J. Plasma Fusion Res. SERIES, Vol. 4 (2001) 564–569

Analytical Description of T. Sato's Mechanism of Transformation of Ion-Acoustic Double Layer into Strong Bunemann's One in Cosmic and Laboratory Nonequilibrium Plasmas

LAPSHIN Vladimir, MASLOV Vasyl* and STOMIN Vadim
NSC Kharkov Institute of Physics & Technology, Kharkov 61108, Ukraine

(Received: 5 December 2000 / Accepted: 27 September 2001)

Abstract

The model of a transformation of ion-acoustic double layer with small amplitude into strong double layer is presented in [1]. In this paper this model and all stages of evolution are described analytically. Earlier by numerical simulation in [2] and analytically in [3] it has been shown that an ion-acoustic instability development leads to a formation of a nonmonotoneous ion-acoustic double layer with small amplitude if an electron drift velocity is a little smaller than an electron thermal velocity. But this double layer accelerates ions. Hence on first front of ion-acoustic double layer an ion density becomes smaller. But into this region the electron flow penetrate with electron velocity only a little smaller than electron thermal velocity. Hence on the first front of the potential hollow the electron drift velocity becomes more than electron thermal velocity due to flow continuity law [1]. Due to Bunemann mechanism interaction of electron flow with this region an electric potential hump of large amplitude is excited with growth rate, proportional to $\omega_{pe}(m_e/m_i)^{1/3}$. Here ω_{pe} is the electron plasma frequency; m_e, m_i are electron and ion masses. This strong electric potential hump can be unstable relative to radiation of electron reversal jump. After that former hump becomes strong double layer.

Keywords:

wave structure, solitary wave, double layer, nonequilibrium plasma

1. Excitation of Ion-Acoustic Solitary Perturbation of Finite Amplitude in Current-Carrying Plasmas

In present paper a plasma with electron current relative to nonpropagating positive and negative ions is considered with electron current velocity a little smaller than electron thermal velocity, $V_0 < V_{the}$. This plasma is nonequilibrium. Ion-acoustic perturbations are excited. It is shown in [4] that homogeneous ion-acoustic turbulence in one-dimensional nonequilibrium current-carrying plasma is saturated on low level. Futher this ion-acoustic turbulence is unstable relative to the modulation. This modulation instability results in turbulence splitting into widely spaced short perturbations. The width of the latter is of order of the wavelength. On nonlinear stage of evolution these short perturbations become the solitary types. Therefore properties of electrostatic potential hollow of solitary kind are investigated in this paper (see Fig. 1a). The hollow reflects electrons with energy smaller than the hollow depth. This leads to hollow depth (amplitude of electric potential, φ_0) growth.

The equation describing shape and time evolution

*Corresponding author's e-mail: vmaslov@kipt.kharkov.ua

of the electric field structure of the hollow is derived in this paper. It is obtained that the ion-acoustic hollow of electrostatic potential is excited due to current-carrying instability. The case of large amplitude of excited perturbation is considered, when there are no traditional small parameters, permitting to describe properties and excitation of perturbation. It is shown that the hollow propagates with velocity which is close to ion-acoustic velocity of positive ions $(T_e/m_{i+})^{1/2}$. Here T_e is the temperature of plasma electrons; m_{i+} is the mass of positive ion.

We use hydrodynamic equations for densities $n_{i\pm}$ and velocities $V_{i\pm}$ of positive and negative ions, Vlasov equation for electron distribution function f_e and Poisson equation for the electrostatic potential φ.

Electrons propagate relative to ions with some current velocity $V_{\acute{o}}$. Due to reflection of resonant electrons, with nonsymmetric relative to hollow velocity $V_{\acute{o}}$ distribution function, from potential hollow the quasineutrality brakes near the hollow: before the hollow the electron density decreases and after the hollow the electron density increases. The quasineutrality is realized due to formation of electrostatic potential jump $\Delta\varphi$ near the hollow (see Fig. 1a).

At increasing of hollow amplitude up to critical value, when inverse time of resonant electron (with velocities $|V - V_{\acute{o}}| \le V_{tr}(\varphi_0)((2e\varphi_0/m_e)^{1/2})$ interaction with the hollow becomes larger than growth rate $\gamma(\varphi_0) \equiv \partial \ln \varphi_0/\partial t$ of hollow amplitude $V_{tr}(\varphi_0) > \gamma(\varphi_0)\delta z(\varphi_0)$ a slow evolution of the hollow starts in comparison with electron dynamics. Here δz is the width of the hollow, e is the electron charge. The resonant electron distribution function changes. The front with this changed distribution function propagates from the hollow with relative velocity equal V_{tr}.

