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Effect of Magnetic Field and Plasma Density on Nonlinear Solitary Waves in a Relativistic Plasma

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Abstract

Modified Korteweg-deVries (mK-dV) equation is derived for perturbed potential and solved in order to study the potential structure for the propagation of nonlinear solitary waves (solitons) in a magnetized spatially inhomogeneous plasma having relativistic ions and electrons. It is found that fast and slow modes occur in the plasma and only the fast modes correspond to the solitons. Solitons are possible for a particular range of wave-propagation angle θ (angle between wave vector and direction of the magnetic field) governed by mass ratio (m_i/m_e), temperature ratio (T_i/T_e) and relativistic speeds of ions and electrons. Effect of magnetic field is to decrease the soliton width and energy. The amplitude and energy of soliton get lower and the width gets wider when it propagates into higher plasma density region. The waves propagating at larger angles with the direction of magnetic field correspond to the solitons of higher amplitude and smaller width.

Keywords:

relativistic ion, relativistic electron, potential, mK-dV equation, magnetic field, inhomogeneity, soliton

1. Introduction

Solitonic excitation is one of the most important aspects of the nonlinear phenomena encountered in the plasmas. Since the soliton structure can trap plasma particles and convect them over large distances, they can contribute to anomalous transport of particles and energy from one region to another in laboratory, astrophysical and space plasmas. Therefore, the subject of soliton propagation in plasmas has received a great deal of interest and the nonlinear solitary waves have been extensively studied in relativistic plasmas as well.

Considering only the relativistic speed of ions, Das and Paul [1] derived a Korteweg-deVries (K-dV) equation for a cold plasma and showed the possibility of breaking of ion acoustic wave into many more solitons. Nejoh [2] studied the effect of finite ion temperature on ion acoustic solitary waves in a collisionless relativistic plasma taking the Boltzmann distribution for electrons. Fully relativistic ion fluid equations were also presented to study the double layers, spiky solitary waves and explosive modes considering the Boltzmann distribution for electrons [3]. However, Kuehl and Zhang [4] suggested that in place of electron Boltzmann distribution the finite electron inertia should be taken into account while studying the propagation of solitary waves in a relativistic plasma, since the wave velocity can be comparable to the electron thermal velocity. Later, taking account of finite electron inertia and ion temperature, a limit on ion drift velocity was obtained for existence of the solitons in a relativistic plasma [5]. Recently, the effect of electron inertia has been shown to decrease the phase velocity, amplitude and energy of the solitons in a relativistic plasma [6].

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Soliton characteristics are altered when a soliton propagates in a nonuniform plasma and the K-dV equation describing the soliton behaviour is modified either by varying coefficients or by some additional terms. Assuming that the soliton width is small compared with the scale length of plasma inhomogeneity, arbitrary amplitude soliton propagation in nonrelativistic plasma [7] and small amplitude soliton propagation in relativistic plasma [8] have been studied. It has been shown that the density inhomogeneity considerably modifies the soliton behaviour in relativistic plasmas [8,9].

It can be noted for most of the analyses on relativistic plasmas that only the ions are taken to be relativistic. However, in space plasmas of solar wind [10], boundary layer of the Earth's magnetosphere [11] etc., high-energy ions as well as electrons are found to be streaming with relativistic speeds. Also, under the influence of high-power laser radiation the plasma particles may attain relativistic speeds. Therefore, relativistic effects of electrons should be considered with the ions to understand truly the behaviour of solitary waves in relativistic plasmas. Considering the relativistic effects of both ions and electrons, however, Nejoh and Sanuki [12] studied the large amplitude Langmuir and ion acoustic waves and investigated the conditions for their existence in a relativistic plasma.

The analysis carried out in Ref. [12] with relativistic ions and electrons was restricted to homogeneous and unmagnetized plasma. But invariably we encounter inhomogeneous plasmas under actual conditions, e.g. more so closer to the sheath regions, and the soliton behaviour is also modified under the effect of magnetic field [13-15]. Therefore, in the present paper, we study the effect of magnetic field and plasma density on the soliton propagation in a plasma having both relativistic ions and electrons. Using reductive perturbation technique, we derive a modified K-dV equation for the perturbed potential and find that it admits the potential structure of compressive solitons for a particular range of wave-propagation angle.

