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The Analysis of Magnetic Island Using Generalized Magnetic Coordinates

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Abstract

The generalized magnetic coordinates system, which describes magnetic fields with and without nested magnetic surfaces, is constructed for a simple analytic helical field involving magnetic islands. In order to analyze magnetic islands, the residue of a tangent map at the fixed points and the Fourier components of the perturbation of the magnetic field are studied.

Keywords:

generalized magnetic coordinate, GMC, magnetic flux coordinate, magnetic island,perturbation, residue, resonance, ABC magnetic field, B-spline function

1. Introduction

Magnetic (or flux) coordinates [1,2] are widely used for the description of magnetically confined plasma. These coordinates assume the existence of nested magnetic surfaces, but they do not always exist because magnetic islands caused by perturbation fields break down good magnetic surfaces. Besides, there might be chaotic or stochastic space of magnetic lines of force. In such cases, the conventional magnetic coordinates are not constructed. Some efforts on the generalization of the flux coordinates for the general magnetic configuration are reported [3].

The Generalized Magnetic Coordinates (GMC) [4], which are independent of the existence of nested magnetic surfaces, have been proposed as a new supplement to the flux coordinates system [5]. The GMC are applicable to the general toroidal magnetic field involving magnetic islands and/or the chaotic or stochastic magnetic lines of force.

In this paper, the GMC are constructed for a helical model magnetic field having magnetic islands. We attempt to investigate the properties of magnetic islands by using the Fourier components of the perturbation field obtained in the GMC. For the purpose, the residue of a tangent map [6,7] at fixed points is computed and compared with those obtained by field line tracing.

2. Generalized Magnetic Coordinates

The GMC are curvilinear coordinates (ξ,η,ζ) in which the magnetic field is expressed as

$$\mathbf{B} = \nabla\Psi(\xi,\eta,\zeta)\times\nabla\zeta + H^{\zeta}(\xi,\eta)\nabla\xi\times\nabla\eta\,, \qquad (1)$$

where the function Ψ is equal to A_ζ, and the GMC are constructed on the condition that $H^\zeta \equiv \sqrt{g}B^\zeta$ does not depend on the periodical toroidal angle ζ, $\sqrt{g}$ being the Jacobian. The GMC do not use the magnetic surface quantity as a variable of the coordinates as do conventional flux coordinates.

When the nested magnetic surfaces exist, the function Ψ becomes independent of ζ and $\Psi(\xi,\eta)$=

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const. expresses the magnetic surface. If the good magnetic surfaces do not exist, Ψ depends on ζ. We shall write Ψ as the ζ-dependent part and ζ-independent part,

$$\Psi(\xi,\eta,\zeta) = \tilde{\Psi}(\xi,\eta,\zeta) + \bar{\Psi}(\xi,\eta)\,, \tag{2}$$

$$\bar{\Psi}(\xi,\eta) = \oint \Psi(\xi,\eta,\zeta)\,d\zeta\,, \tag{3}$$

where $\bar{\Psi}(\zeta,\eta)$ is the averaged magnetic surface. The ζ-dependent part $\tilde{\Psi}$ is related to the breaking of magnetic surfaces such as magnetic islands. The GMC are to be constructed so that Ψ depends on ζ as little as possible.

3. Analysis of Magnetic Island

Magnetic islands are caused by resonant radial magnetic fields on the rational surface where the rotational transform is equal to a rational value n_0/m_0. The GMC system separates the perturbation $\tilde{\Psi}$, which breaks the nested good magnetic surfaces, from the magnetic field. As a measure of magnetic islands, the residue is obtained by using the amplitude of the Fourier mode components of the perturbation separated in the coordinates resonating with the rotational transform on the rational surfaces.

Here we introduce the curvilinear coordinates (ψ,θ,ζ), such that $\bar{\Psi} = \bar{\Psi}(\psi)$, and the magnetic lines of force without the perturbation $\tilde{\Psi}$are straight in the $\theta - \zeta$ plane. For the sake of simplicity, we choose ψ so that $d\bar{\Psi}/d\psi = \iota(\psi)$; ψ is the toroidal flux.

Then, the equations for the line of force are expressed as

$$\frac{d\psi}{d\zeta} = \frac{\mathbf{B}\cdot\nabla\psi}{\mathbf{B}\cdot\nabla\zeta} \quad \frac{d\theta}{d\zeta} = \frac{\mathbf{B}\cdot\nabla\theta}{\mathbf{B}\cdot\nabla\zeta}\,. \tag{4}$$

The perturbing radial magnetic field $\tilde{H}^{\psi}$ is expressed as

$$\tilde{H}^{\psi}(\psi,\theta,\zeta) = \left(\frac{\partial\tilde{\Psi}}{\partial\xi}\frac{\partial\Psi}{\partial\eta} - \frac{\partial\tilde{\Psi}}{\partial\eta}\frac{\partial\Psi}{\partial\xi}\right) / \,\iota\oint H^{\zeta}d\zeta\,, \tag{5}$$

which is expanded in Fourier series as

$$\tilde{H}^{\psi}(\psi,\theta,\zeta) = \sum_{m,n}\tilde{H}^{\psi}_{m,n}(\psi)\exp\left\{2\pi i\,(m\theta - n\zeta)\right\}. \tag{6}$$

