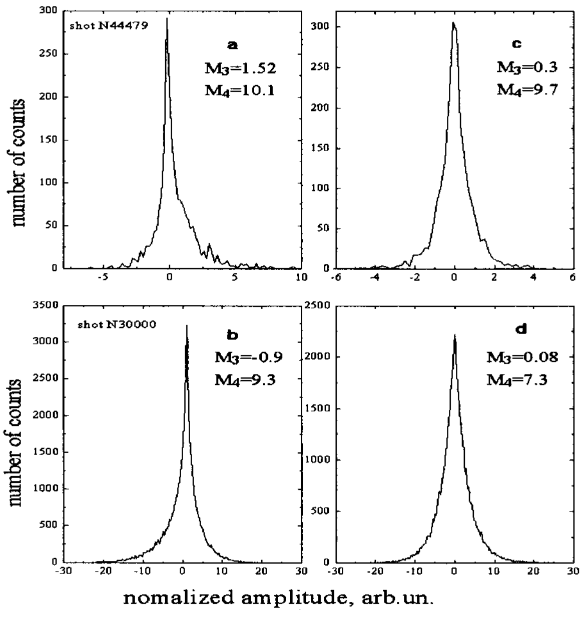

Fig. 2 Probability density function (PDF): (a) PDF of $\tilde{\Gamma}$ for L-2M; (b) PDF for $\tilde{\Gamma}$ for TAU-1; (c) PDF of $\Delta\tilde{\Gamma}$ for L-2M; and (d) PDF of $\Delta\tilde{\Gamma}$ for TAU-1.

[1] For a Ggaussian process represented by the sample $(x_1,...,x_N)$, the kurtosis is $M_3 = 1/N\sum_{j=1}^{N}[x_j - \bar{x}/\chi]^3 = 0$ and the excess is $M_4 = 1/N\sum_{j=1}^{N}[x_j - \bar{x}/\chi]^4 = 3$ (where χ is the standard deviation).

$$L(x)=\frac{1}{\sqrt{2}}\int_{-\infty}^{x} e^{-\sqrt{2}|x|}\,dx = \left\{\begin{array}{ll} \frac{1}{2}e^{\sqrt{2}x}, & x<0 \\ 1-\frac{1}{2}e^{-\sqrt{2}x}, & x\geq 0 \end{array}\right\}, \qquad (1)$$

and $\Phi(x)$ is the standard normal distribution function

$$\Phi(x)=\frac{1}{\sqrt{2}}\int_{-\infty}^{x} e^{-u^2/2}\,du\,, \qquad (2)$$

then the result is

$$L(x)=\int_0^{x}\Phi\left(\frac{x}{\sqrt{\sigma}}\right)e^{-\sigma}\,d\sigma\,, \qquad (3)$$

(see, e.g., ref. [8]). From here, we can conclude that, each increment $\Delta\tilde{\Gamma}=\tilde{\Gamma}_j-\tilde{\Gamma}_{j-1}$ is a result of classical (Brownian) diffusion from the point $\tilde{\Gamma}_{j-1}$ to the point $\tilde{\Gamma}_j$ occurring with its own diffusion coefficient σ_j. This implies that the fluctuating flux varies with time according to the diffusion law. In this case, as j varies, the coefficient σ_j varies randomly, i.e., the diffusion coefficients $\sigma_j (j\geq 1)$ are random values that obey the exponential distribution (see refs. [1]–[3]). Note that the Gaussian distribution has the highest entropy among all the laws with a definite second moment which are concentrated on the non-negative axis; this distribution corresponds to stable states in open systems. Naturally, both plasma devices are open (in the terms of thermodynamics) systems for which the duration of dynamic equilibrium of macroscopic plasma parameters substantially exceeds the characteristic fluctuation times. In the TAU-1 experiment, when the time interval between the successive measurements $\Delta t_j=t_j-t_{j-1}$ is as short as 1–5 μs, the PDF for the incremental values is the Laplacian distribution. Hence, this time interval is characteristic of the local flux associated with the drift-dissipative instability. The variations in the local particle flux determined by drift motions occur on a time scale at least one order of magnitude shorter than the characteristic drift-motion times (the period, the growth rate).

