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Stable Sheath Formation in Magnetized Plasma

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Abstract

The stable sheath formation in an expanding magnetic field to a divertor plate was studied by a one dimensional analysis. The requirement for flow velocity of ions at a plasma-sheath boundary is more restricted than that of the uniform magnetic field, which should be greater than the ion sound speed. The difference, however, between both cases is an order of the Debye length to plasma radius, which is negligibly small. The requirement for ion flow velocity inside a quasi-neutral plasma was obtained by taking into account the effects of the plasma source. Without plasma source in the quasi-neutral plasma, the ion flow velocity at an injection point should be much greater than the ion sound speed. The ion source inside the quasi-neutral plasma, such as electron impact ionization, considerably mitigates this requirement.

Keywords:
plasma sheath, expanding magnetic field, quasi-neutral plasma, divertor

1. Introduction

Formation of the electrostatic potential in a divertor plasma immersed in a nonuniform magnetic field is interesting in many fusion devices. In fusion devices the magnetic field expands in the direction of the plate, i.e. the magnitude of magnetic field decreases towards the plate. In this configuration ions are accelerated towards the plate due to the gradient of the magnetic field strength, called the mirror force. The bombardment of accelerated ions to the plate may cause several severe problems to fusion plasmas, for example, release of large amount of impurities from the divertor plate. Limited research has been carried out describing magnetic field effects on various potential formations and particle and heat fluxes to the divertor plate. The plasma-wall interaction in a uniform and oblique magnetic field to the plate has been studied by means of 1D-PIC numerical simulation [1]. This analysis shows the formation of a quasi-neutral magnetic presheath preceding the electrostatic Debye sheath, which scales to the ion gyroradius at the sound speed and to the incidence angle of the magnetic field. With the use of a two dimensional kinetic analysis, Sato [2] clarified this magnetic presheath is attributed to the ion polarization drift. The potential formation of a presheath in an open magnetic field studied analytically and numerically [3]. The effects, however, of nonuniformity of the magnetic field have not been taken into account for the stable Debye sheath formation. In this paper, we consider a collisionless sheath model between an infinite metal plate and a quasi-neutral plasma in an expanding magnetic field towards the plate. One dimensional kinetic analysis leads to a condition for a stable sheath formation in an expanding magnetic field, which is compared to the criterion for the stable Debye sheath formation in a uniform magnetic field [4,5]. In a quasi-neutral plasma region outside the Debye sheath with a plasma source, the requirement for ion velocity is obtained by limiting the formation of a negative gradient of electrostatic potential.

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2. Stable Sheath Formation in an Expanding Magnetic Field

The magnetic field lines connecting two divertor plates in a SOL (Scrape-Off Layer) region of a toroidal magnetic fusion device consist of a single or multiple mirror configurations. In a divertor region, the magnetic field is expanding towards the plate. Figure 1 shows the strength of the magnetic field between two divertor plates in the LHD (Large Helical Device) [6]. The model geometry for a one dimensional analysis is shown in Fig.2, where the divertor plate is located at the position of $z = L$, and the plasma-sheath boundary is denoted by z_b. The magnitude of magnetic field $B(z)$, which is directed toward the z direction, decreases to the plate so slowly that the magnetic moment of ions is conserved. Inside the sheath ($z_b \leq z < L$), collisions between plasma and neutral particles are neglected because of much shorter sheath thickness (~ Debye length) than the collisional mean-free-path.

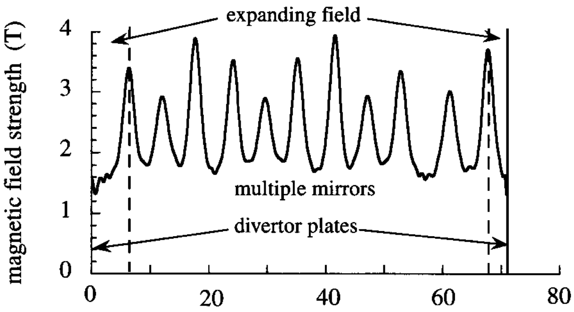

Fig.1 Magnetic field strength between divertor plates in LHD.

