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Analytic Modeling of the Feedback Stabilization of Resistive Wall Modes

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Abstract

Feedback suppression of resistive wall modes (RWM) is studied analytically using a model based on a standard cylindrical approximation. Optimal choice of the input signal for the feedback, effects related to the geometry of the feedback active coils, RWM suppression in a configuration with ITER-like double wall, are considered here. The widespread opinion that the feedback with poloidal sensors is better than that with radial sensors is discussed. It is shown that for an ideal feedback system the best input signal would be a combination of radial and poloidal perturbations measured inside the vessel.

Keywords:
resistive wall mode (RWM), cylindrical approximation, feedback suppression

1. Introduction

Resistive wall modes (RWM) can limit achievable beta below acceptable level in advanced tokamaks with low internal inductances [1-3]. Experiments in the DIII-D tokamak show two directions for stabilizing the RWM in high-beta tokamaks: toroidal plasma rotation and active feedback control of the mode using magnetic coils. At the presence of the uncorrected resonance error field, the rotation alone cannot completely suppress the instability, urging the need of the feedback suppression for achieving high β [4].

Simulations show that a feedback control of RWM can significantly raise the n = 1 ideal MHD beta limit, up to β_N = 5, doubling the no-wall limit [3]. Though theory was always optimistic, it took several years of studies in DIII-D [1,4] before it was reported [5] that, in experiments with active control, it became possible to sustain a discharge at pressures approaching twice the no-wall limit. This success in 2001 has been explained [5] as resulted owing largely to an extensive new set of magnetic sensors installed inside the vacuum vessel.

The problem of proper positioning and orientation of magnetic sensors was earlier studied numerically [2,3,5,6]. The conclusion, stated in ref. [2] and confirmed in refs. [3,5,6], was that the feedback system with sensors measuring the poloidal field is much better than that with the radial-field sensors. With solid proofs, including experimental confirmation [5], the conclusion cannot raise any doubt. But some basic questions remain unanswered. For example, it follows from div$\boldsymbol{B}$ = 0 that poloidal and radial components of the plasma-produced helical magnetic perturbation must be (approximately) equal. Why then 'sensors measuring the poloidal field perturbations are superior to radial sensors' [3]? Is it a general rule or a property of the considered feedback schemes? The problem is analyzed here analytically using a model based on a standard cylindrical approximation.

2. General Equations in Cylindrical Approximation

Feedback stabilization of RWM is often analyzed in cylindrical approximation [6-12]. Such

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analysis is in a good qualitative agreement with the observed feedback characteristics in DIII-D tokamak [6]. The use of cylindrical model is also justified by comparison of the model predictions with results of toroidal computations [9].

The perturbed magnetic field must be calculated with account of currents in all active and passive elements and with proper matching at the dividing surfaces: plasma boundary, walls, and active coils. In the model, the surfaces are coaxial cylinders.

In the vacuum gaps between the dividing surfaces, the perturbed magnetic field can be described by its radial component:

$$b_r = \sum b_k(r)\exp(ik\theta - in\zeta + \gamma t). \qquad (1)$$

Here r, θ and $z = R\zeta$ are the usual cylindrical coordinates ($2\pi R$ is the total length of the system), γ is the growth rate, t is the time.

At the resistive wall, considered as a thin shell $r = r_w$, two conditions must be satisfied:

$$[b_k] = 0, \quad [rb_k'] = \hat{\gamma} B_k. \qquad (2)$$

Here $[X] = X\big|_{r_w-0}^{r_w+0}$ is a jump in the function across the wall, the prime is the radial derivative, $\hat{\gamma} = \gamma\tau_w$, $\tau_w = \mu_0\sigma_w d_w r_w$ is the time constant of the wall, σ_w and d_w are, respectively, its conductivity and thickness, $B_k = b_k(r_w)$ is the amplitude of the k th harmonic of the perturbation at the wall. More details are given in ref. [12].

The contributions of inner and outer sources to b_r have different radial behavior:

$$b_k^{in} = B_k^{in} x^{-\kappa-1}, \quad b_k^{out} = B_k^{out} x^{\kappa-1}, \qquad (3)$$

where $x = r/r_w$, $\kappa = |k|$, and B_k^{in} and B_k^{out} are the values of b_k^{in} and $b_k^{out} = b_k - b_k^{in}$ at $r = r_w$, respectively. These expressions, with continuity at $r = r_w$, describe also the self-field of the wall in the inner ($r < r_w$) and outer regions. Thus, at the outer side of the wall

$$rb_k'\big|_{r_w+0} = -(\kappa+1)B_k + 2\kappa B_k^{ext}, \qquad (4)$$

where $B_k^{ext} = b_k^{out}\big|_{r_w+0}$ is the part of B_k due to the field produced by all sources outside the wall: by other walls at $r > r_w$, if any, and by the feedback system.

