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# Self-Sustained Turbulence of Current-Diffusive Ballooning Mode and Drift Wave Instabilities

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**Abstract**

Starting from two-fluid equations, we derive a set of equations which describes the. current diffusive ballooning mode (CDBM) and the ion temperature gradient (ITG) mode. We numerically solve the equations toobtain real frequency, growth-rate and spatial structure of the eigen mode. The magnitude of transport coefficients in a saturated state are evaluated from the marginal stability condition. We report model dependence of ITG and a contribution of compressibility on CDBM.

**Keywords:**

turbulent transport, ITG, ballooning mode, drift wave

## 1. Introduction

It is important to understand the mechanism of the turbulent transport driven by micro-instabilities to improve confinement of tokamak plasma. Among various turbulent transport modes, we consider a model driven by two kinds of micro-instabilities to explain the particle and the energy transport across the magnetic field. One is the self-sustained turbulence model [1] of current diffusive ballooning mode (CDBM) [2], which has successfully described the L mode transport and improved confinement associated with the formation of the internal transport barriers [3]. The other one is the ion and electron temperature gradient modes (ITG and ETG) on which fluid [4] and kinetic [5] analyses are in progress.

We analyzed the kinetic effect on CDBM discussed in the previous conference [6], but the role of finite drift frequency was not clearly presented. In this paper, we start from full two-fluid equations to analyze both the electromagnetic drift mode, CDBM, and the electrostatic drift mode, ITG, including the turbulence coefficients based on the self-sustained turbulent theory [1] and evaluate the transport coefficients from a saturated fluctuation amplitude. Additionally, we concentrate on the influence of the diamagnetic drift on the CDBM and ignore the kinetic effect due to the finite Larmor radius ($k_\perp \rho_i \ll 1$).

## 2. Basic Equations

### 2.1 Full two-fluid equations

We start from collisionless two-fluid equations, i.e. the equations of continuity, motion and state, and Poisson's and Ampere's eqs. (4), (7)–(10).

$$\frac{\partial n_{1j}}{\partial t} + \nabla_\perp \cdot \left( n_{0j} \boldsymbol{v}_{1j\perp} + n_{1j} \boldsymbol{v}_{0j\perp} \right) + \nabla_\parallel \left( n_{1j} v_{0j\parallel} + n_{0j} v_{1j\parallel} \right) = 0 \tag{1}$$

$$\boldsymbol{v}_{1j\perp} = \frac{\boldsymbol{E}_1 \times \boldsymbol{b}}{B_0} - \frac{\nabla p_{1j} \times \boldsymbol{b}}{q_j n_{0j} B_0} + \frac{n_{1j} \left( \nabla p_{0j} \times \boldsymbol{b} \right)}{q_j n_{0j}^2 B_0} - \frac{1}{\Omega_j} \frac{\partial v_{1j} \times \boldsymbol{b}}{\partial t} \tag{2}$$

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$$m_j n_{0j}\frac{\partial v_{1j\parallel}}{\partial t}+\nabla_\parallel p_{1j}-q_j n_{0j}E_{1\parallel}=0$$

$$\frac{\partial p_{1j}}{\partial t}+v_{0j}\cdot\nabla p_{1j}+v_{1j}\cdot\nabla p_{0j} \tag{3}$$

$$+\gamma_j\left(p_{0j}\nabla_\parallel v_{1j\parallel}+p_{1j}\nabla_\parallel v_{0j\parallel}\right)=0 \tag{4}$$

$$-\nabla^2\phi=\frac{en_{1i}-en_{1e}}{\varepsilon_0} \tag{5}$$

$$\nabla^2\boldsymbol{A}=-\mu_0\boldsymbol{j}_1 \tag{6}$$

where $\boldsymbol{b}=\boldsymbol{B}/B_0$, the unit vector of magnetic field, $\boldsymbol{E}=-\nabla\phi-\partial\boldsymbol{A}/\partial t$, the electric field, $\phi$, the electrostatic potential, $\boldsymbol{A}$, the vector potential, $\boldsymbol{j}_1=\Sigma_j(q_jn_{0j}\boldsymbol{v}_{1j}+q_jn_{1j}\boldsymbol{v}_{0j})$, the fluctuation current, $\nabla_\perp=-\boldsymbol{b}\times(\boldsymbol{b}\times\nabla)$. The subscript $\perp$ denotes a perpendicular component of quantities ($\boldsymbol{a}_\perp=\boldsymbol{b}\times(\boldsymbol{a}\times\boldsymbol{b})$), the suffix 0 the unperturbed quantities, the suffix 1 the fluctuation and the suffix $j$ the species of fluids ($j$ = i, e).