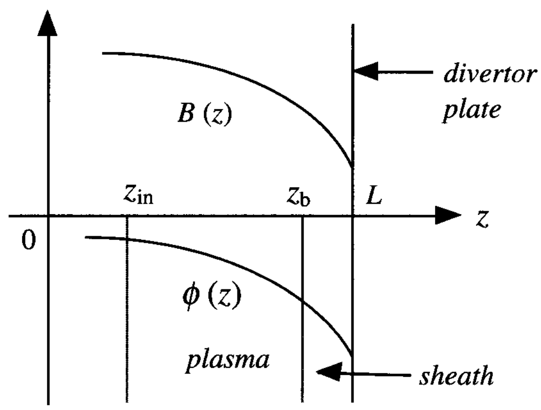

Fig.2 Model and coordinate for one dimensional analysis.

The ion distribution function $f_i(z,v_z,v_\perp)$ governing the steady sheath is determined by the Vlasov equation:

$$v_z \frac{\partial f_i(z,v_z,v_\perp)}{\partial z} + \frac{dv_z}{dt}\frac{\partial f_i(z,v_z,v_\perp)}{\partial v_z} + \frac{dv_\perp}{dt}\frac{\partial f_i(z,v_z,v_\perp)}{\partial v_\perp} = 0. \quad (1)$$

The kinetic equations of an ion are

$$\frac{dv_z}{dt} = -\frac{q}{M}\frac{d\phi}{dz} - \frac{\mu}{M}\frac{dB}{dz}, \quad (2)$$

$$\frac{dv_\perp}{dt} = \frac{\mu}{Mv_\perp}\frac{dB}{dt} = \frac{\mu v_z}{Mv_\perp}\frac{dB}{dz}, \quad (3)$$

where the charge and mass are denoted by q (= Ze) and M, respectively. The magnetic moment of an ion $\mu(z) \equiv Mv_\perp^2/2B(z)$ is one of the constants of motions because of the slow change of the magnetic field. In the case of an expanding magnetic field ($dB/dz < 0$), the second term of RHS in eq. (2) acts as a mirror acceleration for ions. The transformation of variables from $(z,v_z,v_\perp)$ to (z,ε,μ) gives

$$v_z(z,\varepsilon,\mu)\frac{\partial f_i(z,\varepsilon,\mu)}{\partial z} = 0, \quad (4)$$

where ε is the total energy of an ion (= $Mv_z^2/2 + \mu B(z) + q\phi(z)$) and the $v_z(z,\varepsilon,\mu)$ is the particle velocity towards the plate, which should be expressed by the new variables (z,ε,μ), i.e.

$$v_z(z,\varepsilon,\mu) = \sqrt{2[\varepsilon - \mu B(z) - q\phi(z)]/M}. \quad (5)$$

In a decreasing effective potential energy U ($\equiv \mu B(z) + q\phi(z)$), there are no ions with zero or negative velocity $v_z(z,\varepsilon,\mu) \leq 0$. Therefore the Vlasov equation eq. (4) shows the ion distribution function in the (z,ε,μ) space holding constant along the z direction inside the sheath, i.e. $f_i(z,\varepsilon,\mu) = f_i(z_b,\varepsilon,\mu)$ ($\equiv f_b(\varepsilon,\mu)$). This leads to the ion density inside the sheath as:

$$n_i(z) = \frac{2\pi B(z)}{M^2} \int_0^\infty d\mu \int_{U_b}^\infty d\varepsilon f_b(\varepsilon,\mu) / \sqrt{2[\varepsilon - \mu B(z) - q\phi(z)]/M}, \quad (6)$$

where U_b is the effective potential energy at the plasma-sheath boundary (= $\mu B(z_b) + q\phi(z_b)$). In the vicinity of the plasma-sheath boundary z_b, ion density is expressed by the expansion $\Delta\phi \equiv \phi(z_b) - \phi(z)$ and $\Delta B \equiv B(z_b) - B(z)$,

$$n_i(z) \cong n_{ib} \left[\begin{array}{l} 1 - \frac{q}{M} \langle v_z^{-2} \rangle_b \Delta\phi \\ -\left(1 + \frac{1}{2} \langle v_\perp^2 / v_z^2 \rangle_b\right) \frac{\Delta B}{B_b} \end{array} \right], \quad (7)$$

where n_{ib} and $\langle\cdot\cdot\rangle_b$ denote ion density and the average over the ion energy distribution function at the sheath edge z_b, respectively. The electron density, which is assumed to satisfy the Boltzmann relation, is also expanded around the plasma-sheath boundary:

$$n_e(z) \cong n_{eb} \,(1 - e\Delta\phi / T_e), \quad (8)$$

where n_{eb} ($\equiv n_b$) is the electron density at the sheath edge, which is equal to that of ion n_{ib}, and where T_e is the electron temperature, which is assumed uniform in the whole region.