With eq. (4) and eq. (2) lead to

$$(\hat{\gamma} - \Gamma_k)B_k = 2\kappa B_k^{ext} \qquad (5)$$

with Γ_k defined as

$$\Gamma_k \equiv -(r b_k'/b_k)\big|_{r_{w-0}} - (\kappa+1). \qquad (6)$$

In the vacuum gap inside the shell, $r < r_w$,

$$\frac{r b_k'}{b_k} = -(\kappa+1) - \frac{2\kappa\Gamma_k x^{2\kappa}}{2\kappa + \Gamma_k(1 - x^{2\kappa})}. \qquad (7)$$

For plasma-facing wall, this gives Γ_k through rb_k'/b_k on the outer plasma surface, $x = a/r_w$. The radial magnetic field is a continuous function, the same is true for b_k' if surface currents are absent. Then at $x = a/r_w$ the left hand side of eq. (7) is expressed through b_r in the plasma.

In the plasma, b_r must be found from MHD equations. It is known that, in cylindrical geometry, they are reduced to the equation [13] (see also ref. [12] and references therein)

$$\frac{\gamma^2}{F}\nabla_\perp\left(\rho\nabla_\perp\frac{\tilde{\psi}}{F}\right) = -\nabla_\perp^2\tilde{\psi} + \mu_0\frac{mj'(r)}{rF}\tilde{\psi}, \qquad (8)$$

where $\tilde{\psi} = \psi(r)\exp(im\theta - in\zeta + \gamma t)$, the perturbed magnetic field is given by $\tilde{\boldsymbol{b}} = \nabla\tilde{\psi}\times\hat{z}$, $\hat{z}$ is the unit vector along the z-axis of the cylindrical coordinate system r, θ, $z \equiv R\zeta$, $F = (B_J/r)[m - nq(r)]$, $B_J(r)$ is the equilibrium poloidal field, q is the safety factor, ρ is the plasma density, j is the z-component of the current density, $\nabla_\perp = \nabla - \hat{z}(\hat{z}\cdot\nabla)$.

This is the simplest equation for the magnetic perturbation in the plasma. But even the utmost simplification does not help to obtain a general expression for b_r: analytical solutions of eq. (8) are known for uniform or parabolic [14] current profiles only. Therefore, instead of eq. (7), another way of prescribing Γ_k is needed in a general case.

Equation (8) implies that parameters Γ_k are determined by the current density. If its profile is given, Γ_k can be prescribed as some constants characterizing this particular profile. These constants can be evaluated from experimental data or numerical results. We need to know what happens with Γ_k when the feedback is switched on. It seems natural to assume that the feedback field does not change the current density. If so, the solution of eq. (8) and the shape of $b_r(r)$ in the plasma would remain unaffected by the feedback, and Γ_k can be considered as fixed constants independent of the feedback field.

This conclusion is justified by results of toroidal calculations [2,3] showing that the feedback does not strongly modify the perturbation in the plasma. This was confirmed in DIII-D experiments and related numerical analysis, and the observed invariance of the mode structure during the feedback process was called mode rigidity [6].

The system eq. (5) with constant Γ_k is a starting point for analysis. It must be supplemented by an algorithm prescribing the feedback-produced magnetic field as a function of some input signal combined of the magnetic signals measured outside the plasma.

3. One Resistive Qall, Ideal Feedback

If only one resistive wall is present, the external field in eq. (5) is the field produced by the feedback system, $B_k^{\rm ext} = B_k^{\rm f}$. For a single unstable mode $k = m$ eq. (5) gives

$$\hat{\gamma} = \Gamma_m + 2\mu B_m^{\rm f} / B_m , \tag{9}$$

where $\mu = |m|$, $\Gamma_m > 0$ is the no-feedback growth rate, and B_m is the amplitude of the mode.

RWM is suppressed when $\hat{\gamma} < 0$. Assuming a simple proportional control

$$B_m^{\rm f} = -K \times I , \tag{10}$$

we compare the feedback stabilization with input signals I from the radial probes in the mid-plane (I_r), and from poloidal probes measuring $-b_\theta(r,0)$ inside and outside the vessel ($I_\theta^{\rm in}$ and $I_\theta^{\rm out}$).

An ideal feedback system would create only necessary (m,n) harmonic. In this case

$$\begin{aligned} I_r &= B_m , \\ I_\theta^{\rm in} &= (1 + \Gamma_m/\mu)\, B_m , \\ I_\theta^{\rm out} &= I_\theta^{\rm in} - \hat{\gamma}\, B_m/\mu . \end{aligned} \tag{11}$$

The values $I_\theta^{\rm in}$ and $I_\theta^{\rm out}$ are expressed through $B_{\rm m}$ using eqs. (1)–(3) and div$\boldsymbol{B} = 0$.