2. Phase Velocity and Limit on Wave-Propagation Angle

We consider a magnetized collisionless spatially weakly inhomogeneous plasma consisting of relativistically streaming warm ions and hot electrons. A small amplitude ion acoustic wave is assumed to propagate in (x,z) plane at an angle θ with the direction of magnetic field $\boldsymbol{B}$ ($= B_0 z$). The ratio β of particle pressure to magnetic field pressure is taken to be small. Kinetic effects such as Landau damping, heat conduction, viscosity etc. are neglected. The electron Debye length is taken to be much smaller than the dimensions of the system and the plasma is quasineutral. Under these conditions, the following basic fluid equations are obtained in normalized form.

For ion fluid:

$$n_t + (nv_x)_x + (nv_z)_z = 0 \tag{1a}$$

$$(\gamma_i v_x)_t + v_x(\gamma_i v_x)_x + v_z(\gamma_i v_x)_z + \phi_x - (\Omega_i/\omega_{pi})v_y + 2\sigma n_x/n = 0 \tag{1b}$$

$$(v_y)_t + v_x(v_y)_x + v_z(v_y)_z + (\Omega_i/\omega_{pi})v_x = 0 \tag{1c}$$

$$(v_z)_t + v_x(v_z)_x + v_z(v_z)_z + \phi_z + 2\sigma n_z/n = 0 \tag{1d}$$

For electron fluid:

$$(n_e)_t + (n_e u_x)_x + (n_e u_z)_z = 0 \tag{1e}$$

$$(m_e/m_i)\,[(\gamma_e u_x)_t + u_x(\gamma_e u_x)_x + u_z(\gamma_e u_x)_z] - \phi_x + (\Omega_i/\omega_{pi})u_y + (n_e)_x/n_e = 0 \tag{1f}$$

$$(m_e/m_i)\,[(u_y)_t + u_x(u_y)_x + u_z(u_y)_z] - (\Omega_i/\omega_{pi})u_x = 0 \tag{1g}$$

$$(m_e/m_i)\,[(u_z)_t + u_x(u_z)_x + u_z(u_z)_z] - \phi_z + (n_e)_z/n_e = 0 \tag{1h}$$

Poisson's equation:

$$\phi_{xx} + \phi_{zz} - n_e + n = 0 \tag{1i}$$

In these equations, ion and electron densities are normalized by the unperturbed plasma density n_0, fluid velocities by the ion acoustic speed, potential by KT_e/e, where K is the Boltzmann constant. Time and space are normalized by the ion plasma period and electron Debye length, respectively. Subscripts t, x and z represent the differentiation. Ω_i is ion gyrofrequency and ω_{pi} is ion plasma frequency and their ratio $\Omega_i/\omega_{pi} = B_0\sqrt{\varepsilon_0/n_0 m_i}$. Ion to electron temperature ratio (T_i/T_e) is taken as σ. $\gamma_i = (1-v_0^2/c^2)^{-1/2}$ and $\gamma_e = (1-u_0^2/c^2)^{-1/2}$, with v_0 and u_0 as the ion and electron streaming speeds in x-direction with weak relativistic effects, i.e. v_0, $u_0 \ll c$. For the sake of simplicity, we have assumed that the ion and electron speeds are relativistic only in the x-direction. It can be noted from ion momentum equation (1b) that the specific heat ratio for adiabatic ions is taken as 2, since the number of degrees of freedom for the present case is 2. Further, the continuity and momentum equations are used for electron fluid, since the electrons are taken as relativistically streaming species.

