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MHD Stability of 3D Plasma Confinement Configurations with Finite Plasma Current

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Abstract

The ideal magnetohydrodynamic stability properties of reactor-sized sphellamak and quasiaxisymmetric stellarator configurations with finite toroidal plasma current are investigated. In sphellamak systems, a peaked toroidal current profile yields maximum-B conditions in the central region of the plasma as a consequence of the paramagnetic enhancement of the magnetic field induced by the current. The resulting poloidally closed contours of the magnetic field strength prove instrumental in the favourable confinement predictions for α-particles. Under these conditions, the sphellamak is stable or weakly unstable to local Mercier and ballooning modes as well as low order kink instabilities for volume average $\beta^* \simeq 8\%$. Unfortunately, the bootstrap current calculated in the collisionless regime provides only a small fraction of the total plasma current that is required. In a two field-period quasiaxisymmetric reactor device, the bootstrap current in the collisionless limit can enhance the edge rotational transform when reactor-relevant values of $\beta^* \leq 5\%$ are approached. Global external kinks and internal resonant mode structures associated with the rational surface $\iota = 1/2$ can be destabilised. This effect is examined for various pressure profiles indicating the possibility of stabilisation via a judicious profile choice or modest counter current drive.

Keywords:
bootstrap current, QAS, sphellamak, ideal MHD stability, global external kink, β^*, collisionless $1/\nu$ regime rotational transform

1. Introduction

Three dimensional (3D) magnetic confinement configurations such as the sphellamak [1] and quasiaxisymmetric stellarator [2,3,4,5] (QAS) can offer attractive potential for a reactor system such as stable operation with respect to global ideal magnetohydrodynamic (MHD) modes at β^* values in excess of 5%, adequate confinement of α-particles, compactness and acceptable neoclassical transport properties.

The sphellamak system [1] is a coreless device in which helical coils wound on a spheroidal structure combined with a set of vertical field coils to compensate for the current in the connecting arc segments that link two helical windings provide a small initial translation of the field lines. A strong toroidal current is required to provide the confining fields in this type of device which also paramagnetically enhances the toroidal magnetic field. Because the equilibrium magnetic field structure is 3D, the dynamo effect is not required for the generation

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of the toroidal magnetic field component. This system displays features of spherical tokamaks, stellarators, spheromaks and reversed field pinches. The main advantage with respect to spherical tokamaks is the absence of a central conductor and the concomitant shielding requirements against large neutron fluxes expected in a reactor environment. We specifically investigate the the sensitivity of the maximum-B magnetic field structure generated by a peaked toroidal plasma current and the bootstrap current (BC) [6,7] to various pressure profiles at $\beta^* \simeq 8\%$ as well as their dependence with respect to the current in the helical coils (for a toroidal current fixed at 20 MA) and as a function of β^*.

Quasiaxisymmetric stellarator (QAS) reactors are 3D systems in which the magnetic field structure in Boozer coordinates [8] is dominantly axisymmetric like in tokamaks. This property causes the neoclassical transport and the α-particle confinement to behave in a similar way as in a tokamak. For reactor relevant values of β^*, the QAS system can develop a substantial bootstrap current that can significantly alter the rotational transform. We examine the global ideal MHD stability properties with respect to internal and external kink modes using the TERPSICHORE code [9] in a 2-field period QAS at $\beta^* \sim 5\%$ with a self consistent bootstrap current evaluated in the collisionless $1/\nu$ regime and determine the sensitivity to various pressure profiles. In this work, we employ the definition $\beta^* = 2\mu_0\sqrt{V<p^2>}/<B^2>$, where V is the plasma volume, p is the pressure, B is the magnetic field strength and $<A> = \int dVA$, because this constitutes a more relevant parameter with respect to fusion power production. β^* represents the root mean square value of β.

