J. Plasma Fusion Res. SERIES, Vol. 5 (2002) 346–350

Intermittent Ion Trajectories and Focusing in Spherical Inertial Electromagnetic Confinement

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(Received: 11 December 2001 / Accepted: 17 May 2002)

Abstract

Performance of spherical magnetic electrostatic confinement fusion is analyzed. Magnetic field lines in spherical cusps provide a chaotic scattering system that improves the ion focusing due to synergetic effect of intermittent ion trajectories. Confined ions are bounced within electrostatic well reflecting from magnetic barriers shaped by convex field lines. The concept offers a means of enhancing ion reactivity provided by caustic formation that are resulted from an intersection of ion orbits within confinement volume. Parameters of the experimental device are estimated.

Keywords:

magnetic inertial electrostatic confinement fusion, neutron source, spherical ion convergent flow, ion focus

1. Introduction

Magnetic electrostatic plasma confinement (MEPC) is an alternative plasma confinement scheme. In this concept the material grid using as cathode in inertial electrostatic confinement (IEC), is replaced by virtual cathode generated by confining electrons in multicusp magnetic field. Potential advantages of MEPC are that many different fusion reactions (i.e., D-D, D-T, D-^{3}He, p-^{11}B, and p-^{7}Li) may occur from high-energy collisions between the counter-streaming ions oscillating in the electrostatic well formed by electrons. This devices can produce high-energy neutrons, protons, alphas, etc. It may be attractive source (10^6–10^{12} particles/s) suitable for scattering analysis, neutron activation, isotope production, medical therapy, oil well logging. Several decades of small, cheap, pulsed experiments have demonstrated the basic principles of MEPC [1,2]. Further progress is possible by the use of improved schemes. Pseudo-spherical confinement volume of magnetic electrostatic confinement device was considered by Lavrentiev in [2], experimentally studied in Poliwell [3], Penning traps [4]. In this paper the scheme of ion focus in spherical multicusp magnetic electrostatic confinement [5] is analyzed.

2. Spherical Multicusp Magnetic Electrostatic Plasma Confinement

The main idea described in this paper is to transform the plasma volume of the linear Magnetic Electrostatic Plasma Confinement device (see ref. [1]) into pseudo-spherical volume by arrangement of ring cusps on a sphere (Fig. 1, Table 1).

Electrons are produced by electron gun in point cusps, by ionization of neutral gas and by secondary emission from cathodes of linear cusps. Electrons are

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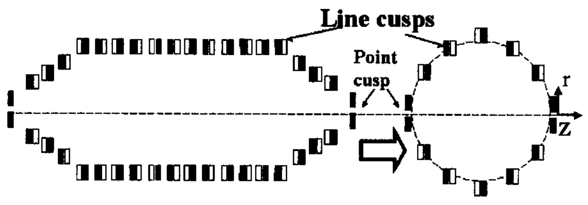

Fig. 1 Schematic of the trap. The main idea is to transform a linear arrangement of the ring cusps to a sherical one. The figure represents a cylinderical symmetric configuration about z axis. The concept offers a means of enhancing ion reactivity provided by caustic formation that are resulted from an intersection of ion orbits within confinement volume.

Table 1 Spherical cusps: experimental device. The plasma parameters are estimated using approximate scalings of density, temperature [1], $T_e \approx T_i \approx 0.05\ \phi_A$, $n \propto B^2$, estimation of confinement time was made in text.

Cusp anode radius, m	0.3
Plasma radius R, m	0.25
Cusp magnetic induciton, T	1
Volume magnetic induction, T	0.2
Cusp anode gap width, cm	0.3
Cusp plugging anode voltage, ϕ_A, kV	25
Electron injection energy, keV	20
Electron injection current, A	0.2
Pulse duration, ms	> 30
Volume plasma density n_0, m^{-3}	~ 10^{18}
Central plasma density, m^{-3}	~ 10^{19}
$T_{i,e}$, keV	~ 1
Energy confinement time τ_e, msec	~ 6

confined in a trap by magnetic fields. Negative space charge of these contained electrons forms an electrostatic well for ion confinement. Ions are born in the edge of the device by the ionization and fall in the electrostatic well. Estimation of space charge needed for negative potential well [6] is of $\delta n^- = 3\phi_p/2\pi eR^2$, where ϕ_p – potential in the center, R-plasma radius. For the experimental trap (Table 1) is needed only a slight imbalance of electron charge of $\delta n^- \sim 10^6$–10^7 1/cm^3. Effects of collisions should be taken into account only in the edge and small central region with dense plasma. In main central part of the confinement volume the ion motion is collisionless. Electrostatic plugging is used to reduce the loss rate of plasma flowing along magnetic field lines out of the cusp gaps. Cathodes outside the cusp gap reflect escaping electrons back into the plasma. The cusp plugging is equivalent to magnetic shielding of the grid wires of an electrostatic-inertial confinement device. Described confinement concept offers a means of enhancing ion reactivity provided by caustic formation within a pseudo-spherical plasma. The dense caustic regions is considered to be formed by bounced ions reflected from convex magnetic barriers.