We use the approximation of a hollow slow evolution for its description, using small parameter $\eta \equiv \gamma(\varphi_0)\delta z(\varphi_0)/V_{tr}(\varphi_0)$. In zero approximation on this parameter, taking into account that the resonant electrons are reflected from the hollow, one can derive from Vlasov equation the expression for electron distribution function on velocity, V,

$$f_e = f_{0e}[-(V^2 - 2e(\varphi \pm \Delta\varphi)/m_e)^{1/2} \pm V_0]\,,$$
$$V \gtrless A(\varphi)\,\mathrm{sign}(z),$$
$$A(\varphi) \equiv [2e(\varphi_0 + \varphi)/m_e]^{1/2}\,. \qquad (1)$$

Here f_{0e} is the initial unperturbed distribution function of electrons; $z = 0$ corresponds to $\varphi = -\varphi_0$.

We use the normalized values: $\phi \equiv e\varphi/T_e$, $N_- \equiv n_{0-}/n_{0+}$, $N_e \equiv n_{0e}/n_{0+}$, $Q_\pm = q_\pm/e$, $V_{s\pm} = (T_e/m_{i\pm})^{1/2}$. We normalize z on Debye radius of electrons r_{de}, V_0 on electron thermal velocity V_{the}, time t on plasma frequency of positive ions $\omega_{p+}{}^{-1}$, velocity of solitary perturbation V_c on ion-acoustic velocity $(T_e/m_{i+})^{1/2}$ of positive ions. n_{0-}, n_{0+} are unperturbed densities of negative and positive ions; $q_\pm$, $m_{i\pm}$ are the charges and masses of positive and negative ions.

Integrating the expression (1) on velocity, one can derive the electron density in first approximation on $V_{\acute{o}}$

$$n_e \approx n_{0e}\exp(\phi)[1 - (2\Delta\phi/\sqrt{\pi})\textstyle\int_0^\beta dx\,\exp(-x^2)$$
$$- 2V_0(2/\pi)^{1/2}\textstyle\int_0^\beta dx(x^2 - \phi)^{1/2}\exp(-x^2)$$
$$\beta \equiv [e(\varphi_0 + \varphi)/T_e]^{1/2}\,. \qquad (2)$$

Far from the hollow the plasma is quasineutral. It means in approximation $n_{0-} = 0$ that far from the hollow we have $N_e(z)|_{z\to\infty} = N_e(z)|_{z\to-\infty} \approx 1$. From here one can

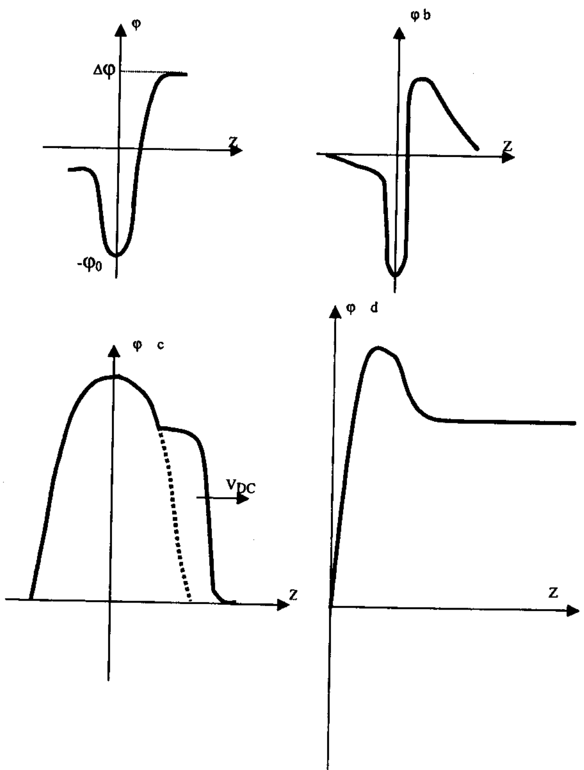

Fig. 1 Transformation of ion-acoustic double layer into strong Bunemann's one in current-carrying plasma: a) small-amplitude ion-acoustic solitary perturbation; b) small-amplitude ion-acoustic solitary perturbation with nonhomogeneous tails; c) electron double layer separation from strong hump; d) strong double layer.

