Compatibility of Different Elements of Confinement Characteristics in Quasi-axisymmetric Stellarators

OKAMURA Shoichi, NISHIMURA Shin, ISOBE Mitsutaka, SUZUKI Chihiro, MURAKAMI Sadayoshi, YOKOYAMA Masayuki, SHIMIZU Akihiro and MATSUOKA Keisuke

National Institute for Fusion Science, Toki 509-5292, Japan
1 Nagoya University, Nagoya 464-8603, Japan

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Abstract

In order to understand the relation between several confinement characteristics of quasi-axisymmetric stellarators, optimization runs were made with a special weight on a selected characteristics of the configuration without considering other aspects. A limit of highly axisymmetric configuration was obtained with helical ripple of 0.12% at the plasma boundary sacrificing the magnetic well and rotational transform. Another design with reduced helical ripple with high rotational transform gave improved transport in highly collisionless regime but it also lost magnetic well. On the other hand the configuration with reduced Shafranov shift and improved MHD stability was designed with slightly smaller helical ripples. The method to obtain simultaneously high quasi-symmetry and good MHD property has not been found.

Keywords:
quasi-axisymmetric stellarator, helical ripple, Shafranov shift, optimization, MHD stability, collisionless transport

1. Introduction

In the magnetic configuration design for the toroidal confinement system, it is generally very difficult to satisfy all important conditions (particle orbits, magnetic shear, MHD stability, etc.) necessary for the good confinement characteristics. Usually we try to find a good compromising point for those conditions following to the objectives of the design work or the experimental plan. One of typical examples is found in the magnetic configuration of heliotron/torsatron device where good particle orbits and the linear ideal MHD stability are contradicting with each other depending on the parameter of magnetic axis position. In this case, the decision of determining a compromising point can be made straightforwardly since the relation between those characteristics is already established at least in the theoretical work.

A quasi-axisymmetric stellarator concept [1,2] is one of new generations of helical systems which are frequently called advanced stellarators. This configuration is actually of a stellarator because it has the rotational transform \((\iota = 1/q)\) given by the external coils. \(\iota\) value typically ranges from 0.2 to 0.5 depending on the individual design. Most important characteristic of this concept is that the drift orbits of trapped particles are almost equivalent to those in tokamaks (negligibly small number of helically trapped particles). Because the high beta stability is a basic requisite for the new advanced stellarators, configuration
designs have been made for a good combination of the axisymmetry and MHD stability for high beta equilibrium [3,4]. From the aspect of designing a good experimental device, present designs might be sufficient for obtaining a good plasma confinement with very small helical ripples because the important issue in the confinement would be anomalous transport.

However, when we consider the prospect to the reactor for quasi-axisymmetric stellarators, the confinement of high energy particles in the highly collisionless regime must be discussed. One solution was found [5] with additional mirror ripples which is even less axisymmetric than the standard designs of quasi-axisymmetric stellarators. In this paper, we tried to find a solution by reducing the helical ripple components as much as possible. We intentionally abandon to consider the MHD stability and high level of rotational transform in this work, in order to find an extreme limit of quasi-axisymmetry with smaller but acceptable level of external rotational transport. Such process is also instructive to find which confinement characteristics are cooperative or contradictory to the quasi-axisymmetry. Other examples of configuration design are also given with the consideration of high level of rotational transform or the MHD stability. Based on these results, the discussions about the compatibility of confinement characteristics of quasi-axisymmetric stellarators are given.

2. Configuration with Very Small Helical Ripples

The characteristic of quasi-axisymmetry is clearly shown when the magnetic field structure is represented in the Boozer coordinates. Since the particle orbits can be described with only the spectra of magnetic field strength in this coordinates [6], the axisymmetry in the Boozer spectrum guarantees the tokamak-like trapped particle motion. The configuration design was made to find out how much axisymmetry in Boozer spectrum can be obtained if we improve the symmetry without considering the MHD stability tentatively. We started from one of quasi-axisymmetric configurations which has toroidal periods N=2 and an averaged aspect ratio 3.9. The rotational transform at the magnetic axis is 0.23 slightly increasing to 0.28 at the plasma boundary. This configuration was designed with care of both quasi-axisymmetry and the magnetic well.

The improvement of configuration was made with 41 variables of Fourier coefficients for the boundary shape. Based on the fixed boundary VMEC calculation for the equilibrium, the Boozer spectrum of magnetic field was calculated. By summing up amplitudes of non-axisymmetric components of the spectrum over all magnetic surfaces, the level of axisymmetry is evaluated. The 41 variables are gradually modified to the direction to decrease this value. The aspect ratio was fixed. In the quasi-axisymmetric stellarator, the rotational transform is created by the non-axisymmetry in the geometric shape of the torus. Since the geometrically more symmetric configuration naturally has better axisymmetry in the Boozer spectrum, the optimization with Boozer spectrum gradually push the
solution to the axisymmetric geometric shape of the torus which has lower value of the rotational transform. We stopped this work when the rotational transform decreased to 0.2 at the boundary.