Magnetic fields. To describe magnetic fields created by the current coils (Fig. 1) we note an axial symmetry of the system. It means that magnetic surfaces will be created to confine the electrons. Only one component of magnetic vector potential describes the magnetic field structure. In cylindrical coordinates (r, φ, z), magnetic field $H(r, z)$ is created by the fields of N_c coils,

$$\vec{H}(r,z) = \sum_{n=1}^{N_c} \vec{H}_n ,$$

as well as magnetic vector potential,

$$\vec{A}_\varphi(r,z) = \sum_{n=1}^{N_c} \vec{A}_{\varphi_n}(r, z - z_n) ,$$

for coils centered at z_n, vector potential of a coil centered at $r = 0$, $z = 0$, defines as [7],

$$A_\varphi(r,z) = \frac{4J}{ck}\sqrt{\frac{a}{r}}\left\{\left(1 - \frac{k^2}{2}\right)K(k) - E(k)\right\};$$
$$k^2 = \frac{4ar}{(a+r)^2 + z^2},$$

where a – coil radius, J-current in the coil, K and E – elliptic integrals. Field lines are defined by equations, $\varphi = const$, $rA_\phi = const$.

$$\vec{H}_r(r,z) = -\frac{\partial A_\varphi}{\partial z}, \quad \vec{H}_z(r,z) = \frac{1}{r}\frac{\partial(A_\varphi)}{\partial r}$$

Field lines topology for spherical system of 5 ring cusps and point cusps at the axes is shown in Fig. 2. It illustrates the minimum-B configuration that offers a good magnetohydrodynamical stability. Radial distribution of beta (a ratio of plasma pressure to magnetic pressure) is defined by this minimum-B configuration. Plasma is a diamagnetic matter and vacuum magnetic fields will be modified by the confined hot plasma. For the effect considered in this paper, it is important only general topology, it is a convex nature, of the magnetic field lines. There are no strong diamagnetic currents and this topology is not expected to be destroyed by confined plasma. The finite-beta effect is expected to be important for the strength of magnetic field inside the confinement volume rather

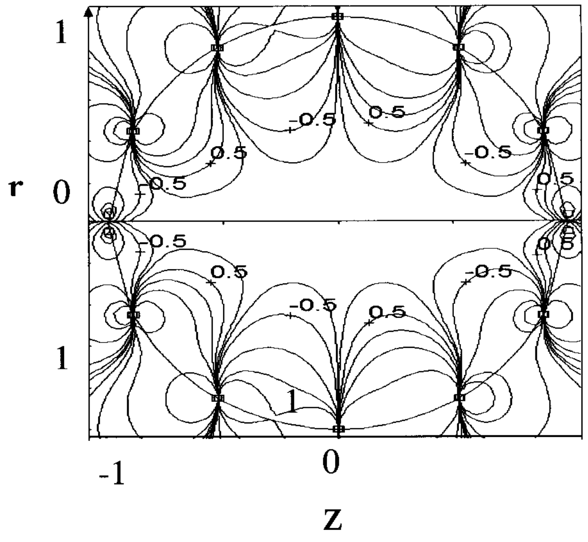

Fig. 2 Magnetic field lines (constant rA_φ) in the spherical cusps system. The plot represents a cylindrical symmetric configuration about z axis. Axis are measured in units of the radius of the spherically confined plasma. Convex magnetic line provide a stochasticity system for ions bounced within.

not for the magnetic lines topology defined only by the topology of coils. The number of ring cusps can be optimized by detailed simulation of the system.