derive, using (2), the expression for potential jump near the hollow

$$\Delta\phi \approx V_0(2/\pi)^{1/2}(1 - \exp(-\phi_0))/ [1 - (2/\sqrt{\pi})\int_0^{\sqrt{\phi 0}} dx \exp(-x^2)] \quad (3)$$

From hydrodynamic equations for ions one can obtain for perturbations of densities of positive and negative ions

$$n_{i\pm} = n_{\pm NL} + n_{\pm\tau},$$
$$n_{\pm NL} = n_{0\pm}/[1 - (\pm q_\pm)2\varphi/m_{i\pm}V_c^2]^{1/2},$$
$$\partial n_{\pm\tau}/\partial z = \pm 2(\partial\varphi/\partial t)(n_{0\pm}q_\pm/m_{i\pm}V_c^3) [1 - (\pm q_\pm)\varphi/m_{i\pm}V_c^2]/[1 - (\pm q_\pm)2\varphi/m_{i\pm}V_c^2]^{3/2} \quad (4)$$

Substituting (2), (4) in Poisson equation one can derive nonlinear evolution equation

$$\partial^3\phi/\partial z^3 + \{Q_+^2V^2_{s+}(1 - 2\phi Q_+V^2_{s+}/V_c^2)^{-3/2} (1 - \phi Q_+V^2_{s+}/V_c^2) + Q_-^2N_-V^2_{s-}(1 + 2\phi Q_-V^2_{s-}/V_c^2)^{-3/2} (1 + \phi Q_-V^2_{s-}/V_c^2)\}(\partial\phi/\partial t)2/V_c^3$$
$$+ ((\partial\phi/\partial z)/V_c^2)\{Q_+^2V^2_{s+}(1 - 2\phi Q_+V^2_{s+}/V_c^2)^{-3/2} + Q_-^2N_-V^2_{s-}(1 + 2\phi Q_-V^2_{s-}/V_c^2)^{-3/2}\}$$
$$- \{\exp(\phi) - \operatorname{sign}(z)V_0(2/\pi)^{1/2} \{(\phi_0/(\phi_0 + \phi))^{1/2}\exp(-\phi_0) - \int_{\sqrt{-\phi}}^{\sqrt{\phi 0}} dy(1 - 2y^2)\exp(-y^2)/(y^2 + \phi)^{1/2}$$
$$+ (1 - \exp(-\phi_0))[1 - (2/\sqrt{\pi})\int_0^{\sqrt{\phi 0}} dx \exp(-x^2)]^{-1} [\exp(-\phi_0)/(\phi_0 + \phi)^{1/2} + 2(\phi_0 + \phi)\exp(-\phi_0) + 4\int_{\sqrt{-\phi}}^{\sqrt{\phi 0}} dy\ y(y^2 + \phi)^{1/2}\exp(-y^2)]/\sqrt{\pi}\}\}$$
$$N_e\partial\phi/\partial z = 0 \quad (5)$$

From nonlinear equation (5), using condition $(\partial\phi/\partial z)|_{\phi=-\phi_0} = 0$, one can show that the hollow propagates with the slow velocity $V_c \approx (T_e/m_{i+})^{1/2}(n_+/n_e)^{1/2}(q_+/e)$. From (5) one can get also the growth rate γ_{nl} of the hollow small amplitude. For that we select in (5) $\phi = -\phi_0$ in approximation of small amplitude.

$$\partial\phi_0/\partial t \approx (\Delta\phi/2\sqrt{\phi_0})\lim_{\phi\to-\phi_0}|\partial\phi/\partial z|/(\phi_0 + \phi)^{1/2} \quad (6)$$

For determination of the expression $\lim_{\phi\to-\phi_0}|\partial\phi/\partial z|/(\phi_0 + \phi)^{1/2}$ we use the equation (5) in quasistationary approximation

$$(\partial\phi/\partial z)^2/2 = (V_c/V_{s-})^2[N_-((1 + Q_-2\phi V^2_{s-}/V_c^2)^{1/2} - 1) + (1 - Q_+2\phi V^2_{s+}/V_c^2)^{1/2} - 1]$$
$$+ N_e\{\exp(\phi) - 1 - 2\operatorname{sign}(z)V_0(2/\pi)^{1/2} [\exp(-\phi_0)\sqrt{\phi_0}((\phi_0 + \phi)^{3/2} - \phi_0^{3/2})2/3 + \exp(-\phi_0)(1 + \phi_0 + \phi_0^2 2/3)$$
$$- \exp(\phi)(1 - \phi + \phi^2 2/3) + (1 - \exp(-\phi_0)) [1 - (2/\sqrt{\pi})\int_0^{\sqrt{\phi 0}} dx \exp(-x^2)]^{-1} [\exp(-\phi_0)((\phi_0 + \phi)^{3/2} - \phi_0^{3/2})2/3$$
$$+ \sqrt{\phi_0}\exp(-\phi_0)(1 + \phi_0 2/3) - \sqrt{-\phi}\exp(\phi)(1 - \phi 2/3) - \int_{\sqrt{-\phi}}^{\sqrt{\phi 0}} dy\exp(-y^2)]/\sqrt{\pi}]\} \quad (7)$$

