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Tearing Mode in a Heliotron Plasma

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Abstract

Tearing mode is studied in a heliotron plasma by varying the net toroidal current. Ideal internal kink mode can be unstable when two resonant magnetic surfaces are generated. The tearing mode is found at the current below the marginal stability value for the ideal internal kink mode. The mode structure is localized around the inner resonant surface like the internal kink mode. The growth rate and the mode structure can be affected by the unstable internal kink mode. Tearing and resistive interchange hybrid mode is also obtained at a finite beta.

Keywords:

tearing mode, heliotron, internal kink mode, resistive interchange mode, reduced MHD equations

1. Introduction

Heliotron configurations have an advantage that no net toroidal current is necessary for generating the confinement magnetic configuration. However, substantial net toroidal currents are observed in the Large Helical Device (LHD) experiments [1]. Such currents have a potential to drive current driven modes. Several theoretical analysis for the ideal internal and external kink modes in the stellarator plasmas have been carried out [2-6]. On the other hand, in the helias configuration, the m = 2 and n = 1 tearing mode was identified in the W-7 AS experiment recently [7]. Here m and n are the poloidal and the toroidal mode numbers, respectively. Thus, it is interesting to know how the tearing mode behaves in the heliotron plasma. In the present paper, we study the feature of the tearing mode in one of the LHD configurations numerically.

The stability of the tearing mode is often studied based on the so-called Δ' analysis [8]. Here Δ' is the jump of the logarithmic derivative of the outer solution for the poloidal flux at the resonant surface. This method is useful in the determination of the stability boundary in a parameter scan and was used in the straight stellarator configurations [3,9]. However, it is difficult to apply it to the 2D or 3D configuration. In the study of the tearing mode in the toroidal geometry, the global mode analysis is useful. Thus, in the present work, the method based on the reduced MHD equations is employed. The mode structure of the tearing mode, the relation with the ideal mode and the finite beta effects are discussed.

2. Numerical Method

In order to examine the global structure of the tearing mode, we utilize the RESORM code [10], which is a linear stability code for ideal and resistive instabilities in the heliotron plasmas. This code solves the linearized equations of the following reduced MHD equations for the poloidal flux Ψ, the stream function Φ and the plasma pressure P as an initial value problem;

$$\frac{\partial \Psi}{\partial t} = -\left(\frac{R}{R_0}\right)^2 \boldsymbol{B} \cdot \nabla \Phi + \eta J_\zeta , \qquad (1)$$

$$\rho_m \frac{\mathrm{d}\nabla_\perp^2 \Phi}{\mathrm{d}t} = -\boldsymbol{B} \cdot \nabla J_\zeta + R_0^2 \nabla \Omega \times \nabla P \cdot \nabla \zeta , \qquad (2)$$

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$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0\,, \qquad (3)$$

where ζ, R, ρ_{m} and η are the toroidal angle, the major radius of the torus, the plasma density and the resistivity, respectively. Here Ω is the averaged magnetic curvature, of which the explicit expression is given in ref. [10]. The subscript 0 means the value at the magnetic axis. The magnetic differential operator and the convective time derivative are given by $\boldsymbol{B}\cdot\nabla = (R_0B_0/R^2)(\partial/\partial\zeta) - \nabla\Psi\times\nabla\zeta\cdot\nabla$ and $\mathrm{d}/\mathrm{d}t = (\partial/\partial t) + (R/R_0)^2\nabla\Psi\times\nabla\zeta\cdot\nabla$, respectively. The toroidal current density is given by $J_\zeta = R^2\nabla\cdot(\nabla_\perp/R^2)\Psi$.

The Fourier transform is employed in the angle directions in the code, and the toroidally averaged 3D equilibrium which is calculated by the VMEC code [11] is examined. Therefore, the toroidal mode number is specified as a parameter, while the poloidal mode coupling appears through the convolution with the averaged equilibrium quantities.