We consider the linear approximation of the field line in the neighborhood of the rational surface ψ_0 of rotational transform $\iota = n_0/m_0$. If we introduce the variable $\phi = m_0\theta - n_0\zeta$, the equations of the field line can be written in the Hamilton's form,

$$\frac{d\psi}{d\zeta} = -\frac{\partial F}{\partial\phi} \quad \frac{d\phi}{d\zeta} = \frac{\partial F}{\partial\psi}\,, \tag{7}$$

where the Hamiltonian F is expressed as

$$F(\psi,\phi) = \frac{m_0\iota'(\psi_0)}{2}\left(\psi - \psi_0\right)^2 + U(\phi)\,, \tag{8}$$

and the potential U is represented as

$$U(\phi) = -\sum_k \frac{1}{k}\,\tilde{H}^{\psi}_{km_0,kn_0}(\psi_0)\exp\{ik\phi\}\,, \tag{9}$$

where the index k is the resonance mode number. The equation (8) is the expression of the magnetic island.

Then, the residue [6,7] is expressed as

$$R = \sin^2\left(\frac{m_0\,\omega}{2}\right), \tag{10}$$

where the proper frequency ω is given by

$$\omega^2 = m_0\,\iota'(\psi_0)\,U''(\phi_0)\,, \tag{11}$$

$$U'(\phi_0) = 0\,. \tag{12}$$

The fixed point for $0 < R < 1$ is the O-point, and the fixed point for $R > 0$, $R > 1$ is the X-point. Therefore if $R \neq 0$, then the magnetic islands exist. The existence of magnetic islands is known by the residue. If the residue is large, the magnetic island is large.

4. Numerical Example

We use the ABC (Arnold-Beltrami-Childress) magnetic field in the (x,y,z) Cartesian coordinates as the model magnetic field,

$$\begin{aligned} B_x &= b\cos(2\pi y) + c\sin(2\pi z)\,,\\ B_y &= c\cos(2\pi z) + a\sin(2\pi x)\,,\\ B_z &= a\cos(2\pi x) + b\sin(2\pi y) + B_0\,. \end{aligned} \tag{13}$$

This field can express a toroidal asymmetric magnetic field with period unity. The constant B_0 in the toroidal direction of z is added to be $B_z > 0$. We use the constant $a = 0.2$, $b = 0.1$, $c = 0.6$, and we choose the field parameters of $B_0 = 0.47$, 0.45 and 0.43, so that the magnetic islands appear. The difference of B_0 influences the size of magnetic islands.

The Cartesian coordinates are expressed in terms of (ξ,η,ζ) as follows:

$$x = \xi + \sum_{l=1}^{M+3} \sum_{m=1}^{M+3} \sum_{n=-N}^{N} \xi_{lmn} B_l(\xi) B_m(\eta) \exp(in\zeta),$$

$$y = \eta + \sum_{l=1}^{M+3} \sum_{m=1}^{M+3} \sum_{n=-N}^{N} \eta_{lmn} B_l(\xi) B_m(\eta) \exp(in\zeta),$$

$$z = \zeta / 2\pi, \tag{14}$$

where B_l, B_m are the cubic B-spline function. In order to treat fields of large variation, the coordinates are expanded in Fourier series in the toroidal direction ζ, and the B-spline function, which has local support, is used to locally follow the variation of the field in the other two dimensions.

We set $M = 60$ as the mesh numbers of ξ, η and ζ, and $N = 30$ as the number of Fourier mode. Figure 1 shows the GMC meshes of $(\xi,\eta) = const.$ at equal intervals constructed for the $B_0 = 0.47$ field on the $z = 0$, 0.25, 0.5, 0.75 planes in the Cartesian coordinates. The Poincaré map of magnetic field lines is also overlapped in Fig. 1.

The residue calculated by the eq. (10) is compared with the value obtained by tracing the field line. These comparisons show very close agreement on both the O-point and the X-point.

The distribution of Fourier amplitudes $|\tilde{H}^{\psi}_{m,n}|$ of $(m,n) = (5,1)$, $(6,1)$, $(7,1)$ and $(12,2)$ with respect to the rotational transform ι is shown in Fig. 2 (for the case of a field with $B_0 = 0.47$). On the rational surface of $\iota = 1/5$, $1/7$, the fundamental resonance mode is the largest of all components and there are large magnetic islands. On the rational surface of $\iota = 1/6$, the secondary resonance mode is larger than the fundamental mode and there are small magnetic islands of (12,2) mode. The non-