Substituting (7) in approximation of small amplitude into (6) one can derive

$$\gamma_{nl} \approx \omega_{p+}(V_0/V_{the})^{3/2}(q_+/e)(n_+/n_e)^{1/2} \{1 + [1/3 - (n_e/n_+)(e/q_+)] (e\varphi_0/T_e)(\pi/2)^{1/2}(V_{the}/2V_0)\} \quad (8)$$

One can see that the hollow of large amplitude is formed at electron current velocity V_0 larger than threshold one. The threshold decreases at decreasing n_e/n_{i+} and equal zero at $n_e/n_{i+} < q_+/3e$. The maximum threshold is realized at $n_{i-} = 0$.

So, we have shown that due to one-dimensional ion-acoustic instability development the electric potential hollow with the potential jump near the hollow is excited.

2. Description of Solitary Perturbation Excitation by Bunemann Instability Development

Now we describe the further evolution of the excited ion-acoustic hollow with jump. The potential jump on the first front of the ion-acoustic hollow is nonmonotonous due to excitation of hollow in time (see Fig. 1b). The potential jump is maximum one near hollow and it decreases slowly up to zero far from the hollow. So, the shape of the electric potential on the first front of the hollow is a wide hump. This hump accelerates ions from its region. Hence on the first front of the ion-acoustic double layer an ion density becomes smaller. For the neutral condition support the electrons, penetrating in this region, should be accelerated. If into this region the electron flow penetrate with electron velocity only a little smaller than electron thermal velocity, hence on the first front of the hollow the accelerated electron drift velocity becomes more than electron thermal velocity due to flow continuity law [1]. Therefore, on the first front of the hollow the condition for Bunemann instability is realized. Due to Bunemann mechanism interaction of electron flow with this region an electric potential hump is excited. Let us describe a solitary perturbation in type of electric potential hump. We will show that it represents a nonlinear perturbation on a slow electron-sound mode. Such nonlinear perturbation has been observed in laboratory experiments and in numerical simulation [5]. As it is slow, resonant electrons can be trapped by such perturbation. Thus, it is possible to expect that kinetics of resonant electrons is of a large role in determination of this perturbation properties. Hump of electric potential provides hollow of electron density in a neighbourhood of maximum potential.

From Vlasov equation the expression for perturbation of electron distribution function follows. Integrating the latter expression on velocities in case of small amplitudes of the solitary perturbation φ_0 one can derive the expression for perturbation of electron density similarly to [6]

$$\delta n' = (\partial\varphi/\partial t)[y + (1-2y^2)(1-R(y))/y] + \varphi' R(y)$$
$$+ \varphi\varphi'[1 - y^2 + (3/2 - y^2)(R(y)-1)]$$
$$R(y) \equiv 1 + (y/\sqrt{\pi})\int_{-\infty}^{\infty} dx \exp(-t^2)/(t-y)\ ,$$
$$y \equiv V_0/V_{\mathrm{the}}\sqrt{2} \qquad (9)$$

Here prime means a spatial derivation; V_0, φ are velocity and electric potential of soliton; $\phi \equiv e\varphi/T_e$; V_{the} is the electron thermal velocity. Substituting (9) in Poisson equation, one can derive, in approximation $n_{0-} = 0$, an equation, describing spatial distribution of electric potential:

$$(\phi')^2 = \phi^2 R(y) - [1 + (2y^2 - 3)R(y)]\phi^3/6 \qquad (10)$$

From (10) and $\phi'|_{\phi=\phi_0} = 0$ one can obtain the expression $V_0 \approx 1.32 V_{\mathrm{the}}$ [6].

Let us determine approximately the soliton width from (10): $\delta z \approx \varphi_0/\varphi'|_{\varphi=\varphi_0/2} \approx r_{\mathrm{de}}(48\ T_e/e\varphi_0)^{1/2}$. The soliton width decreases with amplitude growth.