It follows from eq. (9) that any choice from eq. (11) would allow RWM suppression with algorithm eq. (10). For example, for $I = I_r$ the stability criterion is

$$K > K_0 \equiv \Gamma_m / (2\mu) . \tag{12}$$

The largest of the three signals in eq. (11) is $I_\theta^{\rm in}$. Larger signal can be detected easier, and feedback stabilization can start earlier. This is an advantage of the feedback with poloidal sensors inside the vessel. But it hardly can explain the observed essential superiority [2,3,5,6] of this system over that with radial sensors, since I_r differs from $I_\theta^{\rm in}$ by a constant multiplier only. This difference results in larger gain K for a system with radial sensors, in other respects both feedback systems are equivalent.

This is true for the ideal case. But relations eq. (11) may not be valid for a real feedback.

4. One Wall, Conventional Feedback, Radial Sensors

In the ideal case, the feedback system must produce the same helical harmonic as the plasma-generated perturbation, and nothing more. This would require stellarator-like helical windings. In practice, the correction coils consisting of frame-like rectangular segments [1-9] are always considered for tokamaks. Such coils, called below conventional, allow the desired RWM suppression in theory and in DIII-D experiments, but they are not optimal: RWM has a helical structure while the rectangular active coils are aligned toroidally. Non-optimal active coils generate side-band harmonics, different from the intrinsically unstable principle harmonic. This harmonics can contribute to the measured input signals I, affecting the dispersion relation eq. (9) through eq. (10) or another feedback algorithm.

The magnetic field $\boldsymbol{b}^{\rm f} = \nabla\psi^{\rm f} \times \hat{z}$ produced by a conventional feedback system with correction coils similar to those used in DIII-D has a property $\psi^{\rm f}(\theta) = -\psi^{\rm f}(-\theta)$. Accordingly,

$$b_r^{\rm f}(\theta) = b_r^{\rm f}(-\theta) , \tag{13}$$

$$b_\theta^{\rm f}(\theta) = b_\theta^{\rm f}(-\theta) . \tag{14}$$

It is known that even in the case of DIII-D (six active coils each covering a 60-degree toroidal arc) the feedback field can be modeled by a single harmonic in toroidal angle EMBED [1]. Thus, the toroidal discreteness can be disregarded in the analysis.

A real feedback system in addition to necessary (m,n) harmonic generates a number of side-band harmonics. Conventional feedback has a property eq. (13), so that $b_r^{\rm f}$ spectrum is symmetric, $b_k^{\rm f} = b_{-k}^{\rm f}$. Therefore, even if a system is somehow optimized for stabilizing (m,n) mode, at least $(-m,n)$ side-band harmonic in $b_r^{\rm f}$ must be taken into account. With two harmonics of the field from active coils, eq. (9) must be considered together with

$$\hat{\gamma} = \Gamma_{-m} + 2\mu B_{-m}^{\rm f} / B_{-m} . \tag{15}$$

Instability of a single mode means that without feedback $\Gamma_m > 0$, but $\Gamma_{-m} < 0$.

If the feedback field is described by two harmonics, (m,n) and $(-m,n)$, and $B_{-m}^{\rm f} = B_m^{\rm f}$,

$$I_r = B_m + B_{-m} = 2B_m \frac{\hat{\gamma} - \hat{\gamma}_{cr}}{\hat{\gamma} - \Gamma_{-m}} , \tag{16}$$

where

$$\hat{\gamma}_{cr} = 0.5(\Gamma_{-m} + \Gamma_m) . \tag{17}$$

With eq. (16) and eq.(10), eq. (9) turns into quadratic equation for $\hat{\gamma}$:

$$\hat{\gamma}^2 + 2\hat{\gamma}(2\mu K - \hat{\gamma}_{cr})\Gamma_m \Gamma_{-m} - 4\mu K \hat{\gamma}_{cr} = 0 . \qquad (18)$$

For stability, both roots of (18) must be negative. This requires $\hat{\gamma}_{cr} < 0$ and

$$K > K_0(1 - 0.5\Gamma_m / \hat{\gamma}_{cr}) . \qquad (19)$$

The necessary gain becomes infinitely large when $\hat{\gamma}_{cr} \to 0$ because the measured input signal I_r vanishes at $\hat{\gamma} = \hat{\gamma}_{cr}$. When $\hat{\gamma}_{cr}$ becomes positive, the unstable (m,n) mode cannot be stabilized by the conventional feedback described by (10) with $I = I_r$.

