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The Marginally Stable Pressure Profile and a Possibility of High Beta Plasma Confinement in LHD

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Abstract

It is shown theoretically that LHD has potentiality of the high beta plasma confinement. The chaotic field line-layer, which surrounds the outside of the outermost magnetic surface, plays a key role for an efficient plasma confinement in LHD.

The plasma in the chaotic field line region prevents an immediate cooling of the core plasma caused by neutral atoms in the vacuum vessel. Furthermore, plasma pressure in the chaotic field line region can increase the core plasma pressure, even in the magnetic hill condition. The equilibrium and stability of LHD plasma are analyzed under the ideal MHD model, and a marginally stable pressure profile is derived analytically and also numerically.

Keywords:

LHD, high beta plasma, equilibrium, MHD stability, chaotic field line layer, fusion reactor

1. Introduction

After the first discharge on March 31, 1998, the Large Helical Device (LHD) of National Institute for Fusion Science has extended the parameter range of the plasma confinement, successfully. The electron temperature on the magnetic axis exceeds 10 keV along with the increase of the heating input, and, meanwhile, the observation such as ELM or plasma disruptions, which synchronizes with an increase in the heating power input, has not yet been observed.

Numerical studies of particle orbits show that the loss cone does not exist in LHD, hence there is an excellent containment performance to the high-energy particle in LHD [1].

The characteristics of LHD magnetic field are the high magnetic shear configuration, and the existence of the chaotic field line layer which surrounds the outermost magnetic surface.

High shear magnetic field configuration has following merit for plasma confinement: (1) Unstable resonant modes are localized on the rational surface so that these instabilities do not relate directly to the decay of the entire plasma column. (2) The magnetic shear has a strong stabilizing effect for the convective instability of entire plasma column. (3) The magnetic field line in the chaotic region has extremely long connection length.

Characteristic of chaotic field line region and its role for the plasma confinement in LHD can be summarized as follows [2]: (1) The connection length of the diverter field line which approaches close to the outermost magnetic surface exceeds 10 km. The cold diverter plasma does not cool down the core plasma directly therefore. (2) The lines of force that are disengaged from the chaotic field line region reach the vacuum vessel wall soon. Then, it is expected in chaotic field line region that the plasma pressure can be sustained stably by the line-tying effect of the field

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lines, which are disengaged from the chaotic field region. (3) The chaotic field line region can confine the reflecting particles (particles whose velocity is almost perpendicular to the magnetic field). (4) The plasma pressure of the chaotic field line region can increase the core plasma pressure, even in the case of the magnetic hill configuration. (5) The plasma contained in the chaotic field line region prevents an immediate core plasma cooling down caused by the neutral atom flitting in the vacuum vessel (role of the plasma blanket).

Furthermore, LHD are showing the excellent experimental results as follows: 1) A smooth high-energy proton spectrum of 300 keV or more is observed in the ICRF experiment. 2) Excellent plasma is sustained even when magnetic field strength is decreased to the order of 1/6 (B_{ax} = 0.5 T) of the normal values (B_{ax} = 2.75 ~ 3 T). 3) Plasma exists in the chaotic field line region surrounding the outermost magnetic surface.

According to the fact, that a large plasma current is not needed for the plasma confinement, and the above mentioned theoretical and experimental results, we have guessed that LHD can sustain high beta plasma stably.

In Sec. **2**, we have summarized briefly an algorithm for the numerical treatment of high beta equilibrium of LHD. In Sec. **3**, we have analyzed the stability of LHD plasma under the ideal MHD model, and have obtained numerical results for the marginally stable pressure profile. Section **4** is devoted to summary and discussions.

2. Analysis of Equilibrium of LHD

The first step to analyze characteristics of high beta plasma will be analysis of equilibrium. For the tokamak which is an axisymmetric torus, equilibrium is described by the Grad-Shafranov equation. This equation can analyze the configuration of complete $\beta = 1$ plasma that the magnetic field vanishes in plasma.

Let's extend the Grad-Shafranov equation to 3D configuration in order to solve the high beta equilibrium of a helical plasma. It is assumed that nested magnetic flux function can exist and that the plasma pressure is a function of the flux function.

