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Potential Coherent Structures in Nonuniform Streaming Dusty Magnetoplasma

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Abstract

In this paper we study linear and nonlinear behaviour of modified convective cells and vortices in nonuniform dusty magnetoplasmas with perpendicular and parallel to the magnetic field plasma flows, and in basically two different physical systems, with stationary (corresponding to the case of Shukla-Varma mode) and nonstationary (i.e. taking part in perturbations) dust particles. For the case of stationary dust, by choosing some specific profiles for the sheared plasma flow and the dust density, we analyze the eigenvalue equation in order to deduce the growth rate. A threshold is also obtained for the wavenumber separating spatially damped and convective modes (growing in space) due to its interaction with the sheared plasma flow. In the nonlinear regime, for both stationary and nonstationary dust particles, and in the presence of various plasma flows perpendicular and parallel to the magnetic field lines, a variety of possible nonlinear solutions, driven by the nonuniform shear flow and dust density is presented, i.e., single and double vortex chains accompanied with zonal flows, and tripolar and global vortices.

Keywords:
shear flow, coherent structure, vortex chain, tripole

1. Stationary Dust. The Model and Derivations

We use the model [1] of a nonuniform, three-component electron-ion-dusty plasma with an equilibrium perpendicular (with respect to a uniform magnetic field $B_0\vec{e}_z$ along the z-axis) nonuniform flow (drift) $\vec{v}_0 = v_0(x)\vec{e}_y$. The basic state is then

$$v_0(x) = \frac{1}{B_0}\frac{d\Phi_0(x)}{dx},$$

$$\varepsilon_0 \Phi_0''(x) = e\,[n_{e0}(x) - n_{i0}(x) + Z_d n_{d0}(x)]\,, \tag{1}$$

where Φ_0 is the unperturbed electrostatic potential, $\Phi_0'' = \partial^2\Phi_0/\partial x^2$, n_{j0} is the unperturbed number density of the particle species j (j equals e for the electrons, i for the ions, and d for the dust grains), and Z_d is the unperturbed number of charges residing on the surface of the dust grains which are negatively charged.

From the ion momentum equation, the ion and electron continuity equations, and by using Poisson's equation, we obtain, for $|\partial/\partial t| \ll \Omega_i = eB_0/m_i$, the following expression for the conservation of the charge density:

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0}\vec{e}_z \times \nabla_\perp(\Phi + \Phi_0)\cdot\nabla_\perp\right]$$
$$\left[\alpha(\nabla_\perp^2\Phi + \Phi_0''(x)) - \Phi_0''(x) + \frac{e}{\varepsilon_0}(n_{e0} - n_{i0})\right] = 0\,, \tag{2}$$

or, for the total potential $\Psi = \Phi + \Phi_0$ as

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$$\left[\frac{\partial}{\partial t}+\frac{1}{B_0}\vec{e}_z\times\nabla_\perp\Psi\cdot\nabla_\perp\right]$$
$$\left[\alpha\nabla_\perp^2\Psi-\frac{eZ_d}{\varepsilon_0}n_{d0}(x)\right]=0\,,\quad \alpha=1+\frac{\omega_{pi}^2}{\Omega_i^2}\,. \tag{3}$$

Here the prime denotes derivative with respect to the x-coordinate. The nonlinear equation (2), which describes the evolution of the electrostatic potential in a dusty plasma containing a nonuniform background of stationary dust grains as well as equilibrium density inhomogeneities and sheared plasma flows, is analyzed both in the linear as well as in nonlinear regimes.

2. Linear Analysis

Within the framework of a local analysis, we obtain from Eq. (2) the dispersion equation for the Shukla-Varma (SV) mode [2] that is modified due to the shear plasma flow. In the nonlocal case, for the perturbations of the form $\Phi=\hat{\Phi}\exp(-i\omega t+ik_y y)$, the necessary condition (viz. the Rayleigh type) for the instability can be readily found by assuming a complex frequency $\omega=\omega_r+i\omega_i$, and integrating perpendicular to the flow direction, i.e. in the x direction. We have the Rayleigh condition

$$v_0''(x)-\frac{eZ_d n_{d0}'(x)}{B_0\varepsilon_0\alpha}=0\,, \tag{4}$$

Thus, an absolute instability is possible if at any point x_c in the transverse direction (perpendicular to the direction of the flow), the condition (4) is satisfied. The presence of the dust density gradient modifies the standard condition requiring the existence of an inflection point in the flow profile.