For employing the reductive perturbation technique, we use the following set of stretched coordinates [8,14,16].

$$\xi = \varepsilon^{1/2}(\boldsymbol{k}\cdot\boldsymbol{r}/\lambda_0 - t) = \varepsilon^{1/2}(x\sin\theta/\lambda_0 + z\cos\theta/\lambda_0 - t);$$

$$\iota = \varepsilon^{3/2}(\boldsymbol{k}\cdot\boldsymbol{r}) = \varepsilon^{3/2}(x\sin\theta + z\cos\theta) \qquad (2)$$

Here, λ_0 is the phase velocity of ion acoustic wave in (ξ,ι) space and ε is a small dimensionless expansion parameter. $\boldsymbol{k}$ is unit vector along the direction of wave propagation in (x,z) plane making an angle θ with the direction (z-axis) of magnetic field.

The quantities n, n_e, v, u and ϕ are expanded around the equilibrium state in terms of ε and their expansion is given by

$$f = f_0(x,z) + \varepsilon f_1(x,z,t) + \varepsilon^2 f_2(x,z,t) + \ldots,$$
$$g = \varepsilon g_1(x,z,t) + \varepsilon^2 g_2(x,z,t) + \ldots,$$
$$h = \varepsilon^{3/2} h_1(x,z,t) + \varepsilon^2 h_2(x,z,t) + \ldots,$$
$$s_x = s_0 + \varepsilon^{3/2} s_{x1}(x,z,t) + \varepsilon^2 s_{x2}(x,z,t) + \ldots, \qquad (3)$$

where, $f \equiv (n, n_e, \phi)$, $g \equiv (v_z, u_z)$, $h \equiv (v_y, u_y)$ and $s = v$ or u.

Now we use the stretching [Eq.(2)] and expansion of dependent quantities [Eq.(3)] in basic fluid equations [Eqs.(1)] and find a set of equations in different orders of ε. The equations obtained in first-order quantities n_1, n_{e1}, ϕ_1, v_{x1}, v_{y1}, v_{z1}, u_{x1}, u_{y1} and u_{z1} yield the following phase velocity relation for the ion acoustic wave.

$$\lambda_0 = (v_0 + u_0 m_e/m_i)\sin\theta \pm\sqrt{(1+2\sigma)\cos^2\theta - m_e(v_0-u_0)^2\sin^2\theta/m_i} \qquad (4)$$

This equation indicates that the fast mode, corresponding to plus sign, and the slow mode, corresponding to minus sign, can occur in the plasma. Also, the phase velocity depends on ion and electron streaming speeds (relativistic effects), their mass and temperature ratios and angle of wave propagation. The phase velocity increases with increasing ion streaming speed and wave-propagation angle, but it decreases for higher values of electron streaming speed.

For real phase velocity, the quantity appearing in the square root term of Eq.(4) should be positive, which reads

$$\tan^2\theta \le m_i(1 + 2\sigma)/m_e(v_0 - u_0)^2 . \qquad (5a)$$

For positive λ_0, the first term of R.H.S. of Eq.(4) should be greater than the second term. Considering this point, we find the following condition for positive λ_0.

$$\tan^2\theta > (1 + 2\sigma)/(v_0^2 + u_0^2 m_e/m_i) \qquad (5b)$$

On the basis of Eqs.(5a) and (5b), we can say that the ion acoustic wave will propagate if the following inequality is satisfied in the plasma conditions.

$$(1 + 2\sigma)/(v_0^2 + u_0^2 m_e/m_i) < \tan^2\theta \le m_i(1 + 2\sigma)/m_e(v_0 - u_0)^2 \qquad (6)$$

Now we use some typical values of v_0, u_0 and σ to find the limit on wave-propagation angle θ for existence of the wave in the plasma. For a relativistic space plasma [5,17], we select $v_0 = 70$, $u_0 = 35$, $\sigma = 0.04$ for $m_i/m_e = 1836$ and find that $0.85^0 < \theta \le 51.83^0$. Therefore, it is deduced that the fast and slow modes propagate in the plasma when the angle θ falls in the range assigned by Eq.(6).