In case of large amplitudes, $e\varphi_0/T_e > 1$, from Vlasov equation we have the expression for electron distribution function $f = f_{0e}[(u^2 - 2e\varphi/m_e)^{1/2} + V_0\ \mathrm{sign}(u)]$ for $|u| = |V - V_0| > (2e\varphi/m_e)^{1/2}$. Here f_{0e} is Maxwell distribution function. Thus we obtain an equation for the soliton shape

$$(\phi')^2 = -\phi + (2/\sqrt{\pi})^{1/2}\int_{-\infty}^{\infty} dt (t-y)^2 \exp(-t^2)$$
$$\{[1 + \phi/(t-y)^2]^{1/2} - 1\} \qquad (11)$$

From (11) one can approximately derive

$$\delta z \approx \varphi_0/\varphi'\,|_{\varphi=-\varphi_{0/2}} \approx r_{\mathrm{de}}[2e\varphi_0/T_e(\sqrt{2}-1)]^{1/2} \quad (12)$$

From (12) one can conclude that the soliton width, δz, grows with φ_0. Therefore, it is necessary to take into account electrons, trapped by the soliton field. Assuming distribution of their density as $n_{\mathrm{tr}}(z) = n_2\exp[e\varphi(z)/T_{\mathrm{tr}}]$, we derive similarly to (12), that width and velocity of the soliton grow with amplitude growth (in difference from case of small amplitudes of the solitary perturbation).

Such properties of the soliton and their dependencies on amplitude have been observed in experiments and in numerical simulation [5].

Thus, this solitary perturbation is stationary and electron one, if ion mobility is neglected. However at taking into account of ion mobility it is necessary to expect occurrence of slow growth of the perturbation's amplitude, as a result of Bunemann instability development. In the following order of the theory of disturbances from (9) one can derive the correction of the next order to a spatial derivative from electron density

$$n_{\mathrm{el}}' = (\partial\varphi/\partial t)[y + (1-2y^2)(1-R(y))/y] \quad (13)$$

This expression, as follows from a spatial derivative from Poisson equation, should be equal to a spatial derivative from ion density perturbation n_{i+}'. n_{i+}' is possible to find in linear approximation from ion hydrodynamic equations

$$\partial^2 n_{i+}/\partial t^2 = (m_e/m_{i+})\,\phi'' \qquad (14)$$

Equating the second time derivative from (13) and first spatial derivative from (14), we obtain

$$\partial^3\phi/\partial t^3 = (6m_e/m_{i+})\,\phi''' \qquad (15)$$

The solution of (15) we search as

$$\phi(z,t) = \phi_0(t)\,\eta[z - \int_{-\infty}^{t} dt_1 \delta v_0(\phi_0(t_1))]\ , \qquad (16)$$

$\eta(z)$ is quasistationary shape of the perturbation, assuming, that $\partial\phi_0(t)/\partial t = \gamma\phi_0(t)$. In (16) the change of soliton velocity, δv_0, with change of its amplitude is taken into account.

Substituting $\partial\phi/\partial t$ through $\gamma\phi - \delta v_0\phi'$, we obtain from (15)

$$\gamma \approx (m_e/m_{i+})^{1/3}\phi_0^{1/2} \qquad (17)$$

Thus electrons and ions interaction of Bunemann type has resulted in excitation of the solitary hump of electrostatic potential.

3. Potential Jump Separation from the Hump and Hump Transformation into Strong Electric Double Layer

Let us consider, in approximation $n_{0-} = 0$, the possibility of the electric potential jump formation and its separation from the electric potential hump (see Fig. 1c) and transformation due to this separation of the electric potential hump into strong electric double layer (see Fig. 1d). We will show that it is possible if the distribution function of electrons, trapped by the field of the electric potential hump, has three-hump-type. So, let us assume that the distribution function of these electrons have three-hump-type. It includes low-energy and high-energy electrons. Let us show that densities of low-energy and high-energy electrons should be closed for transformation of potential hump into strong double layer.

We assume that the potential jump separates from the hump and moves with a velocity V_j. Thus, the potential jump may be formed from the value φ_0 to zero in a small space interval.

We investigate the properties of this jump in 1D approximation.