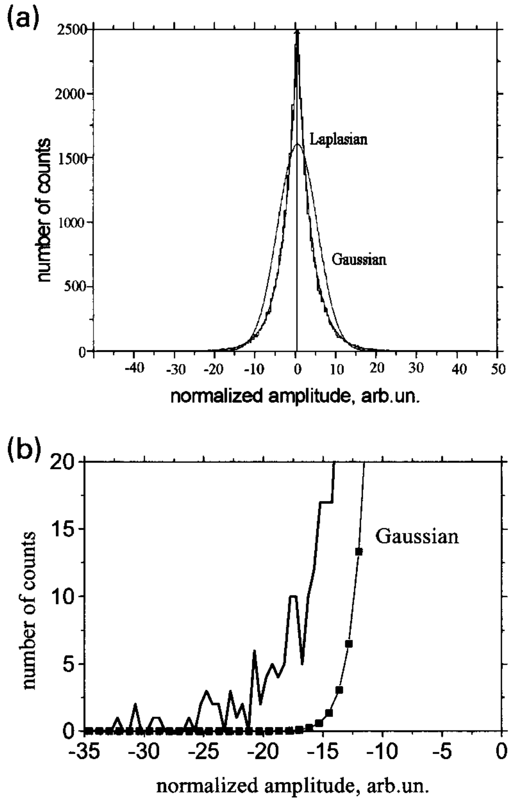

Fig. 3 (a) Approximation of the histogram of the increments of the local flux in TAU-1 by a Laplacian distribution, and (b) the comparison of a Gaussian distribution with the tail of the histogram of the increments.

We can also assume that, in the general case, the fluctuating particle flux $\tilde{\Gamma}$ is a doubly stochastic diffusion process (or, in other terms, the diffusion process with random time). As is known, such processes result from the passage to the limit in the generalized Cox processes [9]. In this case, the incremental distributions of individual processes are the scale mixtures of Gaussians, which is confirmed in our case by the statistical analysis of $\Delta\tilde{\Gamma}$. Another argument is that the PDF of the increments of the drift particle flux in TAU-1 transforms to a Gaussian with increasing Δt_j up to 100 μs. The longer the sampling time, the lower the contribution to the PDF from the processes determined by the influencing functions. The PDF for $\Delta\tilde{\Gamma}$ approaches the normal (Gaussian) distribution, which corresponds to the asymptotic of the generalized Cox processes [10]. The probability for large (more that 3 standard deviations) variations in the local flux increments distributed by the Laplacian law is many times higher than that for the Gaussian law. In other words, the probability of the experimental observation of ultrafast increments of the local flux increases.

Figure 4 shows the time evolution of the wavelet spectra of both the flux driven by the drift instability and the flux increments in TAU-1. These time-dependent spectra are compiled from 100 spectra computed for successive 200 μs time intervals. The amplitude of spectral components is shown by shades of gray. The frequency corresponding to the wavelet duration is plotted on the abscissa, and the time is plotted on the ordinate. The spectra of $\tilde{\Gamma}$ and $\Delta\tilde{\Gamma}$ vary

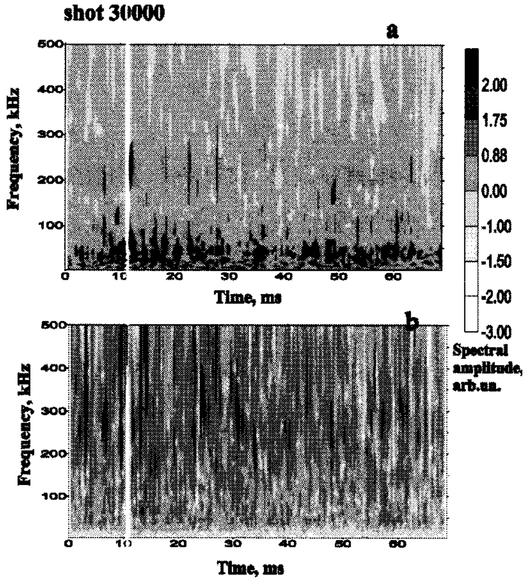

Fig. 4 Time behavior of the wavelet-spectra of (a) the local flux and (b) its increments in the TAU-1 device.