3. Modified Korteweg-de Vries Equation in Perturbed Potential

The reductive perturbation treatment gives different equations in second-order quantities n_2, n_{e2}, ϕ_2, v_{x2}, v_{y2}, v_{z2}, u_{x2}, u_{y2} and u_{z2}. Elimination of these quantities with the help of phase velocity relation and equations in first-order quantities yields the following modified Korteweg-deVries (mK-dV) equation in perturbed potential ϕ_1.

$$\phi_{1_\iota} + \alpha\phi_1\phi_{1_\xi} + \beta\phi_{1_{\xi\xi\xi}} - a_1\xi\phi_{1_\xi}n_{0_\iota} - a_2\xi n_{0_{\iota\iota}} = 0 \qquad (7)$$

The coefficients appearing in the mK-dV equation (7) are given by

$$\alpha = \{C[D^2 - (m_i/m_e)G]\cos^2\theta\}/2\lambda_0^2(v_0 - u_0)DE\sin\theta$$

$$\beta = \{CD[b^2E(m_i/m_e) + (m_i/m_e)n_0C^2(\sin^2\theta + \gamma_i\cos^2\theta) - n_0G(\sin^2\theta + \gamma_e\cos^2\theta)]\}/2b^2(m_i/m_e)\lambda_0^4 n_0(v_0 - u_0)\sin\theta\cos^2\theta$$

$$a_1 = [FCD^2 + F(m_i/m_e)GC - 4\sigma\lambda_0 D^2\cos^2\theta - 2CDEu_0\sin\theta]/2\lambda_0 n_0(v_0 - u_0)DE\sin\theta$$

$$a_2 = \lambda_0[F(v_0 - u_0)CD\sin\theta$$

$$- C^2(Eu_0\sin\theta - FD) - 2\sigma\lambda_0 D^2\cos^2\theta]/$$
$$2n_0CD(v_0 - u_0)\sin\theta\cos^2\theta$$

Here, $b = \Omega_i/\omega_{pi}$; $C = \lambda_0 - v_0\sin\theta$; $D = \lambda_0 - u_0\sin\theta$; $E = C^2 - 2\sigma\cos^2\theta$; $F = 2\sigma\cos^2\theta + Cv_0\sin\theta$; $G = (1 + 2\sigma)\cos^2\theta - C^2$.

Now we solve the mK-dV equation in order to study the structure of perturbed potential ϕ_1. The use of transformation $\zeta = \xi - Vt$, where V is a constant, gives the following solution of Eq.(7) at constant density gradient [8,9].

$$\phi_1(\zeta) = [3(V + a_1\xi n_{0_t})/\alpha]\mathrm{sech}^2\{\zeta\sqrt{(V + a_1\xi n_{0_t})/4\beta}\} \qquad (8)$$

Inspection of Eq.(8) reveals that it describes a potential structure which reaches a peak at $\zeta = 0$ and vanishes at $\zeta \to \pm\infty$, with the peak value $= 3(V + a_1\xi n_{0_t})/\alpha (= \phi_m$, say) and half width $= \sqrt{4\beta/(V + a_1\xi n_{0_t})}$ (= L, say). This structure is known as soliton with amplitude (ϕ_m and width L. Soliton energy E_s [8] can be calculated using Eq.(8) and we find that $E_s = 4\phi_m^2L/3$.

4. Results and Discussion

It is observed for slow ion acoustic mode that the soliton width is not real, since the dispersion coefficient β attains negative values. Therefore, the soliton propagation is not possible for the slow mode. However, β is always positive for the fast ion acoustic mode and thereby gives the real values of soliton width. Further, the nonlinearity coefficient α is positive for the fast mode and therefore it is concluded that the compressive solitons are possible only for the fast mode. For equal speed of ions and electrons (i.e. $v_0 = u_0$), it is observed that α, β and $a_1 \to \infty$ which make the soliton amplitude and width indeterminate, indicating that the soliton cannot occur in the plasma having ions and electrons of equal speeds.

