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Finite β Equilibria of Heliotron J Plasmas

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Abstract

Finite plasma pressure equilibria of Heliotron J is studied using a three-dimensional equilibrium calculation code, HINT, which does not assume the existence of nested flux surfaces. The magnetic island formation investigated for Heliotron J plasmas. In the standard configuration of Heliotron J plasmas, the equilibria with magnetic islands can be appeared at $\beta \geq 1$ %. When beta value on the axis is about 2.7 %, the width of a magnetic island is half of the averaged minor radius. In this study, we also show the approach to improve the convergence of the HINT calculation by using the equilibrium obtained from the VMEC code as an initial guess. The result for applying the improved method to the low beta equilibrium of Heliotron J plasmas is shown.

Keywords:

MHD, equilibrium, magnetic island, HINT code, VMEC code, Heliotron J

1. Introduction

In helical systems, nested flux surfaces can be generated by external coils without plasma current and the analysis of the MHD equilibrium is performed assuming net-current free and existence of nested flux surfaces in many cases. However, the shape of the plasma boundary will be changed by the plasma equilibrium current at the finite pressure. The magnetic island generated at the finite plasma pressure will also break the flux surface in non-axisymmetric system for some cases. Therefore, the analysis including the change of the flux surface at the finite pressure are required without assuming existence of flux surfaces.

Heliotron J device is an $L = 1/M = 4$ helical-axis heliotron, which has large flexibility [1]. The main parameters are $R = 1.2$ m, $\langle a_p \rangle \sim 0.1$–$0.2$ m, $B \leq 1.5$ T and $\iota \sim 0.4$–0.8. Heliotron J has a low shear and the magnetic configuration is very sensitive to the plasma pressure effect. In the previous study, Heliotron J plasma is mainly studied using the VMEC code under the fixed boundary constraint. The VMEC code is an inverse solver based on the energy principle using the flux coordinates [2]. Therefore, the equilibrium including the magnetic island or the stochastic region generated by the plasma pressure was not obtained. In this study, we investigate the finite pressure equilibria in the standard configuration of Heliotron J using the HINT code [3]. At high β, nested flux surfaces is broken by the magnetic island due to the finite pressure.

2. Computation Method of HINT Code

The HINT code is a three-dimensional equilibrium calculation code based on the time-dependent relaxation method (the initial value problem), which uses non-orthogonal helical coordinates (u^1, u^2, u^3) (see ref. [3]). The calculation is performed in the following steps.

The first step (A-process) is a relaxation process along the magnetic field lines under the fixed magnetic filed. This relaxation process calculates the averaged pressure along the field line which starts from a grid point of the helical coordinates (u^1, u^2, u^3),

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$$\bar{p} = \frac{\int p \frac{dl}{B}}{\int \frac{dl}{B}}, \qquad (1)$$

and uses it as updated pressure on a grid. The normal pressure relaxation calculates $\bar{p}$ on all grid points. However, in the low shear configuration which has magnetic islands in the plasma region for example, the field line should be traced for very long distance to obtain the appropriate pressure on a grid. Since it is not efficient to calculate $\bar{p}$ on all grid points, we use the new scheme of A-process in ref. [4] to improve the computational time. This scheme does not trace the field lines on all grid points, but calculates the field lines started from the limited start points. The averaged pressure $\bar{p}$ on a grid point is obtained from the pressure on nearly field lines started from the sample points using the linear interpolation method.

The next step (B-process) is a relaxation of the magnetic field under the plasma pressure distribution is fixed. This process is obtained from the following time evolution equations,

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) = -\nabla p + \boldsymbol{j} \times \boldsymbol{B}, \qquad (2)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B} - \eta(\boldsymbol{j} - \boldsymbol{j}_0)), \qquad (3)$$

where $\mu_0 \boldsymbol{j} = \nabla \times \boldsymbol{B}$ and $\boldsymbol{j}_0$ is the coil current if it exists in a computational domain. The final equilibrium state is solved by these process iteratively until the net-current free condition or the prescribed net-current condition is satisfied. If there are coils in the computational region, the Courant-Friedrichs-Levy conditon (CFL condition) is very severe because the Alfvén velocity is very fast due to the strong field near a coil. Therefore, the factor f_C as,

$$f_C = \begin{cases} 1 & \text{for } B \le B_C \\ (B_C/B)^2 & \text{for } B > B_C \end{cases}, \qquad (4)$$

is calculated on all grid points, where B_c is specified to the value between the plasma region and the region near a coil. The factor, f_C, multiples right side of eq. (2) and the Lorentz force due to the coil current is also excluded from the second term of right side in eq. (2). According to this scheme, the propagation velocity is uniform in the computational region and the CFL condition is also satisfied [5].