These densities give the Poisson's equation around the plasma-sheath boundary,

$$\varepsilon_0 \frac{d^2\Delta\phi}{dz} \cong \frac{en_b}{T_e} \left[\begin{array}{l} 1 - \frac{ZT_e}{M} \langle v_z^{-2} \rangle_b \\ -\frac{T_e}{e}\left(1 + \frac{1}{2} \langle v_\perp^2 / v_e^2 \rangle_b\right) \left.\frac{d(\ln B)/dz}{d\phi/dz}\right|_b \end{array} \right] \Delta\phi. \quad (9)$$

We can easily obtain the condition of a stable or nonoscillatory solution of electrostatic potential at the plasma-sheath edge:

$$\langle v_z^{-2} \rangle_b^{-1} \geq \frac{ZT_e/M}{1-\gamma_b} \quad (10)$$

and

$$\gamma_b < 1, \quad (11)$$

where the factor γ_b indicates the effect of nonuniformity of the magnetic field strength as

$$\gamma_b \equiv \frac{T_e}{e}\left(1 + \frac{1}{2} \langle v_\perp^2 / v_z^2 \rangle_b\right) \left.\frac{d(\ln B)/dz}{d\phi/dz}\right|_b. \quad (12)$$

In the case of a uniform magnetic field ($\gamma_b = 0$), the criterion (10) becomes

$$\langle v_z^{-2} \rangle_b^{-1} \geq ZT_e/M, \quad (13)$$

which is the generalized Bohm criterion [5] for the uniform magnetic field. The decreasing magnetic field and electrostatic potential give a positive value of the factor γ_b, which means the average value of $\langle v_z^{-2} \rangle_b^{-1}$ has to be larger than that of the generalized Bohm criterion eq. (13):

$$\langle v_z^{-2} \rangle_b^{-1} \geq \frac{ZT_e/M}{1-\gamma_b} \geq \frac{ZT_e}{M}. \quad (14)$$

The scale length of electrostatic potential is to the order of the sheath width (~ Debye length). On the other hand, the scale length of the magnetic field is as large as a minor radius of a toroidal plasma. Since the magnitude of γ_b becomes much smaller than unity, the effect of the expanding magnetic field on the condition of the stable sheath formation may be negligible.

The relation between the density gradients of electrons and ions at the sheath edge is obtained from the charge neutral condition,

$$Z\left.\frac{dn_i}{dz}\right|_b = \left.\frac{dn_e}{dz}\right|_b \left[\begin{array}{l} \frac{ZT_e}{M} \langle v_z^{-2} \rangle_b \\ +\frac{T_e}{e}\left(1 + \frac{1}{2} \langle v_\perp^2 / v_z^2 \rangle_b\right) \\ \left.\frac{d(\ln B)/dz}{d\phi/dz}\right|_b \end{array} \right]. \quad (15)$$

This relation means that the requirement for the stable sheath formation, eq. (10), is equivalent to the requirement that the magnitude of the gradient of ion density should be smaller than that of electrons at the sheath edge [7],

$$Z\left|\left.\frac{dn_i}{dz}\right|_b\right| \leq \left|\left.\frac{dn_e}{dz}\right|_b\right|. \quad (16)$$

Since the electron density decreases faster than that of the ion in the decreasing electrostatic potential, ion density will be larger than that of electrons everywhere inside the sheath and the stable sheath will be formed.

3. Ion Velocity Distribution in Quasi-Neutral Plasma

The requirement for ion velocity distribution is investigated in a quasi-neutral plasma outside the Debye sheath ($z_{in} \leq z \leq z_b$ see in Fig. 2). Here the ion source due to electron impact ionization of neutral atoms is considered as well as the injected plasma from the upper stream. In a divertor region ionization by electron impact is effective because of the electron temperature of 10 ~ 100 eV. The distribution function of the ion source is given as:

$$S(z,\varepsilon,\mu) = \frac{M^2}{4\pi T_s^2} n_e(z) n_a \langle\sigma v\rangle_i \,|\, v_z(z,\varepsilon,\mu) \,| \\ \exp[-Mv^2(z,\varepsilon,\mu)/2T_s], \quad (17)$$

where n_a is the density of the neutral atoms, $\langle\sigma v\rangle_i$ is the rate coefficient of ionization by electron impact and T_s is the temperature of source ions, which is assumed uniform. In this distribution function, the ion velocity v_z

