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Eigenmode Analysis of Low Frequency Waves in a Field-Reversed Configuration (FRC)

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Abstract

Global eigenmodes of low frequency waves in a FRC plasma are obtained using MHD model and a simple equilibrium model (1D). Dispersion relation and radial structure of the global wave fields for the azimuthal mode numbers $m = 0,1$ are shown. The results for $m = 0$ are compared with the results of a low frequency wave heating experiment. The possibility of ion heating by the *transit-time magnetic pumping* is discussed.

Keywords:
low frequency waves, eigenmode, MHD, transit-time magnetic pumping, field-reversed configuration

1. Introduction

Low frequency waves have been used for plasma heating. Recently a heating experiment of a Field-Reversed Configuration (FRC) plasma has been performed in Osaka University [1]. In this experiment, low frequency (compared with the ion cyclotron frequency in the external magnetic field) oscillating magnetic field was applied to the FRC plasma. The applied field was homogeneous in the azimuthal direction. As a result, a fluctuation of the magnetic field was observed to propagate in the direction parallel to the equilibrium magnetic field of the FRC plasma. In addition, increase of the plasma energy was observed and comparison of the total temperature and the ion temperature suggests that the increase in the plasma energy was mostly due to the increase in the ion temperature. This implies that the applied oscillating magnetic field could excite low frequency waves and the wave energy was absorbed by the ions. In this study eigenmodes of low frequency waves in a FRC plasma is analyzed for the azimuthal mode number $m = 0$ and 1. The results for $m = 0$ are compared with the experiments.

2. Eigenmode Analysis

To investigate the low frequency waves propagating through a FRC plasma, the single-fluid MHD equations are used:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\mathbf{v} = \mathbf{j} \times \mathbf{B} - \nabla p \tag{2}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{1}{en}(\mathbf{j} \times \mathbf{B}) \tag{3}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \tag{5}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{6}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) p \rho^{-\gamma} = 0 . \tag{7}$$

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Linearizing these equations and assuming that the FRC plasma has no equilibrium flow, we have 6 independent equations for 6 perturbed quantities v_1 and E_1. Since we seek waves propagating in the θ and z-directions with a global radial extension between the geometric axis and the conducting wall, the perturbed quantities are assumed to have the following form in the cylindrical coordinate system:

$$f_1(r, \theta, z, t) = \tilde{f}_1(r) \exp[i(m\theta + kz - \omega t)] \quad (8)$$

This leads to an eigenvalue problem where the wave number k is the eigenvalue and the global wave fields $\tilde{v}_{1r}(r)$, $\tilde{v}_{1\theta}(r)$, $\tilde{v}_{1z}(r)$, $\tilde{E}_{1r}(r)$, $\tilde{E}_{1\theta}(r)$, $\tilde{E}_{1z}(r)$ are the eigenfunctions for a given frequency ω and a boundary condition at $r = 0$ (geometric axis) and $r = r_w$ (perfectly conducting wall). The problem is solved in the following way. Each eigenfunction is approximated in terms of a finite series of basis functions $\phi_n(r)$ and a function which satisfies the boundary condition. For example, $i\tilde{v}_{1r}$ is expressed as

$$i\tilde{v}_{1r}(r) = F_{vr}(r) \sum_{n=1}^{N} C_n^{(vr)} \phi_n(r) \quad (9)$$

$$\phi_n(r) = \cos\left[(n-1)\pi r / r_w\right] \quad (10)$$

where $F_{vr}(r)$ is the function satisfying the boundary condition for $i\tilde{v}_{1r}$, and $C_n^{(vr)}$ are the expansion coefficients. In deriving the equations for the expansion coefficients, we use the *Galerkin method.*

One-dimensional FRC equilibrium model known as the rigid-rotor profile [2], which is homogeneous in θ and z-direction, is used in this study. The equilibrium magnetic field and pressure are expressed as

$$B_0(r) = B_w \tanh\left[K(2r^2/r_s^2 - 1)\right] \quad (11)$$

$$p_0(r) = p_{null} \operatorname{sech}^2\left[K(2r^2/r_s^2 - 1)\right] \quad (12)$$

where r_s and B_w are the separatrix radius and the magnetic field at the wall, and K is a parameter. p_{null} is the pressure at the magnetic field-null point. Figure 1 shows the equilibrium profile for $r_s/r_w = 0.4$, $K = 0.52$.