In this study, we also introduce the method to improve the convergence of the HINT computing by using the equilibrium obtained from the VMEC code as an initial guess. For this purpose, we develop the IDH code, which translates the equilibrium obtained by the VMEC from the flux coordinates (s, θ, ζ) to the helical coordinates (u^1, u^2, u^3) inside and outside the VMEC plasma boundary, respectively. For inside the boundary, the IDH code makes the inverse mapping of the equilibrium from the flux coordinates to the helical coordinate using the method in ref. [6]. It calculates s and θ to coincide with $\mathrm{x}(u^1, u^2)$ under the condition with $u^3 = -\zeta$ fixed for simplicity. On the other hand, for outside the boundary, the field due to the plasma equilibrium currents is calculated using the DIAGNO code [7] and translates to the helical coordinate also.

3. Finite β Equilibria of Heliotron J Plasmas

In Fig. 1, the flux surfaces of standard configuration of Heliotron J plasmas is shown in the vacuum. This configuration has a flat profile of the rotational transform ($\iota \sim 0.55$) and the magnetic well in the vacuum as Fig. 2. The finite pressure equilibrium obtained from the HINT code at $\beta = 2.7$ % is shown in Fig. 3. Here we use $p(s) \propto (1 - s)^2$ as an initial pressure distribution, where s denotes normalized toroidal flux. Large islands which correspond to the resonance of 4/7 are generated in the plasma region and the width of island is half of the averaged radius. In this study, the resonance of 4/7 is appeared when $\beta \ge 0.8$ %. The resonance of $\iota = 4/8$ is not appeared for the standard

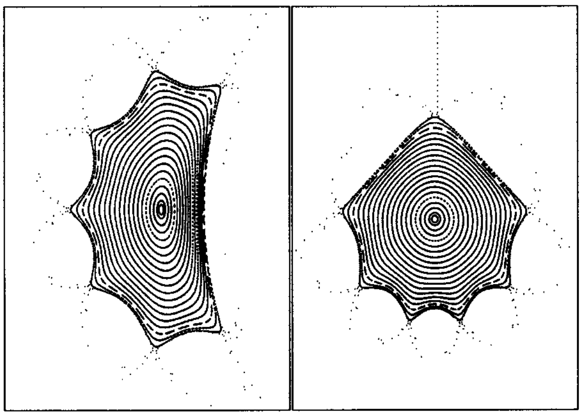

Fig. 1 Flux surfaces of the vacuum configuration. Right side is the cross section at $M\phi = 0$, left side is $M\phi = \pi$.

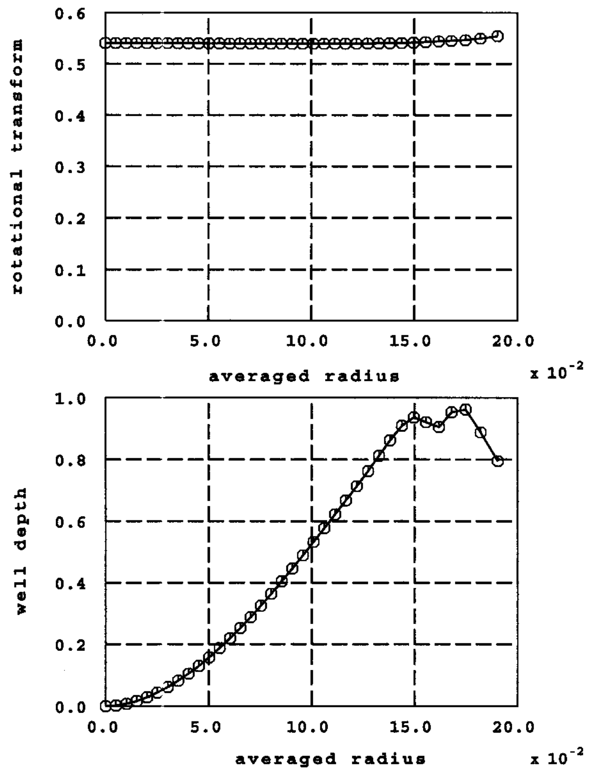

Fig. 2 Profiles of the rotational transform and the magnetic well of standard configuration in Heliotron J plasmas.