and v should be expressed by the function of (z,ε,μ). This velocity distribution of the source ions makes the ion distribution function equal to the Maxwellian distribution [8]. Taking into account the turn of source ions with negative velocity v_z in decreasing effective potential, the density of the source ions is obtained,

$$n_{is}(z) = \sqrt{\frac{\pi M}{2T_s}}\, n_a n_{e0} \langle \sigma v \rangle_i \exp\left[-q\phi(z)/T_s\right] \int_{z_t(\varepsilon,\mu)}^{z_b} dz' \exp\left[(q/T_s - e/T_e)\phi(z')\right], \tag{18}$$

where n_{e0} is the electron density at the position of $\phi = 0$, and z_t is the turning point of an ion with negative v_z, which depends on ε and μ:

$$\begin{cases} z_t = 0 \;:\; \varepsilon \geq \mu B_{\max} \\ \varepsilon - \mu B(z_t) - q\phi(z_t) = 0 \;:\; \varepsilon \geq \mu B_{\max}. \end{cases} \tag{19}$$

For the ions injected from the upper stream of the boundary z_{in}, the ion density is obtained by the same manner stated in the previous section. The local density gradient of these ions is easily obtained,

$$\frac{dn_i}{dz} = n_{in}(z) \left[\frac{q}{M} \langle v_z^{-2} \rangle_{in} \frac{d\phi}{dz} + \left(1 + \frac{1}{2}\langle v_\perp^2 / v_z^2 \rangle_{in}\right) \frac{d(\ln B)}{dz} \right] - n_{is}(z) \frac{q}{T_s}\frac{d\phi}{dz}, \tag{20}$$

where n_{in} and n_{is} are the injected ion and source ion density, respectively, and $\langle .. \rangle_{in}$ denotes the average over the ion velocity distribution function at the plasma injection boundary z_{in}. Electron density, which is assumed to satisfy the Boltzmann relation, and the charge neutrality condition in this region give the relation between the ion flow velocity and the gradients of electrostatic potential and the magnetic field:

$$\frac{ZT_e}{M} \langle v_z^{-2} \rangle_{in} = \frac{1}{1-\delta_s} \left[1 + \delta_s \frac{ZT_e}{T_s} - (1-\delta_s) \frac{T_e}{e} \left(1 + \frac{1}{2}\langle v_\perp^2 / v_z^2 \rangle_{in}\right) \frac{d(\ln B)/dz}{d\phi/dz} \right], \tag{21}$$

where δ_s is defined by the ratio of the density of the source ions to the electron density: $\delta_s \equiv Zn_{is}/n_e$. The temperatures of electrons T_e and source ions T_s are assumed uniform inside the plasma region. The third term of RHS of eq. (21), which designates the nonuniformity of the magnetic field, is the order of unity because of the same scale length of the magnetic field and the electrostatic potential. On the other hand, the second term of RHS of eq. (21), which indicates the effects of the generated ion source, is much larger than unity because of the higher plasma temperature compared to that of the source ions. Therefore, the averaged value of $\langle v_z^{-2} \rangle_{in}^{-1}$ at the injection point becomes much less than the ion sound speed (ZT_e/M). In the case of no plasma source ($\delta_s = 0$), the RHS of eq. (21) becomes less than unity, which implies that the value of $\langle v_z^{-2} \rangle_{in}^{-1}$ is required to be larger than the ion sound speed at the injection boundary.

4. Concluding Remarks

The stable sheath formation in an expanding magnetic field towards the divertor plate was studied by a one dimensional kinetic analysis. The requirement for stable sheath formation is that the averaged ion velocity at the plasma-sheath boundary must be larger than the ion sound speed, which is more restricted than the requirement in a uniform magnetic field (the generalized Bohm criterion [5]). The factor, however, of the difference between both cases is as small as the order of the ratio of the Debye length to the plasma radius. The requirement for the ion velocity inside the plasma was obtained from the quasi-neutral condition of the plasma. If there is no plasma source in the quasi-neutral plasma, the ion flow velocity at the injection point z_{in} into the plasma should be much larger than the ion sound speed. The ion source inside the plasma, such as ionization of neutral particles due to electron impact, considerably reduces this requirement, where the required ion flow velocity becomes lower than the ion sound speed. Future studies in this area of research should include other phenomena, for example Coulomb collision and charge exchange process.

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