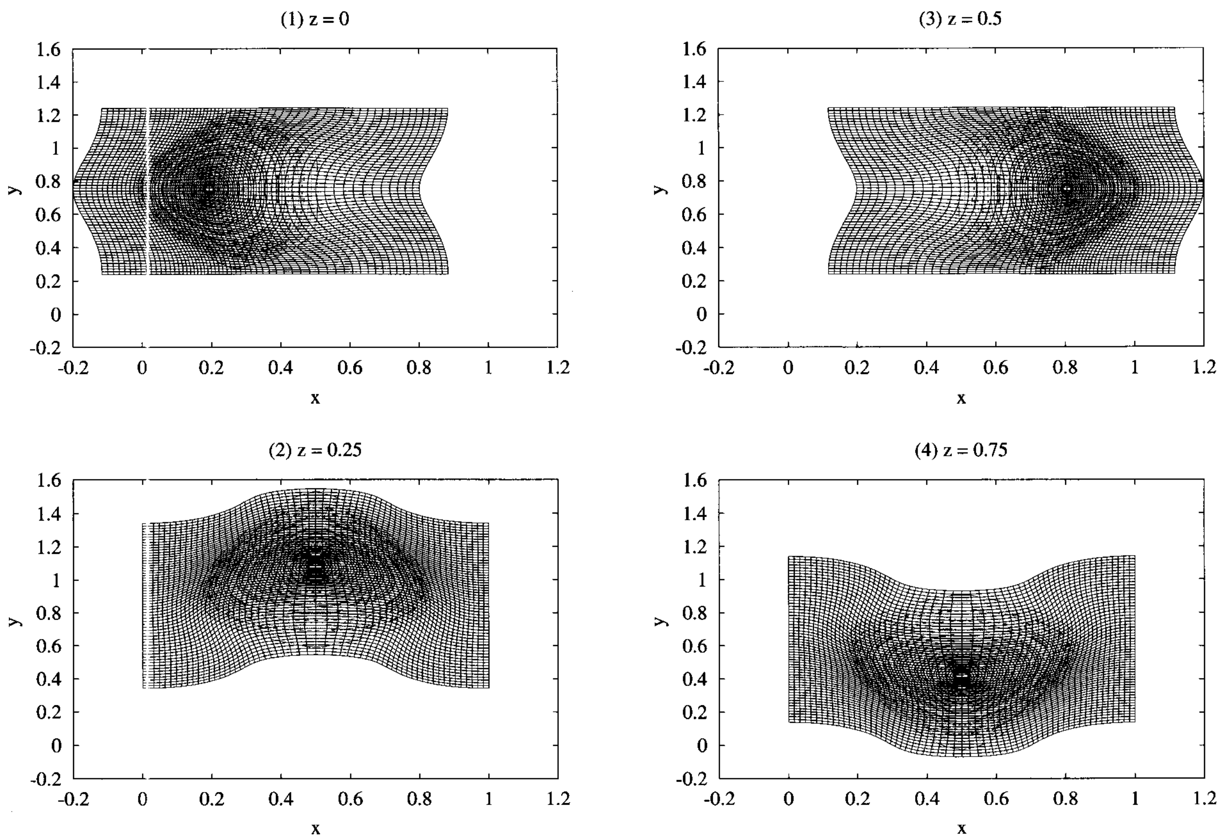

Fig. 1 GMC meshes and Poincaré map in the Cartesian coordinates. (1) z = 0 (2) z = 0.25 (3) z = 0.5 (4) z = 0.75

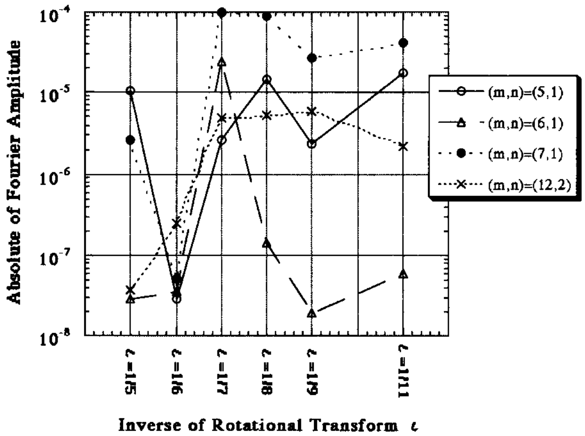

Fig. 2 Distribution of some Fourier amplitudes $|\tilde{H}^{\psi}_{m,n}|$ with respect to the inverse of ι (B_0 = 0.47).

resonant modes are broadly distributed, but they do not contribute to the residue. These characteristics are common in B_0 = 0.47, 0.45, and 0,43 fields.

5. Conclusion

A generalized magnetic coordinates system is constructed on a simple analytic helical field involving magnetic islands. The residue of the tangent map at a fixed point is calculated by using the Fourier component of the perturbation field decomposed by constructing the coordinates. The magnetic island can be described by the resonant mode of the Fourier component of the perturbation on the averaged magnetic surface. The amplitude of non-resonant modes is sometimes of the same order of the resonant modes or larger, but it does not influence on the magnetic island.

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