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A Mechanism of Producing Radial Electric Field

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Abstract

A different physical mechanism of producing a radial current in the plasma is presented here that predicts the time evolution E_r. A two-dimensional Fokker-Planck equation was used for investigation about radial electric field problem. The equation is solved by using the Green's functions and adjoint method.

Keywords:

plasma, tokamak, radial electric field, radial current, H mode, Fokker-Planck equation, Green's functions, adjoint method

1. Introduction

It is well known that a radial electric field is a key factor in the Low-mode to High-mode (L-H) transition in a tokamak. Stabilizing of turbulence by sheared $E \times B$ flow [1,2] is a good working hypothesis to explain the turbulence reduction and confinement improvement that is seen at the plasma edge at the L to H transition. Theories of E_r generation at the plasma edge mostly focus on increase in the main ion pressure gradient or greater main ion poloidal rotation in the electron diamagnetic drift direction. In order to have better theories to compare with experiment, we need theories that can predict the spatial structure and time evolution of E_r.

A different physical mechanism producing a radial current in the plasma is presented here that predicts the spatial and time evolution of E_r, but the problem presently is solved for the time evolution. It is evident that a toroidal electric field can produce a toroidal current, but there is not any work about possibility of producing a radial current by a toroidal electric field. There is only a preliminary work [3] that is assumed a dielectric medium between the plasma and tokamak chamber, where it leads to a capacitance C. This is analogous the coaxial cables problems. This capacitance C represents a radial electric field. By using a RLC model is shown that this radial electric field appears when the toroidal electric field is changed. Here question is: How a toroidal electric field can produce a radial current? To find a reply for this question we shall solve a Fokker-Planck equation by using the Green's functions and adjoint method. By solving the equation with suitable initial and boundary conditions, we will derive the time evolution of radial current density and the situation for producing a radial electric field. It is well-known that a radial electric field is generated by a radial current. Usually a noninductively driven current in tokamaks causes a distortion f_1 in distribution function and drop in loop voltage (it means a drop in toroidal electric field). The H mode is seen in these regimes. Here we try to get the current carried by f_1 in radial direction. Although here our main emphasis will be on rf heating regimes, but this method can be extended to other regimes, too.

2. Fokker-Planck Equation

The evolution of the electron distribution function is described by the Fokker-Planck equation. By substituting $f = f_m + f_1$ into the Boltzman equation for the

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electron distribution function f, the linearized Fokker-Planck equation that will occupy our attention may be written as

$$\frac{\partial}{\partial t}f_1+\frac{qE}{m}\cdot\frac{\partial}{\partial v}f_1-C(f_1)=-\frac{\partial}{\partial v}\cdot S-\frac{qE}{m}\cdot\frac{\partial}{\partial v}f_m \quad (1)$$

where we neglect spatial derivatives, $f_m = n(m/2\pi T)^{3/2}\exp(-\varepsilon/T)$ and $S(v, t)$ is wave-induced flux. Here q and m are the electron charge and mass, respectively and ε is the energy of an electron. Please note here f_1 is a perturbation in distribution function f that can be resulted from an additional heating such as it occurs in rf wave heating. The notational convenience $C(f_1) = C(f_1, f_m) + C(f_m, f_1) + C(f_1, f_i)$ is the linearized collision operator. Initially, at high speed, the current carried by electrons is substantial; when they slow down they carry much smaller current and, because they are colliding frequently by then, even this small current persists only for a very short time. Therefore it is a very good approximation in regimes such as rf heating to assume that the collisions always take place in the high-velocity limit, meaning $v \gg v_T$, where we can simplify

$$C(f)\equiv\Gamma\left[\frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{v_T^2}{v}\frac{\partial f}{\partial v}+f\right)+\frac{(1+Z)-v_T^2/2v^2}{2v^3}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu}f\right] \quad (2)$$

where $\mu = v_\parallel/v$, $\Gamma \equiv nq^4 \ln\Lambda/4\pi\varepsilon_0^2m^2$, $v_T^2 = T/m$ and ε_0 is the dielectric constant of free space; $\ln\Lambda$ is the Coulomb logarithm, and Z is the effective ion charge state.