2. The Sphellamak Reactor System

The sphellamak system has a large toroidal plasma current that can cause it to become susceptible to low order kink instabilities. We have identified a configuration which is stable to localised ballooning and Mercier modes and global $n = 1$ kink modes at $\beta^* \sim 7\%$, but as it has a plasma volume V of less than 500 m^3, we have not pursued it further as a reactor option. We have concentrated on a configuration with $V \sim 1000$ m^3 to perform sensitivity studies of the mod-B^2 structure and the collisionless BC to variations in the width of the pressure profile, the magnitude of the helical coil current and β^*. The toroidal plasma current is prescribed as $2\pi J'(s) = 2\pi J'(0)[3(1-s)^5 + (1-s^5)^2]/4$ such that the total toroidal current $2\pi J(1) = I_p = 20$ MA. This type of current profile produces a maximum-B configuration in the central region of the plasma as shown in Fig. 1 for a case obtained with a standard pressure profile $p(s) = p(0)[0.9(1-s)^2 + 0.1(1-s)]$ at $\beta^* \sim 8\%$. This system is unstable to a low order external kink dominated by a $m/n = 1/1$ component because the rotational transform near the edge of the plasma approaches unity. The spectrum of the dominant components of the radial displacement vector are shown in Fig. 2a. At $\beta^* \sim 8\%$, the collisionless BC contributes a relatively modest 10% to the total toroidal current. Therefore, the bulk of the current must be inductively driven. This constitutes the main shortcoming of the sphellamak concept. The profile of the imposed toroidal current and those of the BC resulting from a pressure profile $p(s) = p(0)(1-s^2)^k$ with $k = 3$, 4 and 5 are displayed in Fig. 2b. For this work, we have considered a single ion and electron plasma of equal temperature and normalised density $N(s) = (1 - 0.999s^2)$. The BC increases from 1.58 MA to 1.68 MA as we increase the pressure profile peakedness factor k from 3 to 5. For the standard $p(s)$ profile, the BC increases linearly from 1.63 MA to 1.97 MA as β^* varies from 7.4% to 9%. The BC varies from 1.7 MA to 1.88 MA as the current in the helical coils I_{HC} changes from 54 MA to 66 MA. In this range, the global kink eigenvalue reaches a minimum at $I_{HC} = 60$ MA. The variation of the rotational transform is minimal with respect to changes in I_{HC} or $p(s)$ peakedness. However, the magnitude of the external kink unstable eigenvalue decreases as the $p(s)$ peakedness parameter varies from 3 to 5. The Mercier criterion is weakly unstable in the inner half of the plasma volume. For the standard $p(s)$ profile, the ballooning modes are unstable in the inner 20% of the plasma volume and also near the edge of the plasma around $s \sim 0.9$. For the pressure profiles $p(s) = p(0)(1-s^2)^k$, the vanishing of the pressure gradient near the axis stabilises the ballooning modes in that region. For $k > 3$, the unstable region near $s \sim 0.9$ disappears, but for $k \geq 5$ a new region of weak instability emerges near $s \sim 0.3$. The confinement of α-particles born near 1/4 of the plasma volume in the region of poloidally closed mod-B^2 contours shows a negligible loss after 0.1 s. For those particles born near half volume (corresponding to a normalised radius of ~ 0.7), the loss pattern approaches 40% corresponding to the bulk of the trapped population as illustrated in Fig. 2c. This fraction of α-particles experiences the full 3D structure of the magnetic field nearer the edge.