Further linear stability will be discussed more closely for some specific profiles of the shear flow and the dust number density. We suppose that the flow profile is described by the following well behaved analytical function $v_0(x)=u+a\kappa\tanh(\kappa x)$, where κ^{-1} defines the characteristic width (slope) of the shear flow. We now proceed calculating appropriate derivatives of $v_0(x)$ and introducing a new variable $\eta=\tanh(\kappa x)$, and by assuming a physically interesting case when $u=\omega/k_y$. The linear equation for the perturbed potential can be written as:

$$(1-\eta^2)\frac{d^2\hat{\Phi}}{d\eta^2}-2\eta\frac{d\hat{\Phi}}{d\eta}+\left[2-\frac{k_y^2+1}{\kappa^2}\frac{1}{1-\eta^2}\right.$$
$$\left.+\frac{1}{\kappa^2}\frac{1}{1-\eta^2}+\frac{eZ_d}{B_0\varepsilon_0\alpha a\kappa^2}\frac{n_{d0}'(\eta)}{\eta}\right]\hat{\Phi}=0\,. \tag{5}$$

For n_{d0} satisfying $n_{d0}=B_0\varepsilon_0\alpha a\log\left[1-\tanh^2(\kappa x)\right]/2eZ_d$, the solutions of Eq. (5) become Legendre functions of the degree 1, and of the order $\nu=(k_y^2+1)^{1/2}/\kappa$. The only localized solutions for $\eta\to\pm1$, i.e. for $x\to\pm\infty$, are obtained if $\nu^2\equiv(k_y^2+1)/\kappa^2=1$. The wave becomes convectively unstable (growing in space) because of the resonant interaction with the flow, provided that the wavenumber satisfies the condition $k_y>(\kappa^2-1)^{1/2}$. In that case, assuming a small deviation δk_y of the stable value of the wavenumber k_y, and taking $\omega\to\omega_r+i\omega_i$, one can find the following expression for the corresponding growth rate

$$\omega_i=\frac{4\delta k_y(\kappa^2-1)}{\kappa\pi(2\kappa^2-1)}\left|\frac{dv_0}{dx}\right|_{x=0} \tag{6}$$

In the opposite limit, the wave is damped.

Now, for the given profiles of $v_0(x)$ and $n_{d0}(x)$, using the Rayleigh condition we find the following connection between the flow width κ^{-1} and the critical points for the instability $\tanh(\kappa x_c)=(1-1/(2\kappa^2))^{1/2}$. Thus, an absolute instability at the point x_c for modes with wavenumbers satisfying this condition can be expected. Furthermore, it follows that there appears a limiting value $\kappa_L\approx0.71$ above which the critical point x_c becomes complex, i.e., the Rayleigh type instability vanishes.

3. Nonlinear Solutions

For a linear dust density profile $n_{d0}(x)=uB_0\varepsilon_0\alpha x/eZ_d$, nonlinear Eq. (3) can be integrated. Further, we search for solutions of the form $\Psi(x,y)=\Psi_1(x)+\Psi_2(x)\cos(k_y y)$, where $|\Psi_1(x)|\gg|\Psi_2(x)|$, which yields the following set of equations

$$\left(\frac{\partial^2}{\partial x^2}-1\right)\Psi_1(x)=\frac{\beta}{2\cosh[\Psi_1(x)-uB_0x]}\,, \tag{7}$$

$$\left(\frac{\partial^2}{\partial x^2}-k_y^2-1\right)\Psi_2(x)$$
$$=-\beta\Psi_2(x)\frac{\sinh[\Psi_1(x)-uB_0x]}{2\cosh^2[\Psi_1(x)-uB_0x]}\,. \tag{8}$$

Equations (7) and (8) can be solved numerically. Well localized (in the x-direction) solutions in the form of a double vortex chain associated with a strong gradient of the potential in the region between two vortex chains, can be found by solving the above system of equations from the point $x=0$. The strong gradient of the potential Ψ in the central region, around the point $x=0$, corresponds to a spatially narrow and strong zonal flow in the direction perpendicular to the basic state gradients. Consequently, apart from being trapped in the links of the two chains formed laterally to the zonal flow, the plasma particles are additionally prevented to move in the x direction by the zonal flow. In a real experimental situation, such a configuration should represent an efficient barrier for the plasma particle transport in the x-direction, i.e., in the direction of the plasma nonuniformities. Moreover, it can be shown that there exists a class of values of the parameters β and k_y for which the above described solutions are possible. More details regarding this numerical procedure can be found in Ref. [3].