Radial scales. (1) The boundary (collisional plasma) layer thickness is estimated by magnetic flux conservation, $\Delta \approx a(B_a R_a/Br)$, a - anode gap half-width, B_a-magnetic induction in cusp gaps, R_a-cusp anode radius. (2) R_c is a radius of central dense region (collisional plasma) assuming a focusing in pure electrostatic well of potential ϕ_p of spherical symmetry. Initial ion energy of the order of ionization energy $E_{\perp 0} \sim 5$ eV, $R_c \approx (E_{\perp 0}/2e\phi_p)^{1/2}R_a$ [1]. (3) The central collisionless plasma region $R_c < r < \Delta$. (4) A radius R_m, when the ion gyroradius $\rho = \upsilon_\perp/\omega$ becomes larger than $\boldsymbol{E} \times \boldsymbol{B}$ drift pass over the gyration time $\sim 1/\omega$, $\rho > \upsilon_{E\times B}/\omega$, $\upsilon_{E\times B} = c\boldsymbol{E} \times \boldsymbol{B}/B^2$. Assuming $\phi(r) = \phi_0(1 - r^2/R^2)$, $E = 2\phi_0(r/R^2)$, maximal angular velocity can be estimated, $\upsilon_\perp(r) \sim (2e\phi_0(r^2/R^2)/m_i)^{1/2}$. Assuming a multipole magnetic field dependence, $B(r) \sim B_b(r/R)^m$, $m = 7$–10, where boundary magnetic field $B_b \sim (2m_e T_{eb})^{1/2}/e\rho_{eb}$, m_e, T_{eb}, ρ_{eb}, electron mass, electron temperature at the edge and electron gyroradius in the edge, respectively. The radius is estimated by $R_m \sim R(\rho_{eb}/R\,(e\phi_0/T_{eb})^{1/2})^{1/m}$. For the experimental device (Table 1), $\Delta \approx 10$–20 mm, $R_c \sim 3$–5 mm., ($m = 7$–10, $T_{eb} \sim 10$ eV, $\rho_{eb} \sim 410^{-4}$ m), $R_m \sim 0.8$ R. It means that for $R_c < r < 0.8\,R$, we need not apply a guiding center approximation for numerical simulation of ion motion.

Time scales. There are several time scales of the process. (1) The ion-electron momentum-transfer collision time. (2) The ion-ion collision time. (3) The ion bounce time τ_b : time that an ion takes to complete a closed orbit in the spherical electrostatic well. $\tau_b \sim 4a/\upsilon_b$, $\upsilon_b = (2\varphi_0/m)^{1/2}$, φ_0 - the ion well depth. For typical operating conditions of experimental device (Table 1), the time scales hierarchy estimated (using formula from [1]): $\tau_b \sim 10^{-6}$ s, $\tau_{ii} \sim 0.1$ s, $\tau_{ie} \sim 100$–200 s. Hence, $\tau_{ii} < \tau_{ie}$ suggesting that the ions and electron physics are decoupled in the ion-ion collision time, accordingly ion-electron collisional interactions can be neglected in the central plasma confinement region.

Power balance issues. Ion thermalization: collisional degradation of the beam-like ion distribution function is a crucial issue in the assessment of the physical feasibility of all IEC schemes, because it may preclude adequate ion concentration at the spherical center. Spherical magnetic electrostatic multicusp considered in this paper, gives a chance to improve the system performance by dense caustic formation in the central region. Synchrotron radiation losses in magnetic IEC have been estimated to be negligible [6]. An important power sink may be from bremsstrahlung radiation losses, that can be minimized by utilizing low-Z fuels (D-D, D-T) [4].

For the ring cusps confinement time is assumed to scale according the classical diffusion estimated by Pastukhov [8], $\tau_{diff} = 2\tau_{ei}\, aV/S\alpha\rho_b\rho_a$ [1]. The non-radiative cross-field electron energy loss time [1], $1/\tau_{E\perp} = 1/\tau_{cond} + 1/\tau_{diff}$, where τ_{cond} is the time for heat loss by conduction $\sim 1.5\ \tau_{diff}$, τ_{diff} is the characteristic time for the diffusion across the magnetic field. For spherical cusps (Table 1), $\tau_{diff} = 0.144\ \tau_{ei}\ aR/\rho_b\rho_a \approx 6.6 \times 10^{-3}$ sec, it is above 50 % higher then in a device of the same scale of linear set of ring cusps (like Jupiter-2M).

The theoretical study in [4] illustrated efficient operating regimes of spherical magnetic IEC (Penning trap) with high ratio of fusion power to ion input power ($Q > 100$) identifying beam-like solution for the ion distribution. Even the pessimistic estimation [9] of fusion breakeven leaves open the development of driven neutron sources in a pure electrostatic well predicting Q-value (fusion power over ion input power) of $Q < 0.21$ for a 50 kV square electrostatic well. More optimistic estimation [10] is considered a scenario of the process of ion-ion collisions providing $Q \sim 1.3$ for the same system. It is needed detailed analysis of the power balance for the spherical MEPC to evaluate a reactivity in dense regions.

3. Ion Focusing and Caustic Formation in Spherical Magnetic Inertial Confinement

Magnetic field lines in spherical cusps provide a chaotic scattering system that improves the ion focusing due to synergetic effect of intermittent ion trajectories. Confined ions are bounced within electrostatic well reflecting from magnetic barriers shaped by convex field lines. The concept offers a means of enhancing ion reactivity provided by caustic formation that are resulted from an intersection of ion orbits within confinement volume. Intermittency of the ion trajectories results in the singular areas (referred as caustics with dense plasma). The ion motion in this system refers to the system known from nonlinear dynamics as a star-shaped billiard, it belongs to the stochastic K-systems and many examples of caustic formation in a stochastic scattering systems are known [11,12].