Instead of solving the Fokker-Planck equation, we solve an adjoint equation for the Green's function. The main physical idea is that in many applications there is no need for complete information on the particle distribution function. Therefore a complete solution of the Boltzman kinetic (or Fokker-Planck) equation is likewise unnecessary. In most applied problems do not require knowledge of the distribution function $f(v, t)$, since all their requirement is the knowledge of several moments of $f(v, t)$. Since in most promising current-generation methods the distribution function of the bulk particles remains Maxwellian, the problem can be linearized by putting $f = f_m(1 + h)$. The radial density of current J_r is expressed by

$$J_r(t)=\int d^3v\, f_m(v,t)h(v,t)qv_\perp \quad (3)$$

According to the references [4,5], in direction of applying adjoint method we first define a commutative operation on the two functions $h(v, t')$ and $g(v, t')$

$$[h,g]_t\equiv\int_V d^3v\int_0^t h(v,t-\tau)g(v,\tau)d\tau \quad (4)$$

We also introduce notation

$$j(v,t,t')=\int d^3v' g(v',v,t,t')qv_\perp \quad (5)$$

The function $j(v, t, t')$ has a meaning of the influence function for the moment J. The corresponding influence function j is the solution of the following equation

$$\frac{\partial j}{\partial t}+\frac{eE_\parallel(t)}{m}\frac{\partial j}{\partial v_\parallel}+C^*(j)=0 \quad (6)$$

where C^* is the operator adjoint to C and may be written as $C^* = C(f_mj)/f_m$. The current density J will be

$$J=\int_0^t dt'\int d^3v\left(S+\frac{eE}{m}f_m\right)\frac{\partial j}{\partial v} \quad (7)$$

We shall solve the equation (6) in a large domain $v = 10v_r$ with initial condition $j(t = 0) = qv_\perp$ and boundary condition $j = qv_\perp$ where v_r is the runaway velocity $v_r \equiv -\,sign(qE)\sqrt{m\Gamma/|qE|}$.

Please note that in the general case Eq. (7) does not make it possible to calculate the current density J_r, even if the solution of Eq. (6) for $j(v)$ is known. The point is that the particle flux $S(v, t)$ under the integral sign in Eq. (7) depends on the unknown solution $f = f_m + f_1$. Here please note in difference between equations (1) and (6). Equation (1) describes the evolution of a group of electrons released at $t = 0$ at velocity v, but $j(v, t)$ in Eq. (6) gives the mean current carried by those electrons at time t later. This current is carried by f_1. How Eq. (6) works is easily seen for toroidal current (it means $j(t = 0) = qv_\parallel$)) by taking $v \gg v_r$ so that the electron only experiences the electric field. In the Boltzman equation (1) the electrons have slowed down to $v - (\Gamma/v_r^2)t$ at time t. Correspondingly in the adjoint equation (6), the initial condition $j_0 = j(t = 0)$ is transported in the reversed direction so that $j(v, t) = j_0(v - (\Gamma/v_r^2)t)$. But here our problem is radial current that the electrons in radial direction can not be affected from toroidal electric field in $v \gg v_r$, so $j(v, t) = j_0$. Although the solution of Eq. (6) and obtaining the Green's function j can give us a good sense about the radial current, but for to get some exact results we show solution of the problem for special case of lower-hybrid waves, too. For lower-hybrid waves the wave-induced flux will be in the toroidal direction $\vec{e}_\parallel$ and the waves interact with particles through landau resonance $\omega - k_{\|}v_{\|} = 0$, where ω and $k_{\|}$ are the wave frequency and parallel wave number. Furthermore, the typical perpendicular velocity of the resonant electrons equals the electron thermal velocity, so that $v_{\perp} \approx v_T \ll v_{\|}$. If we use the definition of $P_d = \int_V d^3v S \cdot (\partial \varepsilon / \partial v)$, where P_d is the absorbed power per unit volume, for narrow waves we can use the following relation

$$J = \frac{P_d}{m} \int_0^t dt' \frac{\partial j / \partial v}{v} \tag{8}$$

where Eq. (8) gives the radial current that must be evaluated with $v_{\|} = \omega / k_{\|}$ and $v_{\perp} = v_T$.

3. Numerical Results

Here we present numerical results. We have solved equation (6) by using a finite element code. Firstly we solve Eqs. (6) and (8) for $E_{\|}$ = const. In this case the radial current J_r will not be produced. In next step we solve Eqs. (6) and (8) for $E_{\|} = E_0(1 + \alpha t)$. In these cases we will see a producing of the radial current. All figures are plotted for $\alpha = 10(\Gamma/v_r^3)t$, and the quantity of T in figures is defined by $T = (\Gamma/v_r^3)t$. The axis of $v_{\|}$ and $v_{\perp}$ show the velocities in toroidal and perpendicular (radial) direction. Figures 1(a), (b), (c) and (d) show the surface of the Green's function of radial current density for times $T = (\Gamma/v_r^3)t = 0$, 0.0002 and 0.0003. Fig. 1(d) is same as Fig. 1(c), only it is zoomed. One may see appearing of the radial current in Figs 1(c) and (d).