A completely different type of non-propagating, i.e., carried by the flow, nonlinear solutions can be found for the perturbed potential Φ starting from Eq. (2), in the case of a quadratic potential $\Phi_0(x)$, i.e., for the linear shear flow profile $v_0(x) = 2a_0x/B_0 + u_0$, and for a quadratic dust number density distribution $n_{d0}(x) = 2a_0\varepsilon_0\alpha/eZ_d - b_0\varepsilon_0\alpha x^2/eZ_d$, where a_0, b_0 are physical parameters that describe the plasma in the basic state. Integrated Eq. (2) in that case becomes $\nabla_\perp^2\Phi + b_0x^2 = \mathcal{F}(\Phi + a_0x^2)$, where $\mathcal{F}$ is an arbitrary function of the given argument, which we take in the linear form as $\mathcal{F} = F_0 + F_1$, $(\Phi + a_0x^2)$, allowing for the constants $F_{0,1}$ to have different values inside and outside of an arbitrary circle with the radius r_0. Thus, we search for the solutions independently outside and inside of the given circle and match the solutions obtained in that way at the circle. The quadrupolar vortex can be found straightforwardly in the form

$$\Phi_{\rm out}(r,\theta) = d_0\,K_0(\lambda_0 r) + d_2\,K_2(\lambda_0 r)\cos(2\theta)\,,\qquad r > r_0\,, \tag{9}$$

$$\Phi_{\rm in}(r,\theta) = \frac{F_0^{\rm in}}{\lambda^2} - \frac{2(b_0+\lambda^2 a_0)}{\lambda^4} - \frac{b_0+\lambda^2 a_0}{2\lambda^2}r^2 + c_0\,J_0(\lambda r) + \left[c_2\,J_2(\lambda r) - \frac{b_0+\lambda^2 a_0}{2\lambda^2}r^2\right]\cos(2\theta)\,,\qquad r > r_0\,. \tag{10}$$

Here, $F_0^{\rm out} = 0$ and we have introduced $F_1^{\rm out} = b_0/a_0\,\lambda_0^2$, $F_1^{\rm in} = -\lambda^2$, and $K_{0,2}$, $J_{0,2}$ are the modified Bessel and Bessel functions, respectively. The constants of integration $d_{0,2}$, $c_{0,2}$, $F_0^{\rm in}$, $F_1^{\rm in}$, and the parameter r_0 are to be found from the appropriate continuity conditions at the given circle, i.e., from the continuity of the function $\mathcal{F}$, the continuity of the gradient of the potential Φ, and from the assumption that the given circle r_0 is an isoline of the argument of $\mathcal{F}$. These conditions comprise continuity of the potential Φ and the Laplacian $\nabla^2\Phi$ at the given border. Although the analytical expressions (9), (10) consist of monopolar and quadrupolar parts, in fact they represent a tripolar vortex generated in and carried by the linear profile of the shear flow, and for a quadratic dust number density (Fig. 1).

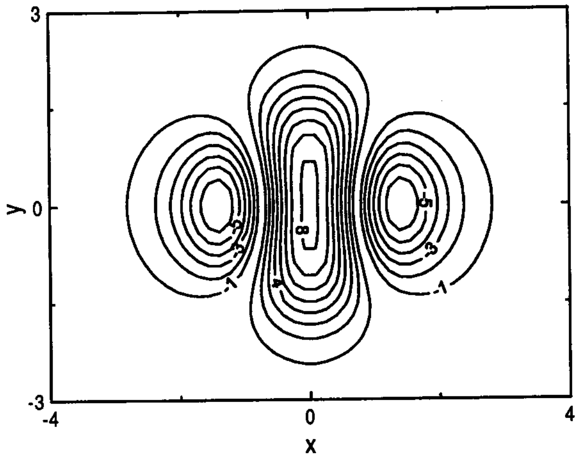

Fig. 1 Tripolar vortex for the perturbed potential Φ, driven and carried by the plasma flow. Lateral vortices have opposite direction of rotation with respect to the vortex core. The step between contours is 1.