Computer simulation. In principle, to model the system accurately, the general form of the Boltzmann transport equation would have to be solved. However, this problem is difficult to solve. Ion-electron collisional interaction can be neglected and the problem can also be modeled by considering the two-dimensional Fokker-Planck transport equation of the ion species alone. The ions and electron physics are decoupled in the ion-ion collision time and below only single ion three-dimension trajectory is tested in vacuum magnetic and electrostatic fields. Potential profile formed by electrostatic plugging of cusps was studied theoretically and experimentally (see ref. [1]). Here is considered an idealized version of spherical symmetry when the perturbation of the ion self space charge on the potential well profile is negligible and the well is parabolic one. In this case, it is assumed an electrostatic potential in the form of $\phi(r) = \phi_p(1 - (r/R)^2)$, R-plasma radius. The three dimension motion equations,

$$m_i\,\partial\vec{v}/\partial t = -e\vec{\nabla}\phi + \vec{v}\times\vec{B}$$
$$\partial\vec{r}/\partial t = \vec{v}$$

were numerically solved using subroutine *ode45* (from toolbox of MATLAB-6 package) based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. It is a one-step solver in a computing for nonstiff problems.

An example of the bounced ion orbit illustrates an intermittency and mixing and indicates a formation of the focusing regions (caustics) near the center $R_c < r < 0.1R$ (Fig. 3). The topology of this focusing is determined by a set of cusps (3D symmetry of system and multipole). The strength of current in the coils effects only the radial position of caustics rather not the formation itself. It reflects a property of magnetic topology being an example of stochastic scattering system. A variation of the confinement characteristics, such as magnetic field strength and the potential profile shape, do not effect on the process of the focusing regions formation: the caustics formation was observed with the cusp magnetic field strength of 0.5 Tesla and the potential profile of the form $\phi(r) = \phi_p(1 - (r/R)^d)$, $d = 3,5,7$. The fusion reactivity is expected to be increased in the dense caustic regions leading to an enhancement of performance.

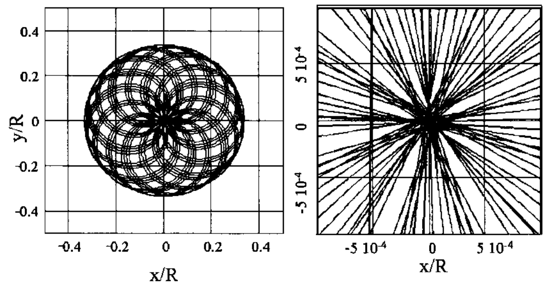

Fig. 3 Single ion test trajectory in the quasispherical electromagnetic multicusp system with a plasma radius of $R = 0.3$ m, cusp magnetic field $B = 1$ T, electrostatic well depth of 1 kV of the parabolic form. 2D cross-section. Intermittency and mixing indicates a formation of the focusing regions (caustics) near the center.

Heuristic comment. There are several issues of caustic formation in a real spherical cusps. One of them is driving of a plasma to a Maxwellian distribution in the edge. In a pure IEC system it leads to the spread of the focus in the geometric center of sphere due to angular momentum change in the edge collisional plasma. In spherical multicusp system the performance enhancement due to bounced ion orbits *topology* and caustic formation is not expected to depend on collisions in edge plasma. According to the theory of singularities and caustics [11], the singularity formation depends on the curvature of reflecting boundary. A stochasticity parameter [12] estimated for the magnetic barriers curvature, is over unity suggesting that magnetic barriers are strong stochastic region for the ion motion within. If the system is characterized as stochastic scattering system, mixing trajectory is expected to produce the dense caustic-like channels in the confinement volume. Other issue is a potential profile forming by cusps. In many experiments performed in MEPC devices, a deep negative potential well has been sustained for many milliseconds [1]. From the theory of caustics [11] it is known that the topology of singularities does not affected by change of potential field of the system. It means, that a deviation of the real plasma potential profile symmetry from spherical one (processed here), *is not expected to lead to a destruction of caustics.*

4. Conclusion

Performance of spherical magnetic electrostatic confinement fusion is analyzed. Magnetic field lines in spherical cusps provide a chaotic scattering system that improves the ion focusing due to the synergetic effect of intermittent ion trajectories. Confined ions are bounced within electrostatic well reflecting from magnetic barriers shaped by convex field lines. The concept offers a means of enhancing ion reactivity provided by caustic formation that are resulted from an intersection of ion orbits within confinement volume. Single ion 3D trajectory tested numerically in idealized version of problem, indicates the formation of dense region referred as caustic. To develop further understanding, the problem should be modeled by considering the Fokker-Planck transport equation. Proposed experimental device parameters are estimated.

This work was partially supported by Japan Society for the Promotion of Science (JSPS).

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