3. Results

3.1 Results for $m = 0$

The eigenvalue problem was solved for the azimuthal mode number $m = 0$ in the FRC equilibrium shown in Fig. 1. Figure 2 shows the dispersion relation for the low frequency waves propagating in z-direction. The frequency is normalized to $\omega_{ci0} \equiv eB_w/m_i$, which is the ion cyclotron frequency in the external magnetic field and the wave numbeáû s normalized to $1/r_w$. The eigenvalues k are plotted for ω/ω_{ci0} = 0.015–0.3 in increments of 0.015. The broken line corresponds to 80 [kHz] which is the frequency of the applied field in the experiment. The solid line shows the Alfvén velocity v_{A0} based on B_w and ρ_{null} which is the mass density at the field-null point. We see from Fig. 2 that the eigenmodes are dense for $\omega/k \geq v_{A0}$ and sparse for $\omega/k \leq v_{A0}$.

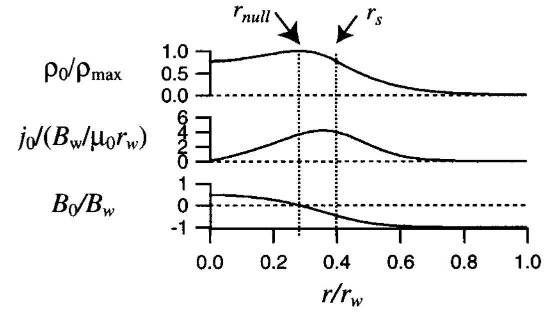

Fig. 1 Radial profile of the One-dimensional FRC equilibrium for $r_s/r_w = 0.4$, $K = 0.52$.

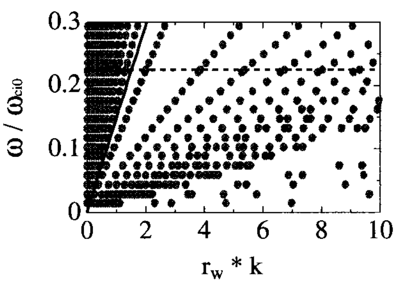

Fig. 2 Dispersion relation for $m = 0$ low frequency waves.

Figures 3 and 4 show the radial structure of the perturbed electric field, mass flow, and magnetic field of the eigenmodes along the broken line in Fig. 2 (80[kHz]). The solid gray, solid black and broken black lines show the r, θ, and z components, respectively. For a wave with the phase velocity much larger than v_{A0} the amplitude appears only outside the separatrix as shown in Fig. 3a. Namely this wave propagates in z-direction only outside the separatrix. As the phase velocity decreases, the amplitudes move to the inner region (Fig. 3b), and finally the amplitude appears inside the separatrix for $(\omega/k)/v_{A0} = 1.4$ (Fig. 3c). In all these modes (Figs. 3a-c), the magnetic field is largest in the θ-direction for $r \geq r_{null}$ as observed in the experiment.

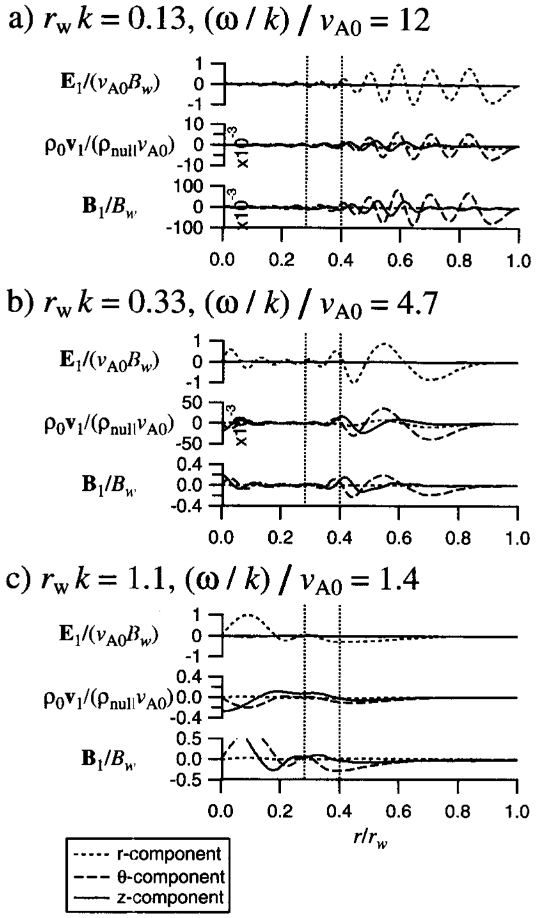

Fig. 3 Radial structure of the eigenmodes for $m = 0$.