Figure 1 shows the Boozer spectrum of the final solution (Sym-1). The amplitudes of the spectra are the relative values to the toroidal field. The largest non-axisymmetric component is the mirror term $(m, n)=(0, 1)$ where $m$ and $n$ are the poloidal and toroidal mode numbers per period respectively. At the half minor radius, the relative amplitudes of non-axisymmetric components are less than 0.013 % of the toroidal field. Figure 2 shows the variation of the magnetic field strength along the field line for three minor radii. All three lines show toroidicity with negligible levels of non-axisymmetric (helical) ripples.

Three poloidal cross sections of magnetic surfaces are shown in Fig. 3. If we compare the boundary shape of this configuration with a standard configuration (2b32) for CHS-qa device design [3], the crescent shapes are similar for the vertically elongated cross sections while the shape of the horizontally elongated cross section is more circular and the direction of the triangularity is opposite to each other (a tip of triangle faces toward the torus center in Fig. 3). The magnetic well is almost vanishing which is a direct consequence of the direction of the triangularity. The Shafranov shift is very large and consequently the equilibrium beta is much lower than a standard configuration of CHS-qa. This solution lost good MHD property in exchange for the good symmetry.

### 3. Configuration with Small Helical Ripples and High Rotational Transform

Even though the solution Sym-1 has a very good quasi-axisymmetry, the calculation of alpha particle
confinement did not show very promising results for its vacuum configuration. Small rotational transform causes relatively large direct losses of alpha particles due to the bigger Larmor radius. In order to solve this problem, we next started from the CHS-qa standard configuration 2b32. It has a toroidal periods \( N=2 \), the rotational transform of 0.4 at the boundary and the averaged aspect ratio 3.2. The configuration improvement was made towards the better quasi-axisymmetry with a condition of keeping the edge rotational transform at 0.4 but again without considering the MHD stability. The magnetic surfaces of the final solution of this optimization work (Sym-2) are shown in Fig. 4. The non-axisymmetric Boozer spectrum components were decreased below 1% of the toroidal magnetic field which is smaller by a factor of 4 than those in 2b32 configuration.

The Monte Carlo calculation was made for this new configuration to evaluate the neoclassical diffusion coefficient. Figure 5 shows the transport coefficients for 1 keV electrons for 2b32 and Sym-2 configurations with additional parameters of 1.5 m major radius and 1.5 T magnetic field. The diffusion coefficient is evaluated by following the motions of electrons that are initially distributed at the magnetic surface of half minor radius and diffuse by collisions. The diffusion coefficients for the conventional helical device CHS are also shown for the comparison. Significant improvement in the confinement for a very low collisionality regime is shown for the Sym-2 configuration.

However, the magnetic well is lost again for this configuration. In Fig. 4, the similar change in the shape of cross section is recognized to the case of Fig. 3, that is, the direction of the triangularity in the horizontally elongated cross section is opposite to 2b32 configuration. By considering these two examples, it is noticed that the shape of the horizontally elongated cross section determines the depth of the magnetic well although the crescent shape of the vertically elongated cross section must also help to create the magnetic well.

## 4. Configuration Improved for the MHD Stability

Finally we try to improve the configuration with more emphasis on the MHD property. The optimization was made, started from 2b32 configuration, for the reduction of the Shafranov shift and the Mercier stability together with the condition of the better quasi-axisymmetry. The improvement was successful for the

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**Fig. 5** Diffusion coefficients for 2b32 and Sym-2 configurations. Those values for two configurations in CHS are also shown.

**Fig. 6** Three cross sections of magnetic surfaces for improved MHD configuration.
MHD properties without affecting the axisymmetry. About 20% reduction of the Shafranov shift is obtained for the new configuration. The Mercier stability is also improved. However the improvement in the quasi-axisymmetry was very small even by making further optimization, as far as these conditions of MHD property were applied. Figure 6 shows the cross sections of magnetic surfaces for the improved configuration. It is very distinct that the triangularity of the horizontally elongated cross section is opposite to the previous cases shown in Figs. 3 and 4.

5. Conclusion
By comparing shapes of cross sections of Figures 3, 4 and 6, it is clear that the good MHD property is determined by the shape of the horizontally elongated cross section. Especially important is the triangularity of these cross sections. From the experiences of several examples of the optimization procedures we have made so far, it was difficult to obtain two results simultaneously, the reduction of helical ripples and the control of the direction of triangularity in the horizontally elongated cross section (facing toward the outside of torus). The remaining problems are 1) to try to obtain a favorable direction of the triangularity starting from the good axisymmetric solution and 2) to try to improve the MHD property of good axisymmetry with the effect of the plasma current.

References