3. Numerical Results

We examine the tearing mode in one of the LHD configurations. The helical coils in the LHD are composed of three layers for providing the flexibility in the minor radius of the helical coils. Here, we choose the configuration with the coil current flowing only in the outer-layer. For the net toroidal current in the plasma column, highly peaked current density, $J = J_0\,(1-\rho)^6$, is assumed, where ρ denotes the square root of the normalized toroidal flux. Figure 1 shows the profile of the rotational transform ι at $\beta_0 = 0$ %. There are two surfaces with $\iota = 1/2$ in the plasma column for the total toroidal current in the range of 110 kA $\leq I \leq$ 150 kA, where the toroidal field at the coil center is assumed to be $B_0 = 3$ T. Only one $\iota = 1/2$ surface exists for $I \leq 100$ kA and no $\iota = 1/2$ surface for $I \geq 160$ kA. We concentrate on the $n = 1$ mode which is resonant at $\iota = 1/2$ in the present paper.

At first, the ideal mode is examined with the RESORM code. Unstable modes are obtained for the range of I corresponding to the existence of the two $\iota = 1/2$ surfaces as shown in Fig. 2, where the growth rate is plotted as a function of I. Here γ is normalized by the poloidal Alfvén time. Figure 3 shows the mode structure of the eigenfunctions for $I = 130$ kA as an example. The $m = 2$ and $n = 1$ component is dominant and the mode structure is localized in the region between the magnetic axis and the inner resonant surface. The stream function is an even function and has a sharp gradient at the surface. The poloidal flux has a null point at the resonant surface. The mode structure of the unstable modes shows the feature of the ideal internal kink mode.

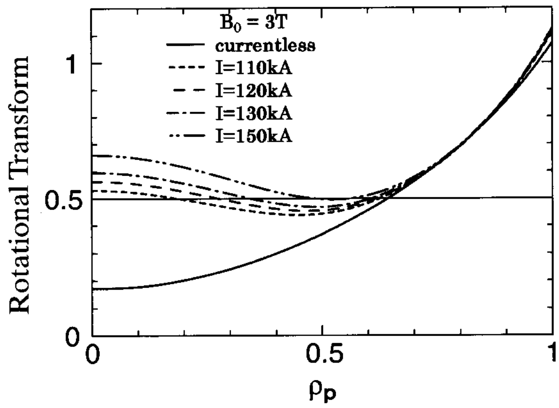

Fig. 1 Profiles of the rotational transform for the various net toroidal currents. The radius ρ_p denotes the square root of the normalized equilibrium poloidal flux, which is also used in Figs. 3, 5 and 6.

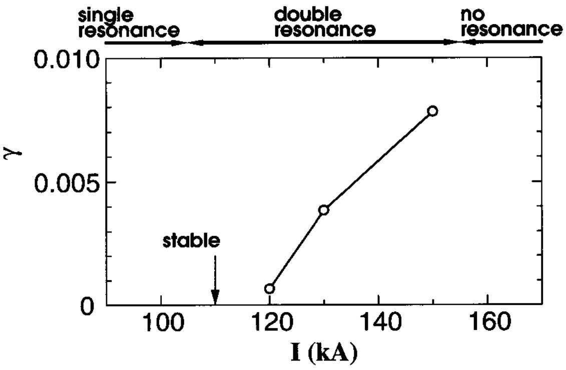

Fig. 2 Growth rate of the $n = 1$ ideal internal kink mode.

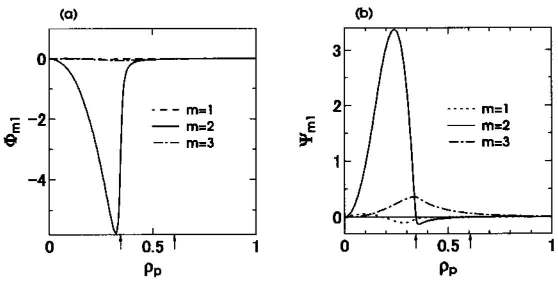

Fig. 3 Mode structures of (a) stream function and (b) poloidal flux of the $n = 1$ ideal internal kink mode for $I = 130$ kA. The arrows indicate the positions of $\iota = 1/2$ surfaces, which are also used in Figs. 5 and 6.

