

# Theoretical Estimate of Electron Energies Accelerated in an Oblique Shock Wave

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## Abstract

Electron acceleration by a magnetosonic shock wave propagating obliquely to a magnetic field is studied by means of a one-dimensional, relativistic, electromagnetic particle simulation code with full ion and electron dynamics. It is demonstrated by simulations that the shock wave can produce ultrarelativistic electrons with their Lorentz factors exceeding 100. Detailed physical considerations for this phenomenon are then given, which clearly indicate that at certain propagation angles the acceleration can become especially strong.

## Keywords:

particle acceleration, ultrarelativistic electron, shock wave, potential formation, particle simulation

## 1. Introduction

High-energy electrons, as well as ions, are often observed in astrophysical plasmas. For instance, in solar flares, electrons can be accelerated to several tens of MeV [1]. From the observations of X-rays and gamma rays, it is now believed that electrons with energies up to  $\sim 100$  TeV are produced by supernova remnant SN1006 [2,3]. Also, in the Crab Nebula high-energy electrons accelerated up to  $\sim 100$  TeV are produced by the pulsar wind [4].

Motivated by these observations, particle acceleration in nonlinear magnetosonic waves has been extensively studied. Even though various ion acceleration mechanisms have been shown by particle simulations [5-15], strong electron acceleration has not been reported.

Quite recently, however, simulations have demonstrated that a magnetosonic shock wave propagating obliquely to a magnetic field can produce ultrarelativistic electrons [16,17]. In this acceleration mechanism, large electric potential formed in the shock wave plays an important role.

In this paper, we study this phenomenon with theory and particle simulations. In Sec. 2, we show simulation results. In Sec. 3, we will discuss the acceleration mechanism in detail. An electron reflected near the end of the main pulse obtains a large amount of energy from both the electric potential and constant electric field appearing in the wave frame. At certain shock propagation angles, speeds of the reflected electrons relative to the shock wave can be quite small; the electrons can then travel a long distance in the direction parallel to the wave front and perpendicular to the external magnetic field. This causes particularly strong electron acceleration.

## 2. Simulation

To study the shock propagation and electron acceleration, we use a one-dimensional (one space coordinate and three velocity components), relativistic, electromagnetic, particle simulation code with full ion and electron dynamics. The shock wave propagates in the  $x$  direction in an external magnetic field  $B_0$  in the  $(x,$

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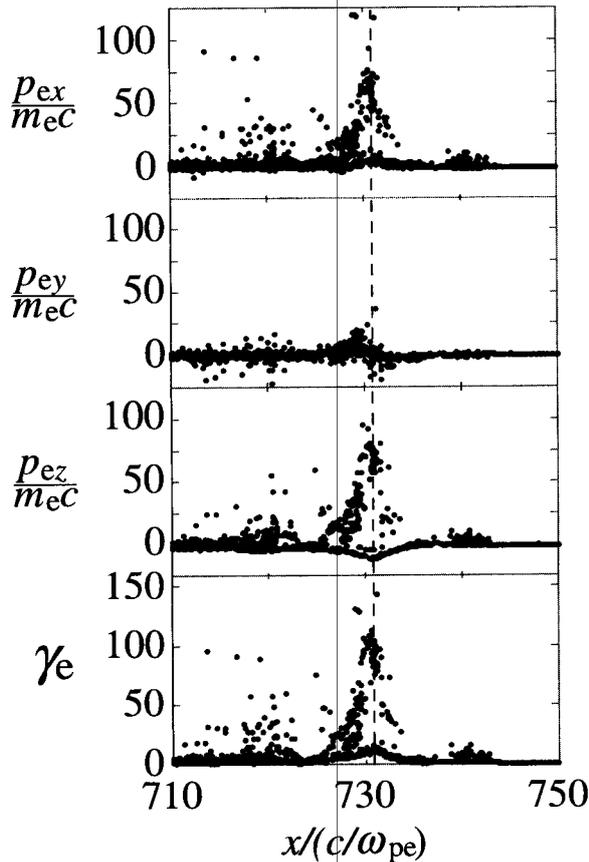


Fig. 1 Phase space plots of electrons. The dotted line shows the position  $x = x_m$ , at which  $B_z$  and  $\phi$  take their maximum values.

$y$ ) plane; for the definiteness, we assume that  $B_{x0} > 0$  and  $B_{z0} > 0$ . The grid size is 4,096; the total numbers of particles are  $N_e = N_i = 262,144$ ; the mass ratio is  $m_i/m_e = 100$ ; the frequency ratio  $\omega_{ce}/\omega_{pe}$  is 3 in the upstream region; and the velocity ratios are  $v_A/c = 0.3$ ,  $v_{Te}/c = 0.1$ , and  $v_{Ti}/c = 0.01$ . The shock propagation angle is  $\theta = 45^\circ$ , where  $\tan\theta = B_{z0}/B_{x0}$ .