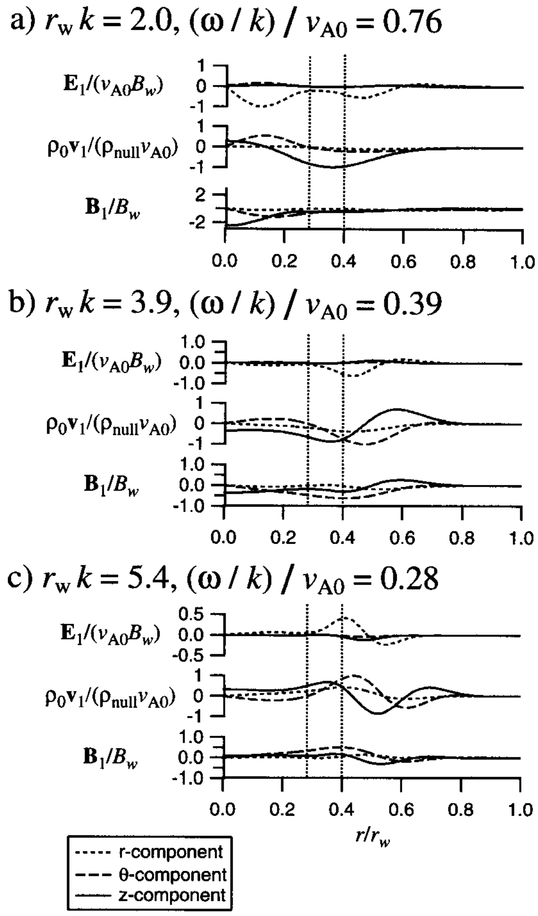

Fig. 4 Radial structure of the eigenmodes for $m = 0$.

These modes may produce the waves observed in the experiment. While for waves with much smaller phase velocities, this tendency is not presented. For $(\omega/k)/v_{A0}$ = 1.76 (Fig. 4a), the amplitudes appear between inside and outside the separatrix for the electric field and mass flow, but *not* for the magnetic field. The magnetic field propagates only inside the magnetic field-null point $r = r_{null}$ and is largest in the z-direction. For more smaller phase velocities, the amplitudes of the magnetic field appear both inside and outside the field-null point, and the number of nodes in the mass flow increases as the phase velocity decreased (Figs. 4b and 4c).

3.2 Results for m = 1

Figure 5 shows the dispersion relation for $m = 1$. Unlike the $m = 0$ case, there is no eigenmode for $0.4v_{A0} < \omega/k < 9v_{A0}$. Figure 6 shows the radial structure of the perturbed fields of the eigenmodes along the broken line in Fig. 5 (95 [kHz]). This frequency is chosen because the wave heating experiment for finite m mode has just been done recently for the frequency. For $(\omega/k)/v_{A0} = 12$ the wave propagates almost $0.7 \le r/r_w \le 1$ where the plasma density is very small (Fig. 6a). For $(\omega/k)/v_{A0}$ = 0.37 the mode shown in Fig. 6b appears. The number of nodes in the mass flow increases as the phase velocity decreases as observed in the modes with $m = 0$ for $(\omega/k)/v_{A0} \le 0.39$.

4. Discussions

In the experiment [1], the magnetic fluctuation measurement was performed only around the separatrix. The fluctuation in the azimuthal direction, $B_{1\theta}$, was larger than the other components. The phase velocity of the observed waves is $\omega/k \sim 1\text{-}2 \times 10^5$ [m/s] and increases with r.

For the eigenmodes shown in Figs. 3a-c, the magnetic field is largest in the azimuthal direction. The mode which propagates near the separatrix has smaller

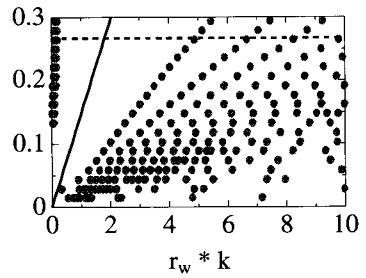

Fig. 5 Dispersion relation for m = 1 low frequency waves.