The jump formation needs a charge separation in space. For selfconsistent charge separation in space two groups of electrons are necessary. Let us consider high-energy electrons with velocity, V_b, and thermal velocity, V_{thb}, satisfying $V_b >> V_{the}$, V_{thb}, and low-energy electrons with velocity, V_{sl}, and thermal velocity, V_{ths}, satisfying $V_{sl} < V_{ths} << V_b$, and with densities n_b and n_s. $V_{the} = (T_e/m_e)^{1/2}$ is a thermal velocity of the plasma electrons. The velocity of low-energy electrons V_{sl} can be chosen equal to V_j.

The density of the low-energy electrons exponentially falls off, resulting in positive charge in the range $\varphi_a < \varphi < \varphi_0$ of the jump electric potential, φ, while the densities of both the high-energy and plasma electrons grow in accordance with the power law. This leads to negative charge at $0 < \varphi < \varphi_a$. Here φ_0 is the amplitude of the jump electric potential. At φ_c the beam starts to be reflected. As a results, the quasineutrality is restored outside the jump. Since the nonresonant high-energy electrons propagate into the plasma, where they are decelerated, their density grows. Therefore, the quasineutrality condition in front of the jump requires $V_j < V_{the}$ (is the thermal velocity of plasma electrons) and the density of the high-energy electrons passing through the jump should be small $n_b^0 << n_e$. n_e is the plasma electron density. Hence φ_0 approximately equals energy of high-energy electrons.

Further we use normalized values. Namely, we normalize particle densities on unperturbed plasma electron density, n_{0e}, electron velocities on electron thermal velocity, electric potential on T_e/e, electron temperatures on plasma electron temperature, T_e.

In the electric field of jump the ion impulse grows. Its flow equals $n_{i+}\varphi_0$. All electrons transfer impulse to jump. Flows of impulse, transfered to jump by plasma electrons, by high- and low-energy electrons equal $2V_j\varphi_0^{1/2}$, $2n_{b0}V_b^2$, n_sT_s, $4V_b^2(n_b - n_{b0})$. Here T_s is the temperature of the slow electrons; n_{b0} is the unperturbed density of the high-energy electrons. Plasma electrons and ions obtain energy from jump. Energy flows, obtained by electrons and ions, equal $2\varphi_0V_j$, $2\varphi_0V_jn_{i+}$. Electrons lose energy at interaction with potential jump. Energy flows, transferred to jump by low- and high-energy electrons, equal $V_jn_sT_s$, $4V_jV_b^2(n_b - n_{b0})$, $\varphi_0V_bn_{b0}$. Using the momentum- and energy-balance equations and also the quasineutrality conditions, we obtain

$$V_j/V_b \approx n_{b0}/(2 + n_{i+}) << 1\,,$$
$$n_s \approx (n_{i+}/2)[1 + T_s/2\varphi_0 - 2(2T_b/\varphi_0)^{1/2}]\,,$$
$$n_b \approx (n_{i+}/4)[1 - T_s/2\varphi_0 + 2(2T_b/\varphi_0)^{1/2}]\,. \qquad (18)$$

From (18) one can see, that $n_s \neq 0$, i.e. the distribution function of electrons, trapped by potential hump, has three-hump-type. From (18) it follows that for jump separation from the potential hump n_b should be large. It is should be noted that during hump formation time the plasma electrons have no time for the response to the field initiation, and are trapped by the hump field. In this way the slow electron group is formed, which is necessary for the jump formation

Similar processes for the potential distribution and particle behaviour have been observed in a numerical simulation [7]. The beam injection into the plasma leads in numerical simulations [7] in certain conditions to the hump formation, to jump separation from the hump and moving it inside the plasma at a thermal velocity of plasma electrons. After that former hump becomes strong double layer (see Fig. 1d).

References

[1] T. Sato, H. Takamaru and the Complexity

Simulation Group, Phys. Plasmas **2**, 3609 (1995).

[2] T. Sato and H. Okuda, Phys. Rev. Lett. **44**, 740 (1980).

[3] V.I. Maslov, *Properties and Evolution of Nonstationary Double Layers in Nonequilibrium Plasma. Proc. of Int. Conf on Double Layers and Other Nonlinear Structures in Plasmas.* Insbruck (Austria). 82 (1992).

[4] A.S. Bakai and V.I. Maslov, Sov. Plasma Physics **13**, 584 (1987).

[5] J.P. Lynov, P. Michelsen, H.L. Pecseli *et al.*, Physica Scripta **20**, 328 (1979).

[6] H. Shamel, Phys. Rep. **140**, 163 (1986).

[7] N. Singh and R.W. Schunk, Plasma Phys. and Control. Fusion **26**, 859 (1984).