Several surface quantities, which are integral constants of equilibrium equations, exist in an equilibrium of plasma. It is necessary to set up appropriately surface quantities beforehand to determine the plasma equilibrium.

Plasma current is not driven actively for the confinement, in LHD. Hence, we adopt a distribution of rotational transform $\iota(\psi)$ and a pressure profile $p(\psi)$, as surface quantitie which provide for equilibrium.

Let's introduce a coordinate system (ψ, χ, ϕ) as shown in Fig. 2. The coordinates χ and ϕ are the poloidal and toroidal angles ($\oint d\chi = \oint d\phi = 2\pi$), which are introduced, appropriately and arbitrarily, to describe a toroidal system. Magnetic flux function ψ is determined by the condition of the equilibrium, numerically.

Let's express the magnetic field $\boldsymbol{B}$, which has a flux function ψ, as follows:

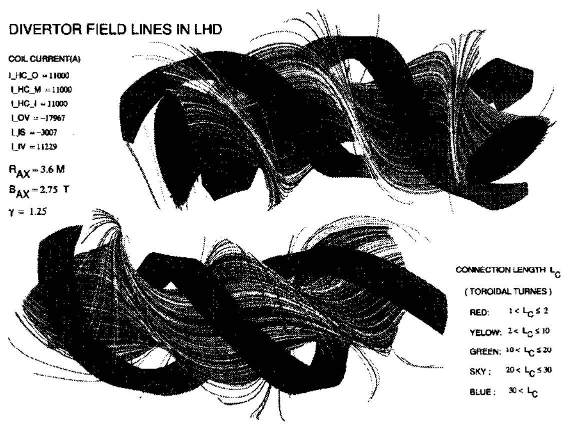

Fig. 1 Chaotic field lines of LHD. The lines of force are classified by the connection length.

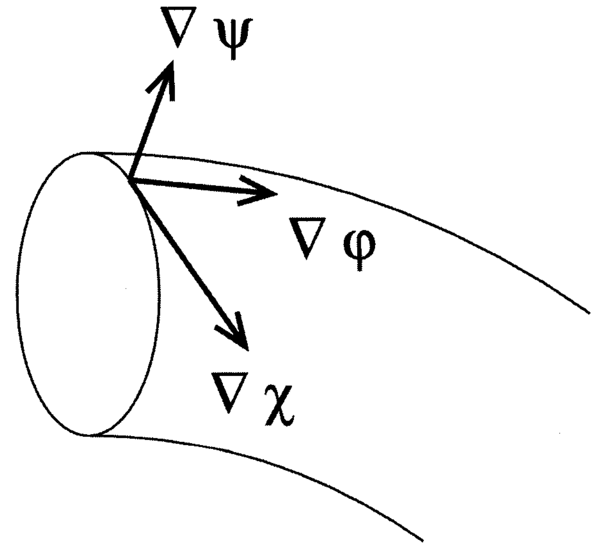

Fig. 2 Coordinate system

$$\boldsymbol{B} = \nabla\psi \times \nabla g. \tag{1}$$

The function g can be written by the use of a periodic function Λ of (χ, ϕ):

$$g = P'(\psi)\chi + Q'(\psi)\phi + \frac{\partial \Lambda}{\partial \psi} + \frac{(P+\Lambda_\chi)\nabla\psi\cdot\nabla\chi + (Q+\Lambda_\phi)\nabla\psi\cdot\nabla\phi}{(\nabla\psi)^2},$$

where $'$ denotes the derivative with respect to ψ and the suffix χ (ϕ) represents the partial derivative with respect to χ (ϕ). Function $P(\psi)$ and $Q(\psi)$ are arbitrary functions of ψ and specify the distribution of rotational transform $\iota/2\pi$ of the line of force.

$$\frac{\iota}{2\pi} = -\frac{Q'(\psi)}{P'(\psi)}. \tag{2}$$

The functions ψ and Λ are determined by the following set of equations, which can be reduced to the Grad-Shafranov equation in the case of axially symmetric plasma.