### 2.2 Reduced set of equations

The main purpose of starting from the full set of fluid equation is to examine the effects of the various approximations, especially quasi-neutrality and incompressibility. Since handiness and accuracy of description are trade-off, we reduce the two-fluid equations in the following. Subtracting the continuity equation of electron from the continuity equation of ion and substituting Poisson's equation for density fluctuations and equation of perpendicular motion, we obtain the vorticity equation,

$$\frac{\partial}{\partial t}\left[\left(\frac{n_{0j}}{\Omega_i B_0}-\frac{n_{0e}}{\Omega_e B_0}\right)\nabla_\perp^2\phi+\frac{\nabla_\perp^2 p_{i1}}{eB_0\Omega_i}+\frac{\nabla_\perp^2 p_{e1}}{eB_0\Omega_e}\right]$$
$$-\frac{\varepsilon_0}{e}\nabla^2\phi-\frac{1}{e}\nabla_\parallel j_\parallel$$
$$=\frac{1}{eB_0}\left(\boldsymbol{b}\times\boldsymbol{\kappa}+\boldsymbol{b}\times\frac{\nabla B_0}{B_0}\right)\cdot\left(\nabla p_{1i}+\nabla p_{1e}\right) \tag{7}$$

where $\Omega_j=q_jB_0/m_j$ is the cyclotron frequency and $\kappa$ is the magnetic curvature. If we assume incompressibility ($\partial p_{1j}/\partial t=\mathrm{i}q_jn_{0j}(1+\eta_j)\omega_{*j}\phi$, where $\mathrm{i}\omega_{*j}=(T_{0j}/q_jn_{0j}B_0)(\nabla rn_{0j})\nabla_\theta$, the density drift frequency and $\eta_j=\mathrm{d}(\ln T_{0j})/\mathrm{d}(\ln n_{0j})$, the temperature scalelength) and quasi-neutrality and ignore the electron inertia term ($|\Omega_i|\ll|\Omega_e|$), the familiar vorticity eqs. (6), (11)–(13) can be obtained. Note that the right hand side of eq. (7) vanishes if the magnetic curvature is absent (i.e. a sheared slab plasma). Remaining equations are the equation of parallel motion, the equation of state and Ampere's equation:

$$m_jn_{0j}\frac{\partial v_{1j\parallel}}{\partial t}=-\nabla_\parallel p_{1j}+q_jn_{0j}\left(\nabla_\parallel\phi+\frac{\partial A_\parallel}{\partial t}\right) \tag{8}$$

$$\frac{\partial p_{1j}}{\partial t}=\mathrm{i}q_jn_{0j}\omega_{*j}\left(1+\eta_j\right)\phi-\gamma_jp_{0j}\nabla_\parallel v_{1j\parallel} \tag{9}$$

$$\nabla^2A_\parallel=-\mu_0en_0\left(v_{1\parallel i}-v_{1\parallel e}\right) \tag{10}$$

where $n_0=n_{0i}=n_{0e}$ is assumed and $v_{0\parallel}$ and $E_0$ are ignored, for simplicity. Now we consider the drift wave approximation, where all fluctuations are expressed as an eikonal type, $f_1(\boldsymbol{r})\simeq f_1(x)\exp(\mathrm{i}(k_yy+k_\parallel z))$, and unperturbed quantities are constant in the direction to $y$ or $\theta$ and $z$. Using the first order of the equation of motion, $v_{0j\perp}=-\nabla p_{0j}\times\boldsymbol{b}/(q_jn_{0j}B_0)$, we can reduce the advection term in eq.(4), $\boldsymbol{v}_{0j\perp}\cdot\nabla p_{1j}+\boldsymbol{v}_{1j\perp}\cdot\nabla p_{0j}$, to $\mathrm{i}q_jn_{0j}\omega_{*j}(1+\eta_j)\phi$ by ignoring the time derivative.

### 2.3 Turbulent transport coefficients

Based on the self-sustained turbulence theory [1], we can evaluate the turbulent transport coefficients due to $\boldsymbol{E}\times\boldsymbol{B}$ drift,

$$\frac{\boldsymbol{b}}{B_0}\cdot\left(\nabla_\perp\phi\times\nabla X_{1j}\right)\to-\chi_j\nabla_\perp^2X_{1j}\,.$$

Non-liner transport rate $\gamma_N$ (i.e. life time of fluctuation) is defined by $\gamma_N=-\mathrm{i}\omega+\chi_jk_\perp^2$ where $\omega$ is the mode frequency. Substituting the above equation to the transport coefficients, $\chi_j=k_\perp^2\langle\phi^2\rangle/(\gamma_NB_0^2)$, in the case of finite real frequency, the transport coefficients are evaluated as a function of the fluctuation amplitude $\langle\phi^2\rangle$

$$\chi_e=\sqrt{\frac{\langle\phi^2\rangle}{B_0^2}-\frac{1}{4}\left(\frac{\omega}{k_\perp^2}\right)^2}+\frac{\mathrm{i}}{2}\frac{\omega}{k_\perp^2}\,. \tag{11}$$

In the following, we neglect the kinetic effect due to the finite Larmor radius for simplicity and assume $\chi_e\simeq\chi_i\simeq\mu_{i\parallel,\perp}\simeq\mu_e$.