Time evolutions of Green's function of the radial current density for times in range of $0 < T < 0.00075$ is shown in Fig. 2 for different velocities. Since the perturbation f_1 is carried in toroidal direction, such as rf heating, usually the parallel velocity $v_{\|}$ is very higher

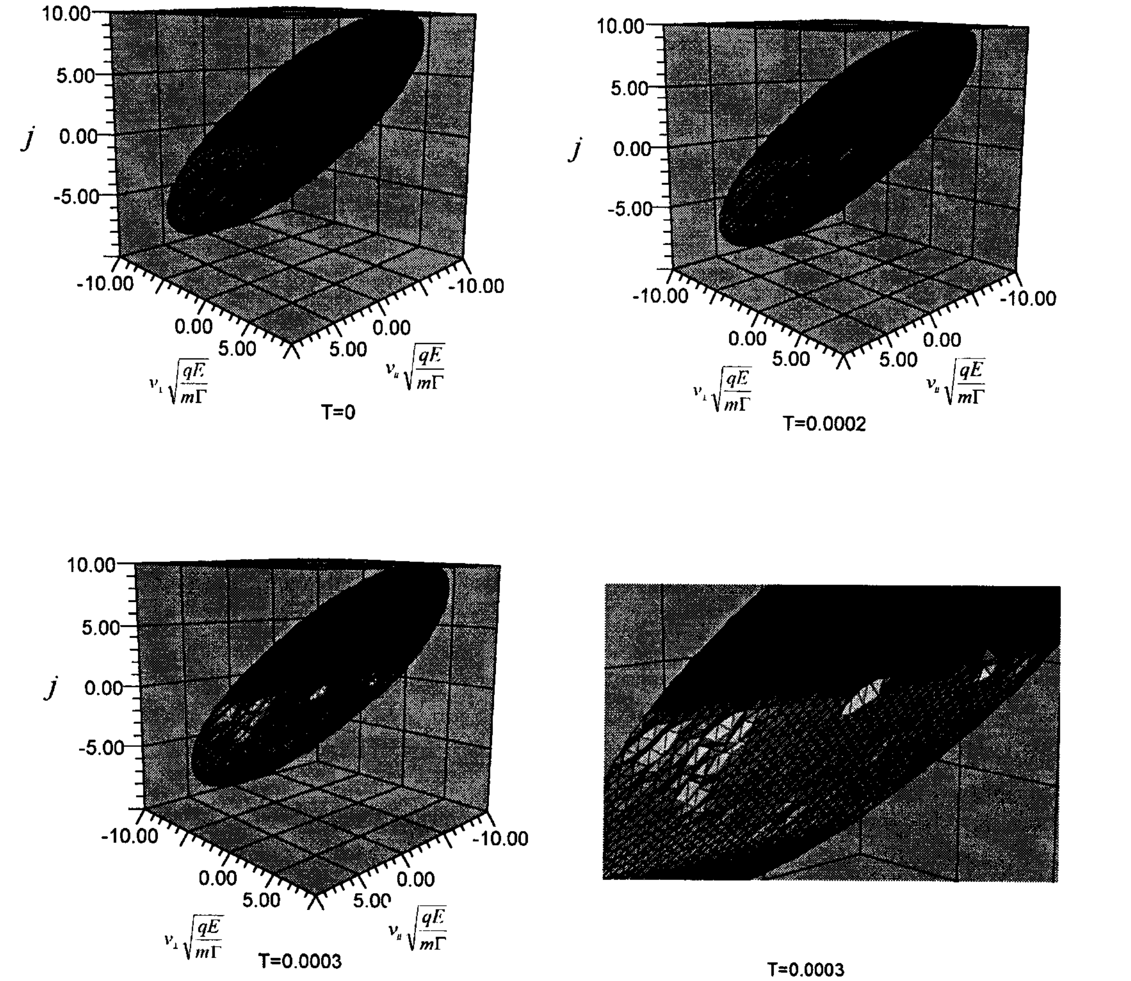

Fig. 1 Surfaces of Green's function of Radial Current Density for different times $T = (\Gamma/v_r^3)\ t = 0$, 0.0002 and 0.0003.