$$p'(\psi) = \frac{1}{\mu_0}\left\{\nabla g\cdot\nabla\times(\nabla\phi\times\nabla g)\right\}$$

$$0 = \frac{\partial}{\partial\chi}\left\{\frac{a_{11}g_\chi - a_{12}g_\phi}{J}\right\} + \frac{\partial}{\partial\phi}\left\{\frac{-a_{12}g_\chi + a_{22}g_\phi}{J}\right\}$$

where a_{11}, a_{12}, a_{22} are given by the following equations.

$$\begin{aligned} J &= (\nabla\psi\cdot\nabla\chi)\cdot\nabla\phi \\ a_{11} &= (\nabla\psi\cdot\nabla\chi)^2 \\ a_{22} &= (\nabla\phi\cdot\nabla\psi)^2 \\ a_{12} &= (\nabla\psi\cdot\nabla\chi)\cdot(\nabla\phi\times\nabla\psi) \end{aligned}$$

3. MHD Stability Analysis of LHD

Let us study the MHD stability of LHD plasma using the potential energy W_f within the plasma, because LHD does not carry a large plasma current and is surrounded by chaotic field line region plasma, which is stabilized by line-tying effect.

$$\delta W_f = \frac{1}{2}\int\left[\gamma p(\nabla\xi)^2 + \xi\cdot\nabla p\nabla\xi + \frac{(\nabla\times(\xi\times\boldsymbol{B}))^2}{\mu_0} - (\xi\times\boldsymbol{J})\cdot\nabla\times(\xi\times\boldsymbol{B})\right]\mathrm{d}V.$$

For simplicity, we assume that the perturbation ξ is localized on some rational surface ($\psi \simeq \psi_0$) and has wave number (m, n) into the (χ, ϕ) direction. Furthermore, we neglect the coupling effects with the inhomogeneity of the equilibrium state.

$$\xi = \xi_1\nabla\chi\times\nabla\phi + \xi_2\nabla\phi\times\nabla\psi + \xi_3\nabla\psi\times\nabla\phi$$

$$\begin{pmatrix} J\xi_1 g_\chi \\ J\xi_2 g_\chi \\ J\xi_3 g_\chi \end{pmatrix} = \begin{pmatrix} \xi(\psi)\sin(m\chi+n\phi) \\ \eta(\psi)\cos(m\chi+n\phi) \\ \zeta(\psi)\cos(m\chi+n\phi) \end{pmatrix}$$

3.1 Stability condition for resonant modes

Stability against the helical type perturbation with finite wave number (m,n) is analyzed, here. The most dangerous mode is the one that resonance condition $m + n\iota/2\pi \simeq 0$ is held. In this case, the stability condition is given by the following form, which can be reduced to the Suydam condition in the case of straight cylinder plasma.

$$\frac{1}{4}\frac{U}{\mu_0}\mu^2\frac{(\boldsymbol{B}_t\times\boldsymbol{B}_p)^2}{\boldsymbol{B}^2} + U\left\{\frac{U'}{U} - \mu\frac{\boldsymbol{B}_p\cdot\boldsymbol{B}}{\boldsymbol{B}^2} - \frac{\mu_0}{\boldsymbol{B}^2}p'\right\}p' \geq 0, \tag{3}$$

where μ_0 is permeability of vacuum. Toroidal (poloidal) component of magnetic field is expressed by $\boldsymbol{B}_t$ ($\boldsymbol{B}_p$), and, μ and U represent the shear parameter and the specific volume, respectively:

$$\begin{aligned} \boldsymbol{B}_t &\equiv g_\chi\nabla\psi\times\nabla\chi \\ \boldsymbol{B}_p &\equiv g_\phi\nabla\psi\times\nabla\chi \\ \mu &= \left(\frac{\iota}{2\pi}\right)^{-1}\frac{\partial}{\partial\psi}\left(\frac{\iota}{2\pi}\right) \\ U &= \frac{1}{2\pi P'}\oint\frac{\mathrm{d}\chi\,\mathrm{d}\phi}{J} = \lim_{N\to\infty}\frac{1}{N}\int\frac{\mathrm{d}\ell}{B}. \end{aligned}$$

Strong stabilizing effects is present against off-resonant perturbations in high magnetic shear configuration. Therefore, in LHD, the whole plasma column will not be collapsed immediately, by an excitation of resonant unstable mode at a rational surface.