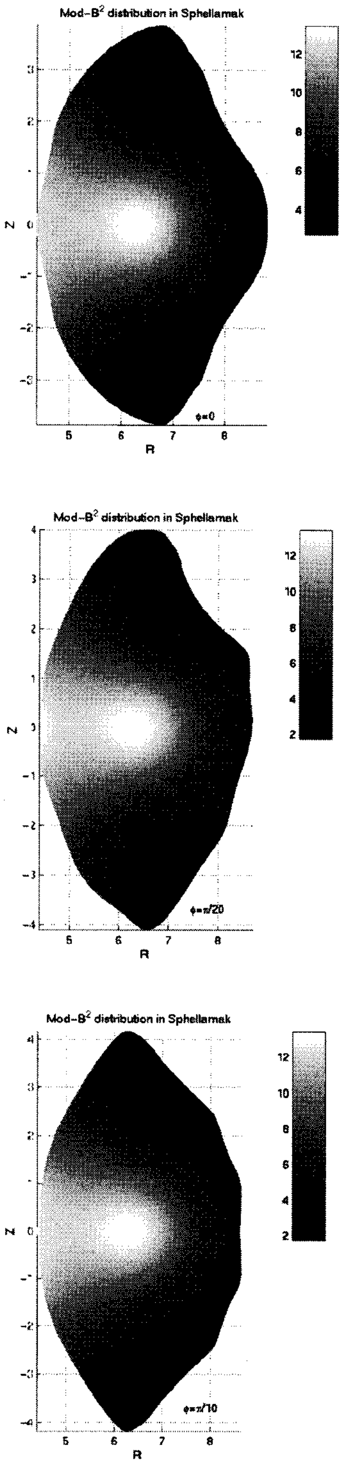

Fig. 1 The mod-B^2 structure in a sphellamak reactor system with $V \sim 1000$ m^3, peaked toroidal current $I_p = 20$ MA and $\beta^* \sim 8\%$ at the beginning of a field period (top), at 1/4 of the period (middle) and at midperiod (bottom). This structure remains resilient to variations of the pressure profile, the helical coil current and β.

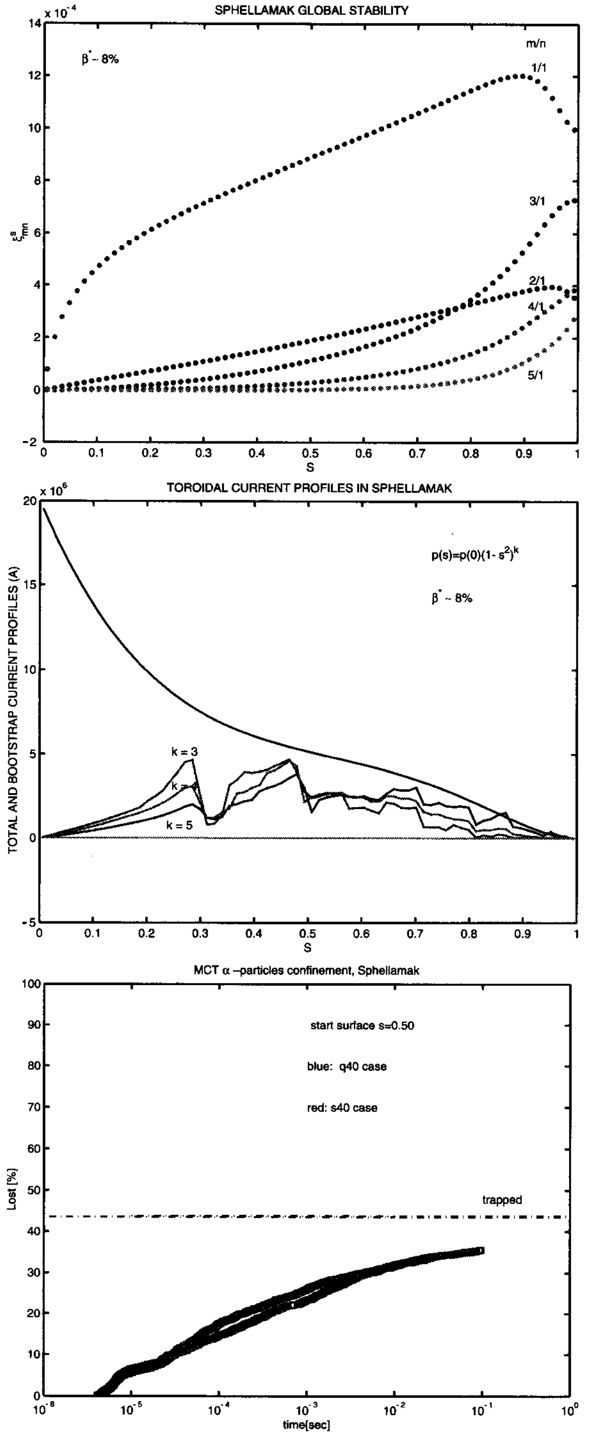

Fig. 2 a) (top) The radial displacement vector spectrum profiles, b) (middle) the prescribed toroidal current and collisionless bootstrap current profiles for $p(s) = p(0)(1 - s^2)^k$, $k = 3 \rightarrow 5$ and c) (bottom) the loss pattern in time of α-particles born at half volume for two $p(s)$ profiles in a sphellamak with peaked toroidal current and $\beta^* \sim 8\%$.