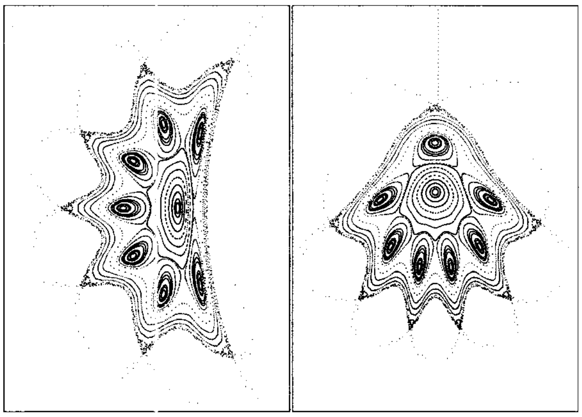

Fig. 3 Flux surfaces of the equilibrium for β = 2.7 %. Right side is the cross section at $M\phi = 0$, left side is $M\phi = \pi$.

configuration.

Finally, we show the equilibrium for β = 1 % using new method described in previous section. This equilibrium uses the result obtained from the VMEC at β =1 % as an initial guess. In Fig. 4, the equilibria obtained from the conventional method and our new method are shown. In both equilibria, the results are very similar, because both the width and the phase of islands are similar.The computational time is also improved about 25 %.

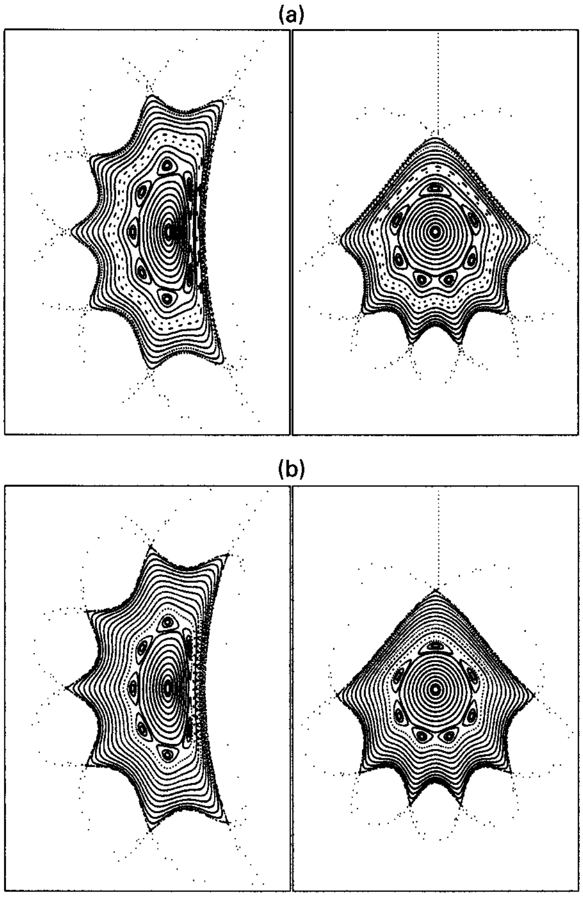

Fig. 4 Flux surfaces of the equilibrium for β = 1 % obtained from the conventional method (a) and new method (b). Right side is the cross section at $M\phi$ = 0, left side is $M\phi = \pi$.

4. Conclusions

We study the equilibria of Heliotron J plasmas using the HINT code. It is found in the standard configuration of Heliotron J plasmas, the magnetic island of ι = 4/7 resonance is generated by finite pressure effect when beta ratio is above 0.8 % ~ 0.85 %. We also improve the HINT computing scheme using the equilibrium obtained from the VMEC code as an initial guess. For this purpose, we develop the IDH code, and we can reduce the relaxation time steps with this new method. In the present study, we study standard configuration only with specified pressure profile.

Studying the effect of pressure profile or the vertical filed to the equilibrium are future subjects.

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