3.2 Stability condition for a convective mode

Since the mode of $m = n = 0$ is not related to the resonance condition at any rational surface, the displacement of this instability grows up traversing the plasma column. Hence, we call this type of instability as convective instability.

Stability condition for convective modes ($m = n = 0$) is reduced to the following form:

$$\gamma p\left(\frac{U'}{U}\right)^2 + p'\frac{U'}{U} + \mu^2\frac{\boldsymbol{B}_\mathrm{p}^2}{\mu_0}U^{-1}\frac{\partial}{\partial\psi}\left\{\frac{\iota}{2\pi}\right\}I'_\mathrm{t} \geq \left(\gamma p\frac{U'}{U} + p' + \mu\frac{\boldsymbol{B}_\mathrm{p}\boldsymbol{B}}{\mu_0}\right)^2\left(\gamma p + \frac{\boldsymbol{B}^2}{\mu_0}\right)^{-1}, \tag{4}$$

where $I_\mathrm{t}(\psi)$ is the toroidal plasma current.

3.3 Numerical example of stable pressure profile in LHD

We show the numerical result for the marginally stable pressure profile given by eqs. (3)and (4). For simplicity, we assume LHD vacuum magnetic field with $R_\mathrm{ax} = 3.6$ m and $B_\mathrm{ax} = 2.75$ T. Toroidal plasma current is assumed to be $I_\mathrm{t} = 0$. Numerical examples of marginally stable pressure profile are shown in Fig. 3, together with the distribution of the rotational transform and the magnetic hill profile. The plasma pressure of the chaotic field line region is assumed to be 0 or to be some finite value.

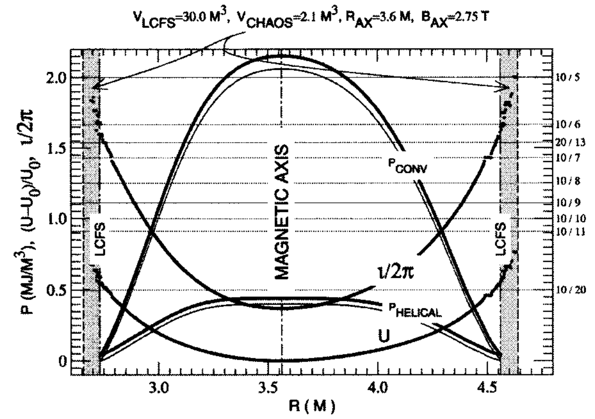

Fig. 3 Numerical examples of marginally stable pressure profile of LHD in the oblong cross section. Axis of abscissas is the major radius *R*. Marginal pressure profile for the helical (convective) mode is shown by P_HELICLA (P_CONV). The plasma pressure of the chaotic field line region ($n_\mathrm{LCFS}T_\mathrm{LCFS}$) is assumed to be 0 or to be $5 \times 10^{19} \times 5$ ($\mathrm{m^{-3}{\cdot}keV}$). LCFS specifies the positions of the last closed flux surface. The distribution of the rotational transform $\iota/2\pi$ and the profile of magnetic hill *U* are also plotted. The chaotic field line region is also shown in the figure.

4. Summary and Discussions

We have shown that the plasma pressure in LHD can be sustained by the magnetic shear and by the plasma pressure in the chaotic field line region. Plasma pressure in the chaotic field line region is sustained stably by the line-tying effect of the lines of force which are disengaged from the chaotic field line region. This pressure specifies the boundary condition for the equation of marginally stable pressure profile given by eq. (3) or eq. (4). The core plasma pressure can be boosted up by the plasma pressure in the chaotic field line region as shown in Fig. 3. From these results and the experimental results we may conclude that LHD has a potentiality of high-beta plasma confinement.

Stability analysis with finite beta equilibrium magnetic field is the next step for high beta plasma study of LHD. Global mode analysis is also important for the high beta plasma stability.

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