Effect of plasma density n_0 on the soliton amplitude ϕ_m and soliton width L is shown in Fig.1, for different values of relativistic electron speed u_0/c. This figure shows that the soliton amplitude gets lower and its width gets wider for increasing values of n_0. This can be explained on the basis of coefficients α, β and a_1. Numerical calculations reveal that β and a_1 decrease with increasing n_0, but α remains constant. Further, the decrease in a_1 is faster than the decrease in β, which causes the soliton amplitude to decrease and the width to increase for higher values of plasma density. Since the soliton energy E_s is directly proportional to ϕ_m^2, it will also decrease with increasing plasma density. The effect of relativistic electron speed u_0/c is found to decrease the soliton amplitude and to increase the soliton width (Fig.1). However, we observe on the basis of expressions for soliton characteristics that the soliton behaviour with relativistic ion speed is opposite to that with the electron speed.

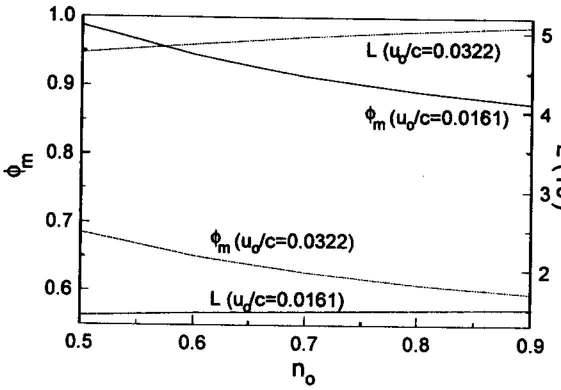

Fig.1 Effect of plasma density n0 on the soliton amplitude ϕ_m and soliton width L. Here, v_0/c = 0.0161 (for u_0/c = 0.0322), v_0/c = 0.0322 (for u_0/c = 0.0161), n_0 = 0.8, n_{0_t} = 0.002, ξ = 0.01, V = 0.0009, b = 0.0015 ($B_0 = 5.8 \times 10^{-7}$T), σ = 0.04 and $\theta = 20^0$.

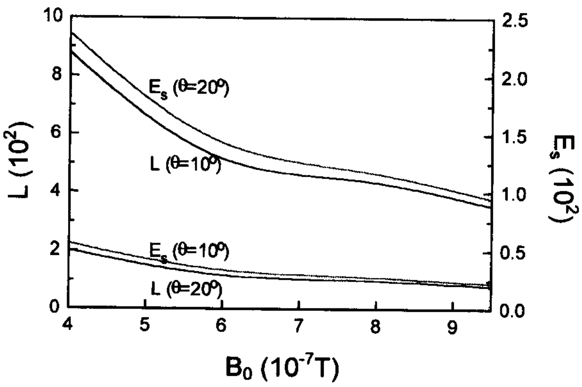

Fig. 2 Effect of magnetic field B_0 on the soliton width L and soliton energy E_s. Here, v_0/c = 0.0322, u_0/c = 0.0161 and other parameters are same as in Fig.1.

Effect of magnetic field B_0 on the soliton characteristics is shown in Fig.2, for different values of wave-propagation angle θ. It is clear from the figure that the soliton propagates with smaller width (L) and lower energy (E_s) under the effect of stronger magnetic field. This can be seen from the expressions of α, β and a_1 that only the dispersion coefficient β is a function of b. It means β depends on the magnetic field, since $b = \Omega_i/\omega_{pi} = B_0\sqrt{\varepsilon_0/n_0m_i}$. It is observed that the dispersion coefficient decreases for higher values of B_0, showing that the soliton experiences less dispersion in the plasma under the effect of stronger magnetic field and therefore the soliton width is decreased with increasing B_0. The soliton energy also decreases with increasing B_0, since it is directly proportional to width. Further, Fig.2 shows that the soliton width becomes smaller and the energy gets higher for increasing wave-propagation angle θ. On the basis of numerical calculations, it is found for increasing θ that the soliton grows with higher amplitude.

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