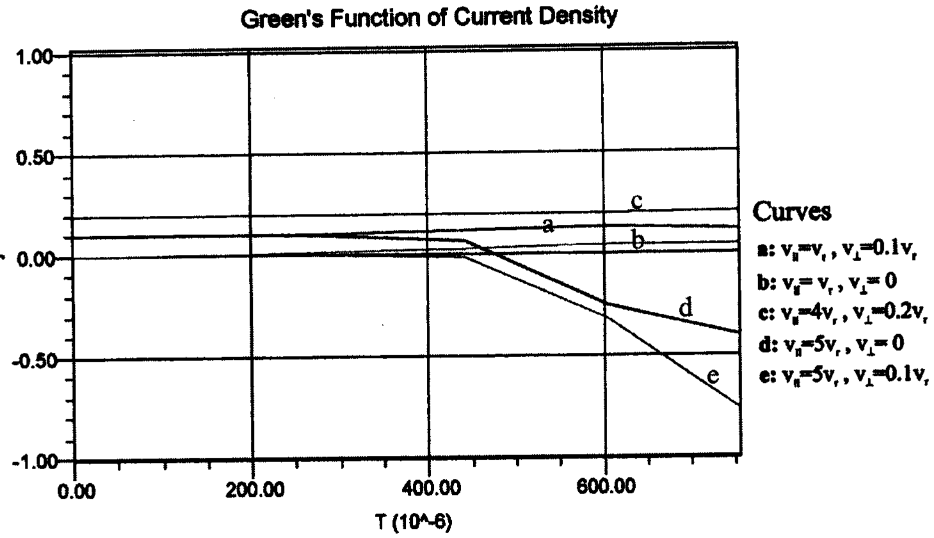

Fig. 2 Time evolution of Green's function of Radial Current Density for different velocities. (here $T = (\Gamma / v_r^3) t$).

than transverse velocity $v_\perp$. For example, as mentioned before, in rf heating we have $v_\parallel = \omega / k_\parallel$ and $v_\perp = v_T$. Therefore in Fig. 2 we have considered some cases that are close to real situation. One may see from Fig. 2 that radial current for higher parallel velocities starts earlier than low velocities.

4. Discussion

It is well known that the tokamak plasma has the ohmic and induction properties. So usually we use an *RL* circuit for description of tokamak plasmas. In ref. [3] an *RLC* model is used for describing behavior of the tokamak plasmas. This is analogous to the coaxial cable and transmission lines used in engineering. In fact according to ref. [3] is assumed that there exists a dielectric medium (diluted plasma) between the plasma and tokamak chamber. When toroidal electric field is constant ($E_\parallel$ = const.), this capacitance property does not play a role and radial current will be zero. But when the toroidal electric field changes the capacitance property affects on plasma circuit and a radial current will be produced.

Here our results are approximately similar to above-mentioned physical phenomena. The plasma shows a dielectric property when electric field changes. The polarization effect in a plasma is similar to that in a solid dielectric, but dipoles in a plasma are ions and electrons separated by a distance. But since ions and electrons can move around a preserve quasineutrality, the application of a steady E field does not result in a polarization field. However, if E changes, this polarization results. After polarization, plasma shows a dielectric property and this leads to a perpendicular current.

5. Conclusion

The effect of an additional heating of plasma (such as rf heating) is to distort the distribution function $f(v, t)$. For to determine the radial current density carried by f_1 (distortion part of $f(v, t)$), we can solve an adjoint equation (6). In fact since, in this case, we are only interested in specific moment of f_1, we may hope to reduce the computational requirements substantially by using a method that gives only this specific moment. For case of a constant toroidal electric field, the Eq. (6) does not give any radial current, but in case of changing the toroidal electric field, this equation gives us the radial current. The reason of producing a radial current (and radial electric field also) may be related to the dielectric property of the plasma in case of changing the toroidal electric field. Usually a noninductively driven current in tokamaks causes a drop of loop voltage, so it will change the toroidal electric field and result will be producing a radial current. Since H-modes are seen in noninductively driven currents, this radial current must be considered for investigation of H-modes.

Although here our result was derived under the simplifying assumptions of rf current drive case and high velocity form of collision operator, but the method applies equally well without such assumption.

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