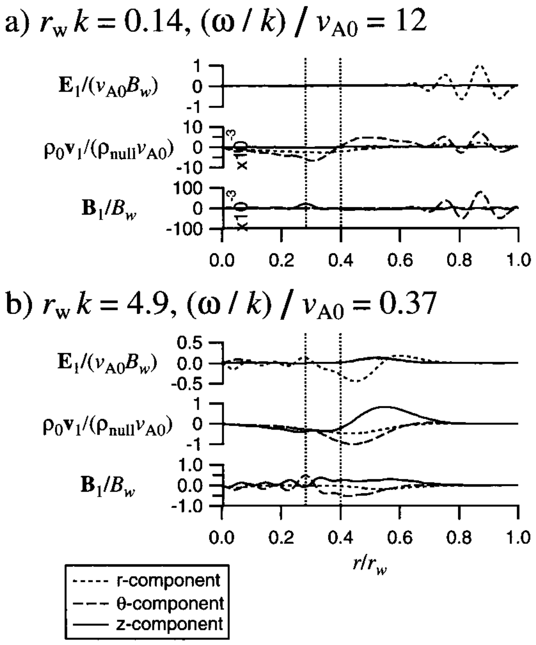

Fig. 6 Radial structure of the eigenmodes for m = 1.

phase velocity than that propagates away from the separatrix (The mode in Fig. 3b has smaller phase velocity than the mode in Fig. 3a, and the mode in Fig. 3c has the smallest phase velocity). This trend is consistent with the experimental result. The Alfvén velocity $v_{A0} \sim 10^5$[m/s] for the experimentsl parameters $B_w = 0.04$ [T] and $n_{max} \sim \bar{n} = 4 \times 10^{19}$ [m^{-3}]. Thus the phase velocity of the mode in Fig. 3c is $\omega/k \sim 1.4 \times 10^5$ [m/s]. This value is roughly the same as the experimental result.

For a typical FRC plasma, $T_i \sim 2T_e$ and $\beta \sim 1$. Thus the thermal velocity of the ions is $v_{th,i} \sim \sqrt{2/3}v_{A0} \sim 0.82v_{A0}$. The mode in Fig. 4a, whose phase velocity is similar to $v_{th,i}$, can have a strong interaction with the ions. In addition, this mode has significant amplitude of B_{1z} inside the magnetic field-null point $r = r_{null}$. Thus there is a possibility that this mode heats the ions by the *transit-time magnetic pumping* [3]. Averaged change in kinetic energy of a plasma ion by this mechanism in a unit time is [4]

$$\left\langle \frac{d}{dt}\frac{m_i v^2}{2} \right\rangle = -\frac{\pi \mu_m^2 |k|}{2m_i} |B_{1z}|^2 \left(\frac{\omega}{k}\right)\left(\frac{\partial f(v_0)}{\partial v_0}\right)_{v_0 = \omega/k}$$

where μ_m is the magnetic moment of the ions in the equilibrium magnetic field, $f(v_0)$ is the velocity distribution function. The total change in the ion's kinetic energy is estimated *quite roughly* under the assumptions: (1) Ions inside the magnetic field-null point are heated for $\Delta t \sim 50$ μs (the wave was applied for about 50 μs in the experiment); (2) The distribution function is the Maxwellian; (3) The amplitude of the perturbed magnetic field B_{1z} is set to be equal to the amplitude of $B_{1\theta} \sim 0.008$ [T] observed in the experiment, because in the experiment we could not measure the magnetic field far inside the separatrix. The total change is

$$\bar{n}\left(\pi r_{null}^2 \ell_s\right)\Delta t \left\langle \frac{d}{dt}\frac{m_i v^2}{2} \right\rangle \sim 75 \text{ [J]}$$

where $\ell_s \sim 3.6$ [m] is the plasma length in z-direction. This value is comparable to the experimental results (85 [J]), although the estimation is quite rough because the magnitude of B_{1z} is *uncertain*. However, this result shows that the transit-time magnetic pumping could have a significant effect on the ion heating.

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