3. 2-Period QAS Reactor

We investigate in this section the global ideal MHD stability properties of a 2-field period QAS reactor in which the rotational transform results from the combined action of the 3D plasma shaping and the self consistent bootstrap current (BC) computed in the collisionless $1/\nu$ regime. [6,7] In particular, we examine the sensitivity of the BC and the global kink properties with respect to various types of pressure profiles at reactor relevant values of $\beta^* \sim 5\%$. The computation of the VMEC equilibria [10] and the BC is undertaken iteratively until a converged BC and corresponding profile are achieved typically in $6 \to 12$ iterations. The pressure profiles are chosen to have vanishing gradients near the edge of the plasma to prevent discontinuities in the BC at the plasma-vacuum interface. The BC profiles for various pressure profiles are presented in Fig. 3a at $\beta^* \sim 5\%$. The toroidal component of the BC (multiplied by the permeability of free space μ_0) as a function of β^* for a nearly parabolic pressure profile $p(s) = p(0)[1 - s - 0.1(1 - s^{10})]/0.9$ is shown in Fig. 3b. The structure of $\sqrt{g}\delta B^s$, where δB^s is the radial component of the perturbed magnetic field, dominated by a $m/n = 2/1$ contribution near the edge of the plasma is displayed at $\beta^* \sim 5\%$ in Fig. 3c. To further understand the sensitivity of the ideal MHD instability properties with respect to the BC, we multiply the toroidal component of the BC that is used as input for the VMEC code by an artificial enhancement/suppression factor T. For a case with a nearly parabolic pressure profile, the global kink eigenvalues as a function of T at $\beta^* = 4.9\%$, the rotational transform profiles for different values of T and the spectrum of the radial component of the displacement vector (that shows a dominating $m/n = 2/1$ term) are presented in the left hand column of Fig. 4. It can be seen that a modest 20% suppression of the bootstrap current can stabilise the external kink mode. A corresponding set of graphs associated with a slightly more peaked pressure profile $p(s) = p(0)[1 - s - 0.2(1 - s^5)]/0.8$ at $\beta^* = 5.2\%$ appears in the right hand column of Fig. 4. For this profile, the BC must be decreased by one half to guarantee stability. Enhancement of the BC destabilises more localised $m/n = 4/3$, $6/5$, $7/5$, $9/7$, etc mode components. This also happens when more peaked pressure profiles such as $p(s) = p(0)(1 - s)^2$ are considered.

4. Conclusions

We have examined the ideal MHD stability properties of two 3D magnetic confinement reactor