### 2.4 Ballooning expression

Using the ballooning transformation [14], we obtain from eqs. (7)–(9),

$$\frac{n_{0i}}{\Omega_i}\left(-\mathrm{i}\omega-\mu_{i\perp}\nabla_\perp^2\right)\nabla_\perp^2\phi+\frac{\left(-\mathrm{i}\omega-\chi_i\nabla_\perp^2\right)}{\Omega_i}\nabla_\perp^2p_{1i}$$
$$+\frac{n_0B_\theta}{r}\frac{\partial}{\partial\xi}\left(v_{1i\parallel}-v_{1e\parallel}\right)=-\frac{imB_\varphi}{erR_0B_0}H(\xi)\,p_{1s} \tag{12}$$

$$m_jn_{0j}\left(-\mathrm{i}\omega-\mu_{j\parallel}\nabla_\perp^2\right)v_{1j\parallel}$$
$$=-\frac{B_\theta}{rB_0}\frac{\partial p_{1j}}{\partial\xi}-q_jn_0\left(\frac{B_\theta}{rB_0}\frac{\partial\phi}{\partial\xi}-\mathrm{i}\omega A_\parallel\right) \tag{13}$$

$$\left(-\mathrm{i}\omega-\chi_j\nabla_\perp^2\right)p_{1j}+\mathrm{i}q_jn_0\,\omega_{*j}\left(1+\eta_j\right)\phi$$
$$=-\frac{\gamma_j p_{0j}B_\theta}{rB_0}\frac{\partial v_{1j\parallel}}{\partial\xi}, \qquad (14)$$

where $m$ is the poloidal mode number, $p_{1s}=p_{1i}+p_{1e}$, $B_\theta$ the poloidal magnetic field, $B_\varphi$ the toroidal magnetic field, $r$ the minor radius, $R_0$ the major radius and $\xi$ the ballooning coordinate. Taking account of the Shafranov shift of the magnetic surface, $\nabla_\perp^2 \to -m^2f^2/r^2$, we take $f^2=1+(s\xi-\alpha\sin\xi)^2$ and $H(\xi)=\kappa+\cos\xi+(s\xi-\alpha\sin\xi)\sin\xi$, where $s=(r/q)\partial q/\partial r$ the magnetic shear, $\alpha=-q^2R\partial\beta/\partial r$ the normalized pressure gradient and $\kappa=-(r/R)(1-q^{-2})$ the average magnetic curvature.

With the marginal stability condition, $\mathrm{Im}(\omega)=\gamma=0$, we numerically solve the ballooning mode eqs. (10), (12)–(14) to find the lowest eigen mode and evaluate the average amplitude of fluctuation $\langle\phi^2\rangle$. Then we seek the maximum $\chi_e$ which isneeded to stabilize the fluctuation for all $m$.

## 3. Numerical Results

### 3.1 ITG mode in a sheared slab plasma

Figure 1 shows $k_\perp\rho_i$ dependence of the eigen frequency for several models of the slab ITG mode. A solid line illustrates the result of the one-fluid model derived by Lee and Diamond [4] (3 equations). A dashed line indicates the result of the electromagnetic full two-fluid model described by eqs. (2)–(6) (14 equations). A doted line shows those of reduced model obtained from eqs. (7)–(10) without curvature (6 equations). These modes have a phase velocity directed to the ion drift motion and the real frequency decreases with the increase of $k_\perp\rho_i$. The growth rate in the one-fluid model once increases and then decreases with the increase of $k_\perp\rho_i$, while that of the two-fluid models are monotonically decreasing function.

From this figure, we find that the differences of growth rate among several models are sufficiently small for qualitative analysis and the electromagnetic models can describe the ITG mode in a sheard slab plasma. In the 6 eqs. model, even if we neglect the electron inertia termand assume the quasi-neutrality in eq. (7), the result changes little. The difference between the reduced 6 equations model and the full 14 equations model is quite small. Therefore, we may adopt the reduced two-fluid model to consider the coupling between the drift instabilities and the ballooning mode.