The finite resistivity influences the stability. Figure

4 shows the S-dependence of the growth rate, where S is the magnetic Reynolds number and proportional to $1/\eta$. An unstable mode appears at I = 110 kA where the ideal mode is stable. This mode is identified as a tearing mode with the following feature. The S-dependence of the growth rate calculated with the values for $S = 10^8$ and $S = 10^9$ scales $\gamma \propto S^{-0.54}$. This dependence is close to that for the tearing mode derived with the Δ' theory, $S^{-3/5}$ [8]. Figure 5 shows the mode structure for I = 110 kA and $S = 10^7$. The m = 2 component is dominant and the mode structure is localized near the inner ι = 1/2 surface as is in the ideal internal kink mode. The stream function is odd with respect to the resonant surface and the poloidal flux has a finite value at the resonant surface which corresponds to the generation of the magnetic island. These are the typical characteristic of the tearing mode.

The deviation from the $S^{-3/5}$ dependence in the small S region seen in Fig. 4 is due to the cylindrical geometry effect. In the Δ' theory, the inner region is expanded around the resonant surface for the asymptotic matching so that the geometry should be slab. However, if the mode structure is localized near the magnetic axis, the distance from the axis can be comparable with the resistive layer width for small S. In this case, neglecting the cylindrical geometry is not a good approximation. In the present case, as is shown in Fig. 5, the position of the resonant surface is close to the axis. Therefore, the deviation from the dependence of $S^{-3/5}$ becomes prominent as S decreases.

As shown in Fig. 4, the S-dependence is varied when the current is increased. This is because the tearing mode is affected by the ideal internal kink mode. As the current increases and γ of the internal kink mode increases, the dependence becomes weak. This tendency is quite similar to that in the case of the ideal and the resistive interchange modes [12]. The increase of the current also influences the mode structure. The stream function for $I \geq$ 120 kA is the even function with respect to the inner ι = 1/2 surface like the ideal internal kink mode. On the other hand, the poloidal flux at the surface is finite like the tearing mode. This implies that the mode structure shows the feature of both the internal kink and the tearing modes. Hence, this mode has an island with the flow like the internal kink mode.

The behavior of the tearing mode at finite beta is also examined for I = 110 kA and $S = 10^7$. The pressure profile of $P = P_0(1 - \rho^4)^2$ is assumed. The tearing mode is dominant for $\beta_0 <$ 0.32 %, and the resistive interchange mode becomes dominant for $\beta_0 >$ 0.32 %. At β_0 = 0.32 %, a tearing and resistive interchange hybrid mode is obtained. The mode structure is shown in Fig. 6. In both of the stream function and the poloidal flux, the typical structures of the tearing mode and the

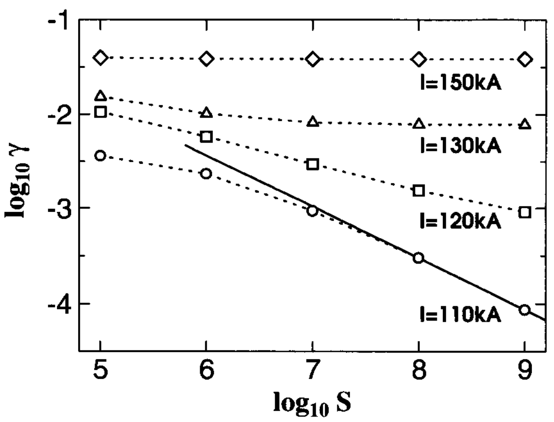

Fig. 4 S-Dependence of the growth rate of the n = 1 resistive mode. Solid line shows the dependence determined with $\gamma(S = 10^8)$ and $\gamma(S = 10^9)$, which corresponds to $\gamma \propto S^{0.54}$.

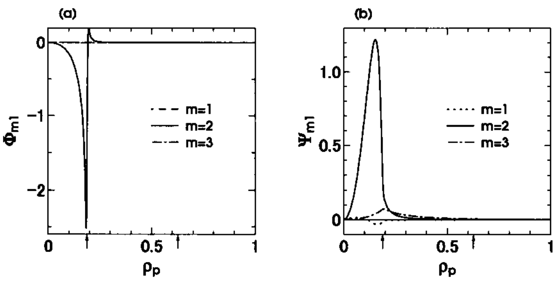

Fig. 5 Mode structures of (a) stream function and (b) poloidal flux of the n = 1 tearing mode with $S = 10^7$ for I = 110 kA.