5. One Wall, Conventional Feedback, Internal Poloidal Sensors

If the feedback field is described by two harmonics with $B^{f}_{-m} = B^{f}_{m}$,

$$I_\theta^{\text{in}} = B_m \left(1 + \frac{\hat{\gamma}}{\mu}\right) \frac{\Gamma_m - \Gamma_{-m}}{\hat{\gamma} - \Gamma_{-m}} . \qquad (20)$$

With this input signal and proportional control (10), eq. (9) turns into

$$(\hat{\gamma} - \Gamma_m)(\hat{\gamma} - \Gamma_{-m}) = -2K(\hat{\gamma} + \mu)(\Gamma_m - \Gamma_{-m}) . \qquad (21)$$

The growth rate $\hat{\gamma}$ becomes negative when $K > \max\{K_1, K_2\}$, where

$$K_1 = \frac{K_0}{1 - \Gamma_m / \Gamma_{-m}} , \qquad K_2 = \frac{\hat{\gamma}_{cr}}{\Gamma_m - \Gamma_{-m}} , \qquad (22)$$

K_0 is given by eq. (12) and $\hat{\gamma}_{cr}$ by eq. (17).

This scheme is much better than the feedback with radial sensors since it allows suppression of RWM without restrictions on plasma parameters, and smaller gain is required.

6. Two Walls, Ideal Feedback, Radial Sensors

Predictions are quite encouraging for tokamaks with a single resistive wall. However, the ITER is designed with two separate resistive walls (p. 2593 [15]). So, it is natural to analyze the difference between the one-wall and double-wall cases.

The second wall increases the total resistive decay time decreasing thereby the RWM growth rate. Also, the second wall acts as a screen affecting penetration of the field from the active coils to the first wall. This can weaken the stabilizing influence of the feedback system.

If there are several resistive walls, eqs. (5) and (6) must be applied successively to each wall. For the outermost wall, B^{ext} is the field produced by the feedback, while for the walls inside, B^{ext} also includes a field produced by all walls screening a given wall from the active coils. The radial magnetic field is a continuous function described by eq. (3) between the walls.

For a plasma column surrounded by two resistive walls of radii r_1 and r_2 the dispersion relation is [12]

$$\hat{\gamma}^2 - \hat{\gamma}(\Gamma_m - w\Delta\Gamma) - 2\mu\Delta\Gamma(w-1)(K_0 + B_f / B_\Sigma) = 0 . \qquad (23)$$

Here $B_\Sigma = b_m(r_1)$ is the perturbation amplitude at the first wall, B_f is the part of B_Σ created by the feedback system, K_0 is defined by eq. (12),

$$w \equiv 1 + x_2^{2\mu}\tau_1/\tau_2 , \quad \Delta\Gamma \equiv 2\mu/(x_2^{2\mu} - 1) , \qquad (24)$$

and $x_2 \equiv r_2/r_1$. One-wall case eq. (9) corresponds to the limit $w \to \infty$ in (23).

It follows from eq. (23) that, with radial sensors and proportional control eq. (10), the condition of the RWM suppression is again eq. (12), but now it is valid for modes only with

$$\Gamma_m - w\Delta\Gamma < 0 . \qquad (25)$$

In the opposite case, $\Gamma_m - w\Delta\Gamma > 0$, suppression of RWM is impossible. If this is true for the ideal feedback, it must be even more true for the conventional feedback with radial sensors.

7. Summary

The analysis gives a natural explanation of the observation that, for the RWM feedback control, 'sensors measuring the poloidal field perturbations are superior to radial sensors' [3].

The poloidal component of the perturbation inside the vessel is always larger than the radial one, see eq. (11). Therefore, internal poloidal sensors are better for the RWM feedback control since a measured (input) signal must be above some detection level. From this viewpoint, the best input signal would be a combination $I_\theta^{\text{in}} + I_r$.

An ideal feedback system could be effective in suppressing RWM with either I_r or I_θ^{in}, just different gains K would be needed. But realistic feedback system generates side-band harmonics that influence the measurements. As a result, the conventional feedback system with radial sensors cannot suppress RWM when $\hat{\gamma}_{cr} > 0$. This value eq. (17) is determined by plasma parameters and can vary during the discharge evolution. It can lead to increase in the necessary gain eq. (19)

above the permissible level. That may be a reason of the loss of the control observed in DIII-D experiments with radial sensors [1]. The weak point of such a feedback is aggravated, even for the ideal feedback, when a second resistive wall is present.

However, with internal poloidal probes the conventional feedback allows suppression of RWM without fail and at rather modest gains, see eq. (22). The difference between two cases, related to properties eqs. (13) and (14), may be a reason of the dramatic improvements in active control in DIII-D experiments in 2001 [5].

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