substantially with time, whereas the macroscopic plasma parameters do not change. It is seen that the local flux in TAU-1 and the increments of this flux are intermittent and exist as "long" events with pauses between them. In Fig. 4, these events correspond to intense random dark zones that represent the wavelet harmonics. The characteristic duration of the local flux events is ~1 ms, and the pauses between them are shorter. The physical mechanisms responsible for the random character of the dynamical characteristics of the local flux (such as the growth and damping rates) and the values of the local flux increments are described in ref. [11].

3. Conclusions

(i) The correct statistical analysis of the characteristics of fluctuating particle fluxes in plasma should be carried out with an equidistant sample of the process $\Delta\widetilde{\Gamma}$, namely, the sample of the flux increments.

(ii) The increments of the local fluxes in the L-2M and TAU-1 devices are stochastic in character and the PDFs of increments are the scale mixtures of Gaussians. The PDF of the increments corresponds to a Laplacian distribution in case of the drift turbulence.

(iii) The correct statistical analysis of the local fluctuating particle flux carried out with an equidistant sample of the increments made it possible to determine the characteristic (dynamic) time of the local particle fluxes in the L-2M and TAU-1 plasmas. In both cases, this time turns out to be one order of magnitude shorter than the characteristic correlation time.

(iv) Physical mechanisms responsible for the random character of the time-dependent parameters of the local flux in plasmas can be related to the nonlinear processes suppressing the growth of unstable oscillations, stochastic particle heating, and the formation of nonlinear structures.

Taking account of the processes mentioned above, we can formulate several problems for future investigations. What are the characteristics of the local turbulence in the transport barriers in toroidal confinement systems? Do the turbulent plasma states under study belong to the systems with dynamic chaos? Can the transitions in such systems be controlled with the help of regular waves? As an example, when a regular controlling wave at the characteristic drift frequency and phase was launched into the plasma, a broad drift mode spectrum transformed to a single-mode spectrum [12]. Finally, we note that, although we have considered the processes of particle diffusion, the approach proposed can apparently be used to analyze heat transport processes in toroidal confinement systems.

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References

[1] J.W. Connor *et al.*, Plasma Phys. Control. Fusion **41**, 693 (1999).

[2] M.A. Pedrosa, M.A. Ochando, J.A. Jimenez *et al.*, Plasma Phys. Control. Fusion **38**, 365 (1996).

[3] Batanov, O.I. Fedyanin, N.K. Kharchev *et al.*, Plasma Phys. Control. Fusion **40**, 1241 (1998).

[4] B.A. Carreras, Phys. Rev. Lett. **83**, 3653 (1999).

[5] G.M. Zaslavsky *et al.*, Plasma Physics **7**, 3691 (2000).

[6] G.M. Batanov *et al.*, JETF Letters **72**, 174 (2001).

[7] G.M. Batanov *et al.*, Plasma Phys. Rep. **27**, 58 (2001).

[8] I.S. Gradshtein and I.M. Ryzhik, *Tables of Integrals, Sums, Series, and Products*, GIFML, Moscow 1962.

[9] V.V. Uchaikin and V.M. Zolotarev, Chance and Stability. Stable Distributions and their Applications published in 1999 by VSP, Utrecht.

[10] B.V. Gnedenko and V.Yu. Korolev, *Random Summation: Limit Theorems and Applications* (1996 by CRC Press, Boca Raton, FL).

[11] G.M. Batanov *et al.*, Plasma Phys. Rep. **28**, 128 (2002).

[12] A.E. Petrov *et al.*, Plasma Phys. Rep. **23**, 654 (1997).