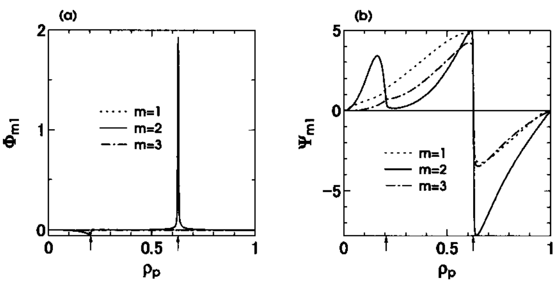

Fig. 6 Mode structures of (a) stream function and (b) poloidal flux of the n = 1 tearing-interchange hybrid mode with $S = 10^7$ for I = 110 kA at β_0 = 0.32 %.

resistive interchange mode are seen around the inner and the outer ι = 1/2 surfaces, respectively. The side bands due to the toroidicity are relatively large in the poloidal flux structure.

4. Summary and Discussion

An unstable tearing mode with $m = 2$ and $n = 1$ is found in a heliotron plasma with two ι = 1/2 surfaces for the toroidal current below the marginal stability value for the ideal internal kink mode. The mode structure is localized around the inner resonant surface as is in the internal kink mode case, not like a double tearing mode. In the region for the unstable internal kink mode, the growth rate and the mode structure is affected by the ideal mode. No unstable tearing mode can be obtained for the current corresponding to a single resonant surface or no resonant surface in the present configuration.

The properties of the tearing mode structure obtained here can be understood in the relation with those in the ideal internal kink mode. In the heliotron plasma with a positive ι' in vacuum, the potential energy for $P = 0$ in the cylindrical limit shows that the region with $\iota > n_0/m_0$ is necessary in the plasma for the destabilization of the ideal internal kink mode with $m = m_0$ and $n = n_0$ [3]. Because the stabilization effect due to the field line bending is large near the plasma edge, the unstable region should exist near the magnetic axis. Therefore, the ι profile with two resonant surfaces is necessary for the unstable internal kink mode, and the mode structure is localized inside of the inner resonant surface.

The resistivity relaxes the 'frozen-in' condition and enlarges the unstable parameter regime. Therefore, the marginally stable internal kink mode can be easily destabilized by the resistivity to be a tearing mode. In this case, the tearing mode almost inherits the mode structure of the internal kink mode because the structure minimizes the stabilizing effect. In the present case, the tearing mode is obtained at a current value which is just below the marginal value for the internal kink mode. Hence, the mode structure is localized around the inner resonant surface as is in the internal kink mode. Because the configuration with single resonant surface or no resonant surface case is quite stable for the ideal mode, such configuration cannot be easily destabilized with the finite resistivity. Therefore, the configuration with two resonant surfaces may be necessary also for the destabilization of the tearing mode in heliotron configurations.

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References

[1] K.Y.Watanabe *et al.* and LHD Experimental Group, ECA (*Proc. 27th EPS Conference on Controlled Fusion and Plasma Physics*, 12–16 June 2000, Budapest, Hungary) **24B**, 1316 (2000).
[2] R.M. Sinclair *et al.*, Phys. Fluids **8**, 118 (1965).
[3] K. Matsuoka *et al.*, Nucl. Fusion **17**, 1123 (1977).
[4] M. Wakatani *et al.*, J. Phys. Soc. Jpn. **47**, 974 (1979).
[5] T. Matsumoto *et al.*, Nucl. Fusion **36**, 1571 (1996).
[6] J.L. Johnson *et al.*, Phys. Plasmas **6**, 2513 (1999).
[7] A. Weller *et al.*, Fusion Energy 2000 (*Proc. 18th Fusion Energy Conference*, Sorrento, 2000), IAEA, Vienna (2001) (CD-ROM file EX7/4).
[8] H.P. Furth *et al.*, Phys. Fluids **6**, 459 (1963).
[9] B.N. Kuvshinov *et al.*, Plasma Phys. Rep. **20**, 252 (1994).
[10] K. Ichiguchi *et al.*, Nucl. Fusion **41**, 181 (2001).
[11] S.P. Hirshman *et al.*, Comp. Phys. Comm. **43**, 143 (1986).
[12] K. Ichiguchi *et al.*, Nucl. Fusion **29**, 2093 (1989).