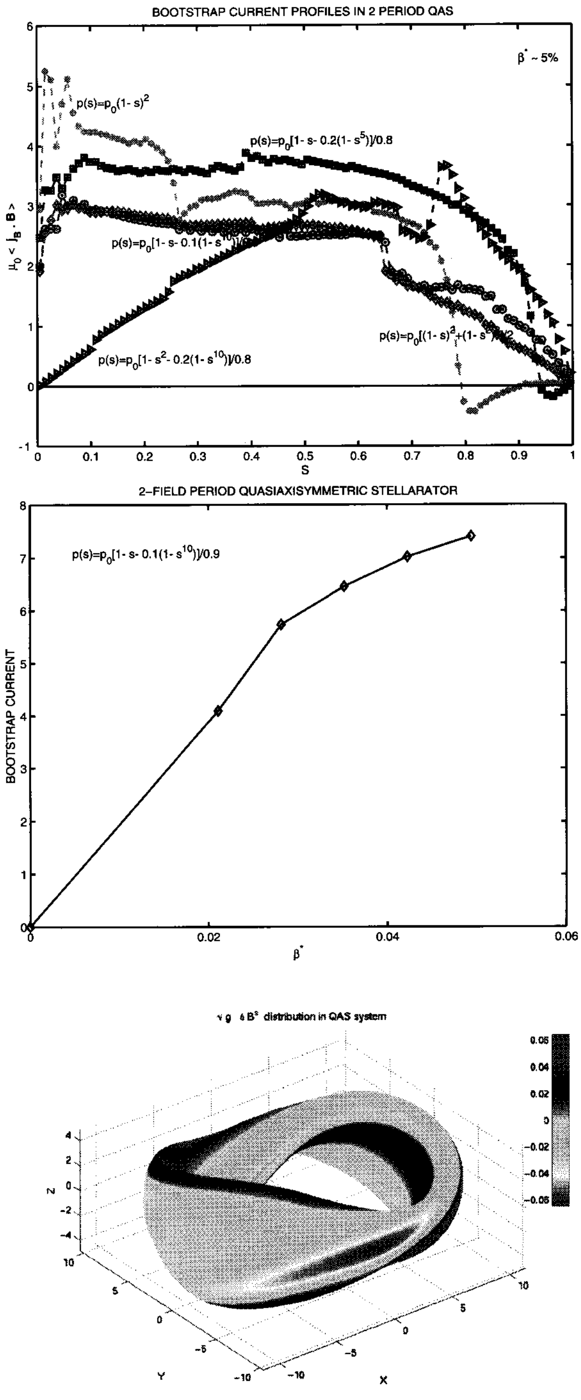

Fig. 3 a) (top) The bootstrap current profiles in the collisionless $1/\nu$ regime at $\beta^* \sim 5\%$ for various pressure profiles. b) (middle) The bootstrap current as a function of β^* for a nearly parabolic $p(s) = p(0)[1 - s - 0.1(1 - s^{10})]/0.9$ pressure profile. c) (bottom) The $\sqrt{g}\delta B^s$ distribution near the edge of the plasma in a 2-period quasiaxisymmetric stellarator at $\beta^* \sim 5\%$ that shows a $m/n = 2/1$ dominated unstable external kink obtained with a $p(s) = p(0)[1 - s - 0.2(1 - s^5)]/0.8$ pressure profile.