Figure 1 displays electron phase space plots  $(x, p_{ex})$ ,  $(x, p_{ey})$ ,  $(x, p_{ez})$ , and  $(x, \gamma_e)$  for a shock wave with a propagation speed  $v_{sh} = 2.26v_A$ ; here,  $p_e$  is the electron momentum and  $\gamma_e$  is the Lorentz factor. The dotted vertical line represents the location where the magnetic field  $B_z$ , transverse electric field  $E_y$ , and the electric potential  $\phi$  take their maximum values; we denote this position by  $x_m$ . At this location, some electrons have very high energies such that  $\gamma_e > 100$ . Figure 2 shows phase space plots  $(x, \gamma_e)$  at various times. Some electrons are reflected near the end of the main pulse and move in the positive  $x$  direction; their energies are increasing. At  $x = x_m$ ,  $\gamma_e$  becomes the maximum. It then

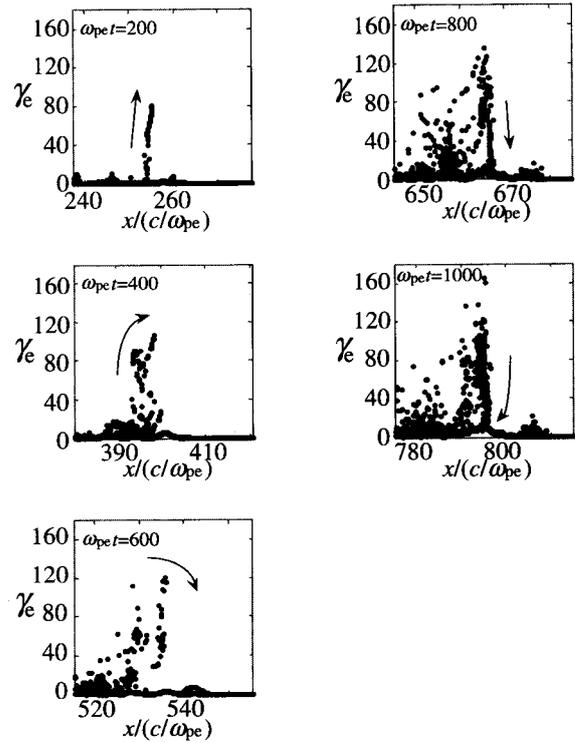


Fig. 2 Phase space plots of electrons,  $(x, \gamma_e)$ , at various times.

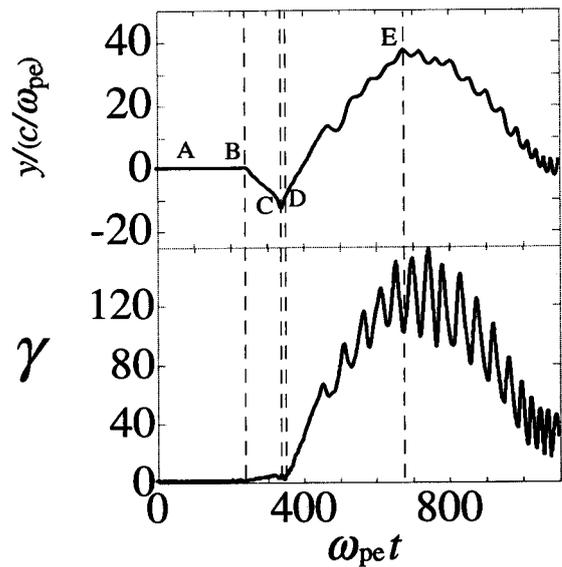


Fig. 3 Time variations of  $y$  and  $\gamma$  of an accelerated electron.

decreases rapidly and the electrons move back to the point where they were reflected. They repeat this oscillatory motion. Figure 3 shows time variations of  $y$

and  $\gamma$  of an accelerated electron. These quantities as well as  $x$  are oscillating after the first reflection. Near point E where the electron is at  $x = x_m$ , both  $y$  and  $\gamma$  take their maximum values.

### 3. Physical Considerations

We now discuss the mechanism of the acceleration. We show in Fig. 4 a schematic diagram of the trajectory of electron guiding center drawn in the wave frame. Here, the reflection takes place at point D; the dotted line shows the orbit of a passing electron. As an electron moves from point A to C, it moves in the negative  $y$  direction because of the  $E_x \times B_z$  drift. It gains kinetic energy  $\Delta E_1$  from the electric potential

$$\Delta E_1 = e\phi(x_C) - e\phi(x_A) \quad (>0). \quad (1)$$

At the same time, it loses energy  $\Delta E_2$  because of the electric field  $E_{y0}$

$$\Delta E_2 = -eE_{y0}(y_C - y_A) \quad (<0). \quad (2)$$

The net change in the energy is therefore

$$\Delta E = \Delta E_1 + \Delta E_2. \quad (3)$$

Even though the magnitudes of  $\Delta E_1$  and  $\Delta E_2$  are quite large, they almost cancel when an electron moves with drift approximation; in particular, in a perpendicular pulse they have exactly the same magnitude and  $\Delta E = 0$ . However, if an electron is reflected and moves from D to E, then it would gain energies from both  $E_x$  and  $E_{y0}$ . As a result, the increase in energy,