4. Dust-Modified Drift Waves

In the previous analysis the dust particles are taken as stationary, i.e., non-taking part in the perturbed motion of the electron-ion fluid. In some limits however, i.e., for very low-frequency processes their motion should be taken into account [4]. Assuming Boltzmann distribution for electrons and using plasma quasineutrality condition, and continuity and momentum equations for ions and dust particles in a cylindrical plasma system with perpendicular flow, we obtain:

$$\left[\frac{\partial}{\partial t} + \vec{e}_z\times\nabla_\perp(\Phi+\Phi_0)\cdot\nabla_\perp\right]\left[(1-\frac{a_0\rho^2}{R^2}\nabla_\perp^2)(\Phi+\Phi_0) - \Phi_0(x) - \xi_0(x)\right] = 0\,, \tag{11}$$

Here we have defined $\xi_0'(x) = -\Omega_i B_0 r^2 n_{e0}'/n_{e0}$, $\rho^2 = T_e/m_i\Omega_i^2$, $\alpha_0 = n_{i0}/n_{e0} + m_d/Z_d m_i$, R is the radius of the plasma column, V_0 is some characteristic shear flow velocity, and we use the normalization:

$$\frac{\partial}{\partial t} \equiv \frac{R}{V_0}\frac{\partial}{\partial t}, \quad \nabla_\perp \equiv R\nabla_\perp,$$

$$(\Phi, \Phi_0, \xi_0) \equiv \frac{(\Phi, \Phi_0, \xi_0)}{B_0 R V_0}.$$

Apart from possible solutions similar to those found in the preceding text, steady state solutions of Eq. (11) can be found after integration which yields the following equation:

$$(\nabla_\perp^2 + k^2)\Psi = \mathcal{F}(r), \tag{12}$$

where k is a constant of integration, and $\mathcal{F}(r) = -R^2(\Phi_0 + \xi_0)/\alpha_0\rho^2$. For a quadratic function $\mathcal{F}$, after some algebra the solution of Eq. (12) can be written as

$$\Psi(r,\theta) = \sum_n c_n J_n(kr)\cos(n\theta) + c_0 J_0(kr) + cr^2 + \text{const.}, \tag{13}$$

where c_0 and c are some constants. Keeping the third harmonic in the summation only, we obtain the contour plot of the vortex that is presented in Fig. 2. The structure resembles experimental result which is obtained in a DC discharge in an unmagnetized dusty plasma [5]. In the latter, the dust vortex flow is generated by a small biased metal plate.

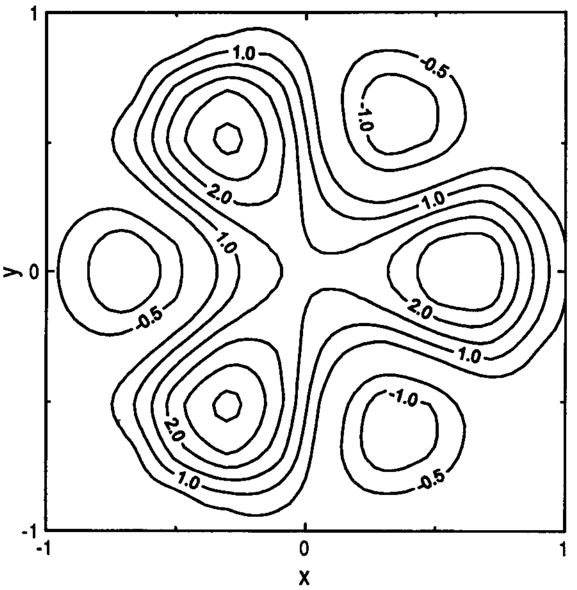

Fig. 2 Global vortex representing a cylindrical chain of counter rotating steady state vortices for the total potential in a cylindrical dusty plasma. The step between contours is 0.5.