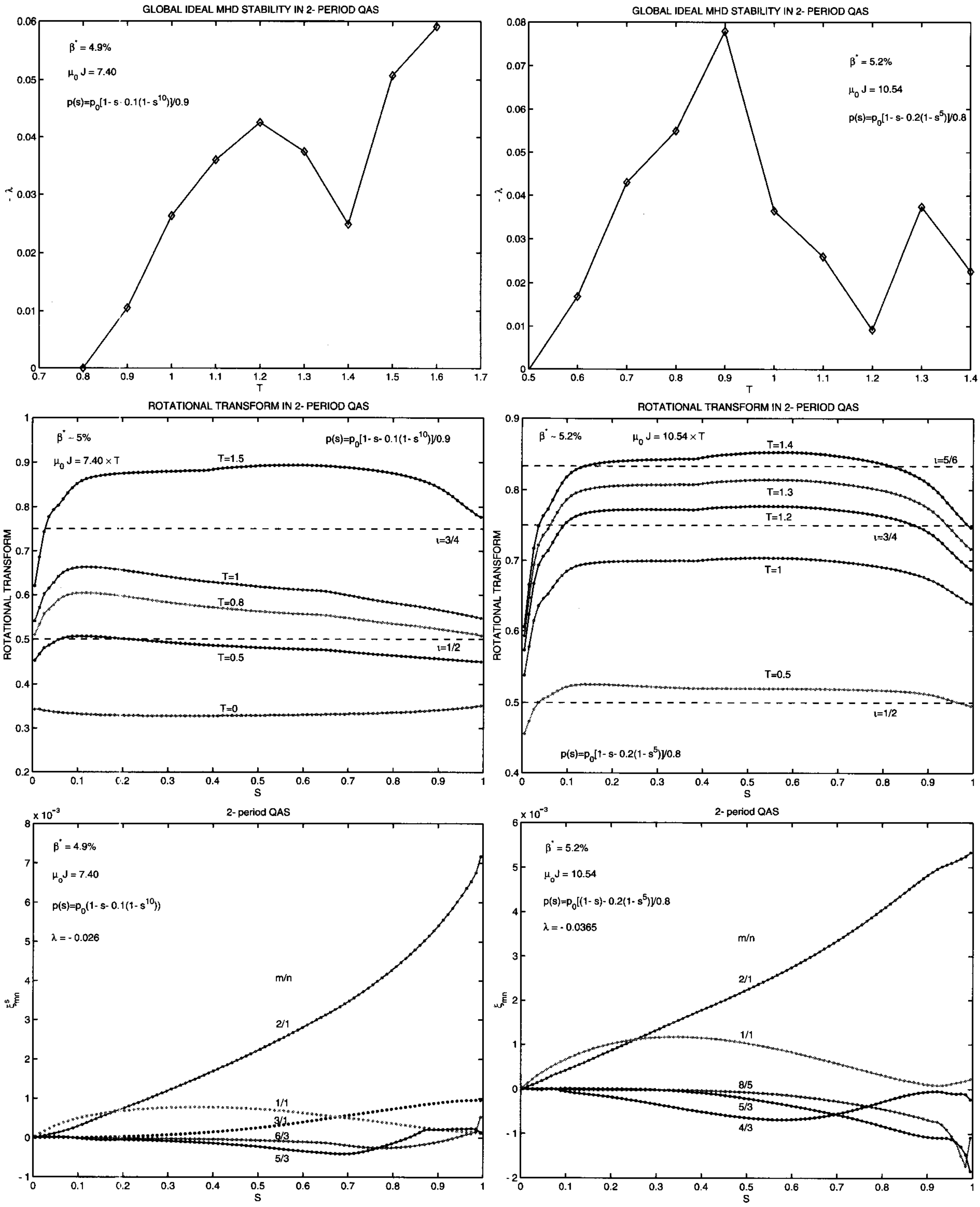

Fig. 4 The global kink eigenvalue as a function of the toroidal bootstrap current enhancement/suppression factor T (top row), the rotational transform profiles for different values of T (middle row) and the spectrum profiles of the radial component of the displacement vector dominated by $n = 1$ external kink modes for T=1 (bottom row) in a quasiaxisymmetric stellarator reactor with a nearly parabolic pressure profile $p(s) = p(0)[1 - s - 0.1(1 - s^{10})]/0.9$ (left hand column) and for a somewhat more peaked pressure profile $p(s) = p(0)[1 - s - 0.2(1 - s^5)]/0.8$ (right hand column).

systems with finite plasma current which constitute the most compact 3D devices proposed so far. A detailed comparative study between the two configurations is not attempted here because they display very different characteristics. A peaked toroidal current in the sphellamak device guarantees a maximum-B configuration near the central region of the plasma. Energetic α-particles born within the domain of closed mod-B contours remain extremely well confined, while the trapped fraction born closer to the edge of the plasma experiences the full 3D structure of the magnetic fields and consequently drift out within a slowing down time. We have analysed the sensitivity of the magnetic field structure and the bootstrap current in a sphellamak configuration that is mildly unstable to global external ideal MHD modes dominated by the m/n = 1/1 component because the rotational transform under the conditions tested approaches the critical value of unity at the edge of the plasma. The bootstrap current in the collisionless $1/\nu$ regime contributes only about 10% of the total toroidal plasma current in the device at $\beta^* \sim$ 8%. This bootstrap current increases with pressure profile peakedness, helical coil current and β^*, while the mod-B structure is essentially unaffected by the variations of these parameters. Though the Mercier criterion is weakly unstable in the central region of the plasma, the ballooning modes are stable for $p(s) = p(0)(1 - s^2)^4$ and only marginally unstable for other pressure profiles with small gradient on axis. Although it may be difficult to control the current and pressure profiles in an actual experiment, the global external kink properties may force a peaked toroidal current and the ballooning properties may adapt the pressure profile to that which we have found to be nearly optimal.

We have furthermore analysed the effect of a self consistent bootstrap current in the collisionless $1/\nu$ regime on the global ideal MHD stability properties of a 2-field period quasiaxisymmetric stellarator device. The bootstrap current increases the rotational transform above the critical value of $\iota = 1/2$ at $\beta^* < 5\%$ which can destabilise a m/n = 2/1 external kink mode. We have investigated the sensitivity of the external kink modes in this reactor system to the BC by altering the pressure profile and by artificially enhancing or decreasing the BC by multiplying its profile by a fixed factor T. We have found that a nearly parabolic pressure profile that suppresses the BC by 20% can stabilise global external kinks. For a slightly more peaked pressure profile, the suppression factor required increases to 50%. This suggests that a more judicious choice of pressure profile could reasonably yield fully stable conditions at reactor relevant values of β^*. Alternatively, some modest counter current drive in the range of 1 MA could not only realise external kink stability but could also assist in profile control. Enhancement of the BC or more peaked pressure profiles cause the destabilisation of internal/external kinks of higher order. We could, nevertheless, hope and anticipate that the plasma will naturally adopt a pressure profile that does not violate kink and ballooning stability conditions under real experimental operation.

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