$$\Delta E = \Delta E_1 + \Delta E_3, \quad (4)$$

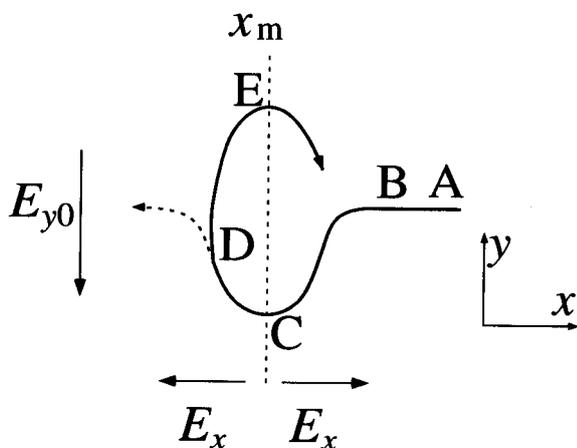


Fig. 4 Schematic diagram of electron guiding center orbit.

can be quite large, where  $\Delta E_3$  is defined as

$$\Delta E_3 = -eE_{y0}(y_E - y_A) \quad (>0). \quad (5)$$

Next, we show a theoretical expression for the maximum energy that electrons can obtain and then discuss another important process in the acceleration mechanism. From the energy equation for an electron particle,

$$m_e c^2 \frac{d\gamma_e}{dt} = -eE_x v_{ex} - eE_{y0} v_{ey}, \quad (6)$$

and from the calculations of wave structure, we find the maximum electron energy as

$$\gamma_{em} = \frac{eF_m + m_e \gamma_{e0} c^2 - m_e c (v_{sh} B_{z0} / c B_{x0}) \gamma_{e0} v_{ez0}}{m_e c^2 [1 - (v_{sh} B_{z0} / c B_{x0}) (B_{zm} / B_m)]}, \quad (7)$$

where the subscript 0 refers to the values in the far upstream region. The quantity  $F_m$  is the maximum value of  $F$ , which is defined as

$$F = - \int E_{\parallel} \frac{B}{B_{x0}} dx, \quad (8)$$

with  $E_{\parallel}$  being the electric field parallel to the magnetic field. The quantity  $F$  takes its maximum value  $F_m$  at  $x = x_m$ . An electron reflected near the end of the main pulse can have this amount of energy at the location where  $B_z$  takes its maximum value.

Equation (7) indicates that  $\gamma_{em}$  can be extremely large when the denominator

$$h = m_e c^2 [1 - (v_{sh} B_{z0} / c B_{x0}) (B_{zm} / B_m)], \quad (9)$$

is close to zero. We discuss the physical meaning of  $h \rightarrow 0$ .

The guiding center velocity  $v_g$  of an electron may be written as

$$v_g \approx v_d + v_{\parallel} B / B, \quad (10)$$

where  $v_{\parallel}$  is the velocity along the magnetic field and  $v_d$  is the drift velocity, which can be approximated as

$$v_d \approx c E \times B / B^2, \quad (11)$$

in the present case. The  $x$  component of  $v_g$  is therefore given by

$$v_{gx} = \frac{c E_{y0} B_z}{B^2} + v_{\parallel} \frac{B_{x0}}{B}. \quad (12)$$

The first term on the right-hand side is always negative, while the second term is positive if  $v_{\parallel} > 0$ . Usually,  $v_{\parallel}$  is negative in the wave frame [16]. However, reflected electrons have positive  $v_{\parallel}$  and have positive  $v_{gx}$ . In this case, the particle can move from point D to point E. If

the condition

$$\frac{cE_{y0} B_z}{B^2} + c \frac{B_{x0}}{B} \sim 0, \quad (13)$$

holds (here,  $v_{\parallel}$  is approximated by  $c$ ), then  $v_{gx}$  becomes quite small. Substitution of the relation  $E_{y0} = -v_{sh} B_{z0}/c$  in Eq. (13) yields

$$1 - \frac{v_{sh}}{c} \frac{B_z}{B} \frac{B_{z0}}{B_{x0}} \sim 0. \quad (14)$$

This gives  $h \sim 0$ ; note that  $B_{zm} \sim B_m$  for a large-amplitude shock wave. That is,  $h$  becomes close to zero when  $v_{gx} \sim 0$ . The condition  $h \sim 0$  can be rewritten as

$$c \cos \theta \sim v_{sh}. \quad (15)$$

If  $v_{gx}$  is small, then it takes a very long time for the particle to move from  $x_D$  to  $x_E$  (or from point D to point E). Because the  $y$  component of  $v_d$  is given by

$$v_{dy} = \frac{cE_z B_{x0}}{B^2} - \frac{cE_x B_z}{B^2}, \quad (16)$$

(in the wave frame, the first term vanishes because  $E_z = 0$ ), the particle moves in the  $y$  direction during this period by

$$\Delta y \sim -c \int_{t_D}^{t_E} cE_x B_z B^{-2} dt. \quad (17)$$

The particle gains energy from  $E_{y0}$  by  $-eE_{y0}\Delta y$ . It can be quite large if the time ( $t_E - t_D$ ) is long. Hence, when  $v_{gx} \sim 0$  (or  $h \sim 0$ ), reflected electrons can have large energies.

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