5. Dust Drift Waves

In this section we analyse very low-frequency perturbations in dusty plasma with a nonuniform plasma flow $\vec{v}_0 = v_0(x)\vec{e}_z$ which is almost parallel to the lines of a sheared magnetic field $\vec{B}_0 = B_0\vec{e}_z + f(x)B_0\vec{e}_y$, $|f(x)| \ll 1$, and with the dust density gradient in x-direction, assuming Boltzmann distribution for electrons and ions, and using quasineutrality condition. Parallel momentum for dust particles can be written as

$$\left(\frac{\partial}{\partial t} + \frac{1}{B_0}\vec{e}_z \times \nabla_\perp\Phi \cdot \nabla_\perp\right)[v_{z1} + v_0(x)] + v_0(x)\frac{\partial v_{z1}}{\partial z} = \frac{eZ_d}{m_d}\frac{\partial \Phi}{\partial z}. \tag{14}$$

Eq. (14) can be integrated yielding $v_{z1} = F \cdot \Phi$, provided that

$$v_0(x) + f_2(x) = F \cdot [p(x) - B_0 u_y x],$$
$$f_2'(x) = \Omega_d f(x), \quad p'(x) = B_0 v_0(x) f(x). \tag{15}$$

Using Eqs. (14), (15), the dust continuity equation can be written in the form

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0}\vec{e}_z \times \nabla_\perp(\Phi + p(x)) \cdot \nabla_\perp\right] [(1 - \rho_d^2\nabla_\perp^2)\Phi - \Psi(x) - \xi(x)] = 0. \tag{16}$$

Here $\rho_d = C_d/\Omega_d$, $C_d^2 = Z_d^2 T_{ef}/m_d$, $T_{ef} = Z_d^2 n_{d0} T_i T_e / [m_d(n_{i0}T_e + n_{e0}T_i)]$, $\Psi(x) = B_0\Omega_d\rho_d^2 \log n_{d0}$, $\xi(x) = cf_2(x)$, $c = m_d T_{ef} F/e^2$, and F is a constant of integration.

Tripolar vortex solution of Eq. (16), similar to Eqs. (9), (10), and traveling with the velocity u_y in y-direction, can be found for specific profiles of the basic state functions $f(x)$, $v_0(x)$, $n_0(x)$ given by the following equations

$$f(x) = \frac{bx}{B_0 v_0(x)} + \frac{u_y}{v_0(x)},$$

$$v_0'(x) + \frac{\Omega_d}{v_0(x)}\left(\frac{bx}{B_0} + u_y\right) - Fbx = 0,$$

$$[\log n_0(x)]' = \frac{ax}{B_0\Omega_d\rho_d^2} - \frac{c}{B_0\rho_d^2}\left[\frac{bx}{B_0} + u_y\right]\frac{1}{v_0(x)}. \tag{17}$$

For some profiles of the basic state functions a single chain type solution, resembling Kelvin-Stuart cat's eyes, can be written in the form

$$\Phi(x,y) = u_y B_0 x - p(x) + A \log 2\left[\cosh(k_y y) + \left(1 - \frac{1}{\beta^2}\right)^{1/2}\cos(k_y y)\right]. \tag{18}$$

A solution in the form of a double vortex chain can be found provided that

$$f(x)=\frac{c_1}{B_0}\frac{1}{v_0(x)},$$

$$v_0'(x)+\frac{c_1\Omega_d}{B_0}\frac{1}{v_0(x)}+F(B_0u_y-c_1)=0,$$

$$[\log n_0(x)]'+\frac{cc_1}{B_0\rho_d^2}\frac{1}{v_0(x)}=0,$$

where c_1 is some constant.

To conclude, plasma nonuniformities studied here, serving as sources of free energy for linear instabilities, are responsible for the creation of some specific stationary nonlinear solutions, i.e., vortex chains, tripoles and global vortices. Tripolar vortices are novel structures in a dusty magnetoplasma, although they have been found earlier in several experiments with ordinary rotating fluids [6] and they are also observed in the seas of our planet [7].

References

[1] J. Vranješ, D. Petrović and P.K. Shukla, Phys. Lett. A **278**, 231 (2001).

[2] P.K. Shukla and R.K. Varma, Phys. Fluids B **5**, 236 (1993).

[3] J. Vranješ, Phys. Rev. E **58**, 931 (1998).

[4] P.K. Shukla, M.Y. Yu and R. Bharuthram, J. Geophys. Res. **96**, 21343 (1991).

[5] N. Sato, G. Uchida, R. Ozaki and S. Iizuka, in *Physics of Dusty Plasmas*, edited by M. Horányi *et al.*, (AIP, New York, 1998), p. 239.

[6] G.J.F. Van Heijst, R.C. Kloosterziel and C.W.M. Williams, J. Fluid. Mech. **225**, 301 (1991).

[7] R.D. Pingree and B. Le Cann, J. Geophys. Res. **97**, 14353 (1992).