### 3.2 CDBM with compressiblity

Next we consider the CDBM in the toroidal geometry including the effect of diamagnetic drift using eqs. (10), (13)–(15). Figure 2 shows the $s-\alpha$ dependence of the turbulent transport coefficient divided by the $\alpha^{3/2}$ for the several magnetic shear parameters in

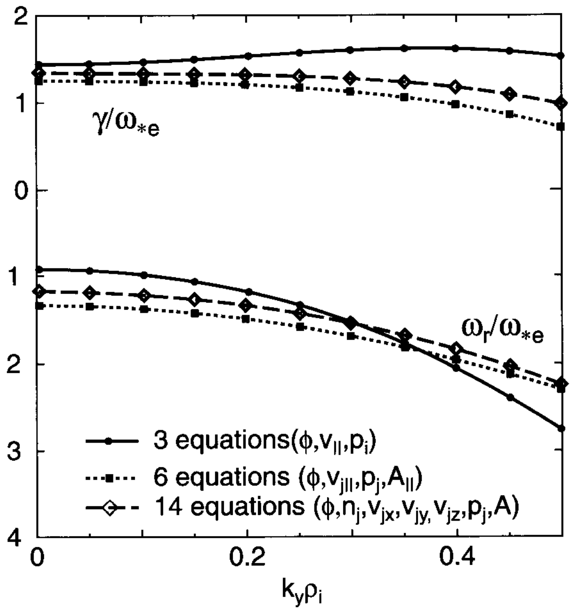


Fig.1 $k_\perp\rho_i$ dependence of the mode frequency of the lowest order calculated by several models, where $n_{0i}=n_{0e}=2\times10^{19}$ m$^{-3}$, $T_{0i}=12.8$ keV, $T_{0e}=3.9$ keV, $B_0=4.7$ T, $L_n=0.38$ m, $\eta_i=5$, $\eta_e=0$ and $L_s=2.75$ m.

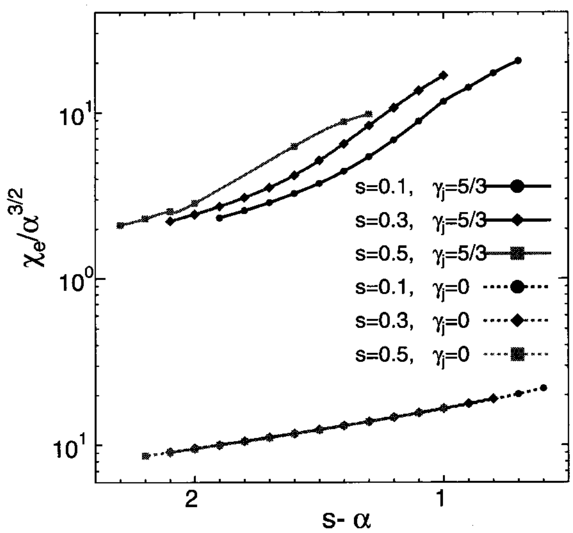


Fig. 2 $s-\alpha$ dependence of $\chi_e/\alpha^{3/2}$ to various values of the magnetic shear parameter ($s=-0.5\sim-0.1$), where $B_0=1$ T, $n_0=1\times10^{19}$ m$^{-3}$, $q=8$, $R_0=3$ m, $T_i=1$ keV, $T_e=1$ keV, $L_n=3$ m and $r=0.17$ m.

the case without compressibility ($\gamma_j$ = 0), doted lines, and with compressibility ($\gamma_j$ = 5/3), solidline. Incompressible result corresponds to the previous analysis [3,6] and are expressed by the relation, $\chi = F(s, \alpha)\alpha^{3/2}$ [3]. With compressibility, the turbulent transport coefficients are about one order higher than those without compressibility. The net value of $\chi_e$ reaches 5 ~ 10[m$^2$/s] and is close to the experimental results. The increase of $\chi_e$ comes from that the role of the electron thermal diffusivity changes from suppression of the fluctuation to excitation. In the case of incompressibility ($\gamma_j$ = 0), parallel fluctuation current is determined by the extended Ohm's law and the ballooning mode are excited by the current diffusivity associated with the electron viscosity. When we include the compressible term $\gamma_j \nabla_{\|} v_{\| j}$, the parallel fluctuation current are modified by the equation of state and the contribution of the electron thermal diffusivity reduces the stabilizing term and excites the fluctuation.

## 4. Summary and Discussion

To describe both the electromagnetic (CDBM) and the electrostatic (ITG) drift waves, we obtain a reduced set of equation starting from the two-fluid equations. We confirm that the full and reduced two-fluid models correctly describe the ITG mode in asheared slab plasma. Taking account of the tokamak configuration and magnetic curvature, we derive the current diffusive ballooning equation with compressibility. From preliminary numerical results, the compressibility enhances $\chi_e$ by an order of magnitude and may have a very important role for the non-linear interaction.

In this paper, we ignore the kinetic effect due to the finite Larmorradius. When we consider the reduction of the electric field felt by the ion, we need to solve the Poisson's equation. For general analysis, it is necessary to start from the gyro-kinetic equations including the turbulence effect. Additionally, the coupling between toroidal ITG mode and the CDBM is an interesting problem in progress.

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