Reduction of Viscous Damping in a Quasi-Axisymmetric Stellarator CHS-qa Configuration

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Abstract
Viscous damping of plasma flow in a quasi-axisymmetric stellarator CHS-qa has been estimated by calculating modulation of magnetic field strength. The modulation along the toroidal direction is drastically reduced by a factor of 5–20 in comparison with a conventional stellarator CHS, which is low enough to make shear viscosity determine toroidal rotation velocity. Along the poloidal direction the modulation is slightly lower than that in the equivalent tokamak configuration. The effect of higher-order bumpy ripple arising from discrete modular coils in an actual device is also discussed.

Keywords:
viscous damping, quasi-axisymmetric stellarator, CHS-qa, modulation of magnetic field strength, plasma rotation, Boozer spectrum, shear viscosity, modular coils, bumpy ripple

1. Introduction
Plasma rotation has recently attracted much attention from the viewpoint of shear of flow or radial electric field closely associated with L-H transition found in both tokamak and helical devices [1,2]. Viscous damping of the plasma rotation is caused by magnetic pumping resulting from modulation of magnetic field strength along stream lines. Actually H-mode in W7-AS stellarator was observed in narrow windows of the edge rotational transform which correspond to minima of the poloidal viscosity [2]. Viscous damping in a quasi-axisymmetric (QA) stellarator is considerably lower than that in conventional stellarators due to its tokamak-like field structure. The improved confinement regimes based on shear of radial electric field driven by fast plasma rotation is expected because of low viscosity in the QA stellarator. Hence it is important to estimate how much viscous damping is expected in the physics design of the QA stellarator.

In this article viscous damping is estimated for a QA stellarator CHS-qa which has been designed as a candidate for the next satellite machine at National Institute for Fusion Science (NIFS) [3,4]. This study mainly focuses on the discussion about the directional dependence of modulation of magnetic field strength in a QA stellarator based on comparisons with the equivalent tokamak and a conventional helical system, which has not been reported previously. In addition, the effect of higher-order bumpy ripple arising from discrete modular coils in an actual device is also discussed.

2. Calculation Method
In the neoclassical theory, viscous force against plasma flow is expressed with a diagonal viscosity tensor \( \eta \) as \( \langle \hat{u} \nabla \cdot \eta \rangle = \langle (p_+ - p_\perp) (\hat{u} \cdot \nabla B) / B \rangle \), where \( \hat{u}, p_+, p_\perp \), and \( B \) are unit vector along a stream line,

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perpendicular pressure, parallel pressure, and magnetic field strength, respectively, and ( ) means taking flux surface average. The precise calculation of \((p_\perp - p_\parallel)\) in a non-axisymmetric torus \([5,6]\) shows that it is proportional to \((\hat{u} \cdot \nabla B)/B\) like axisymmetric tokamaks in the Pfirsch-Schlüter regime. The effect of difference in transit frequency for each Fourier component of \(B\) appears in the plateau regime. However, if one Fourier component is dominant and others are negligible, the proportionality is maintained. Therefore it is acceptable to say that the viscous force is roughly proportional to the square of modulation of magnetic field strength \(\gamma\) defined by \(\gamma^2 = \langle |\hat{u} \cdot \nabla B|^2/B^2 \rangle\) except for banana regime. In this study we consider \(\gamma\) as a measure of viscous damping caused by variation of field strength, and calculated \(\gamma\) by using the results of three-dimensional equilibrium code VMEC (Variational Moments Equilibrium Code) \([7]\). The problem of the effects of low collisionality on viscous force are subject for future studies.

After transforming VMEC coordinates into Boozer ones, magnetic field strength \(B\) and shape of flux surfaces \(R, Z\) and \(\Phi\) in the cylindrical coordinates around the major axis are solved in terms of Fourier series in Boozer angle variables. In this calculation 84 Fourier components are taken into account. When the flow across a flux surface is neglected, variation of field strength along a stream line is expressed as

\[
\hat{u} \cdot \nabla B = \frac{\partial B}{\partial \Phi} d\Phi + \frac{\partial B}{\partial \Theta} d\Theta,
\]

where \(ds\) is line element along the stream line, and \(\Phi\) and \(\Theta\) are toroidal and poloidal angles in Boozer coordinates, respectively.

Actual plasma flow pattern on a flux surface is expected to take on complicated structure. For simplicity, however, \(\hat{u}\) was taken along the direction of a given pitch angle with respect to the toroidal direction in Boozer coordinates. Namely, variation of field strength along lines \(\Theta = \Phi \tan \alpha + \theta_0\) (\(\theta_0\) is an arbitrary angle) is averaged over a flux surface, and dependence of \(\gamma\) on \(\alpha\) was calculated. Using the relation \(d\Theta/d\Phi = \tan \alpha\), the derivatives of toroidal and poloidal angles \(d\Phi/ds\) and \(d\Theta/ds\) are expressed by partial derivatives of \(R, Z,\) and \(\Phi\) which are analytically obtained from the Fourier series. Hence we can calculate \(\hat{u} \cdot \nabla B\) everywhere on a flux surface. After averaging over a flux surface, \(\gamma\) is obtained as a function of minor radius and \(\alpha\).

Note that only vacuum magnetic field configurations are examined throughout this study.

### 3. Boozer Spectrum

Figure 1(a) displays Boozer spectrum of magnetic field for a representative CHS-qa configuration called 2b32 which has an aspect ratio of \(A_r = 3.2\) and 2 toroidal field periods. This configuration was optimized by giving priority to local ballooning stability. Since the optimizer does not presume coil configurations, bumpy ripples due to discrete modular coils are not included. The largest 5 ripple components normalized by \(B_\text{hom}\) for each flux surface are shown in Fig. 1. Toroidal mode number \(n\) is represented by multiples of field period. In a QA stellarator, axisymmetric component \(B_{10}\) (toroidicity) is dominant as shown in Fig. 1(a). Though residual ripples other than the toroidicity are suppressed as much as possible, the largest one \(B_{00}\) (bumpy ripple) reaches 4% of \(B_{00}\) at the edge. These \(n \neq 0\) residual ripples cause finite \(\gamma\) in the toroidal direction.

In order to compare it with a conventional helical...
device, Boozer spectrum of magnetic field for a representative configuration of CHS (Compact Helical System) is shown in Fig. 1(b). Since CHS is the \( f = 2 \) heliotron/torsatron with 8 field periods, helical ripple \( B_{2s} \) is dominant (25% of \( B_{00} \) at the edge) [8]. The major radii of the CHS-qa and CHS are 1.5 and 1 m, respectively.

4. Viscous Damping

Figure 2(a) shows pitch angle dependence of modulation of magnetic field strength \( \gamma \) in CHS-qa, CHS and a tokamak equivalent to the CHS-qa configuration for the outermost flux surfaces. Instead of \( \alpha \), actual pitch angle averaged over a flux surface \( \beta = \tan^{-1}(\rho A_p \gamma \frac{1}{1 + \tan \alpha}) \) is used as the horizontal axis. The equivalent tokamak here is defined as a tokamak which has the same axisymmetric components of the boundary shape as CHS-qa in the cylindrical coordinates.

Though the difference of major radius between CHS-qa and CHS is only a factor of 1.5, the absolute values of \( \gamma \) in CHS-qa are about one order smaller than those in CHS, and are the same level as the equivalent tokamak. This means that CHS-qa be free from large damping of plasma rotation due to field ripples as observed in the past CHS experiments [9].

Though the variation of \( \gamma \) with the pitch angle in CHS-qa is similar to that of the equivalent tokamak, toroidal \( \gamma \) is finite and the angle to minimize \( \gamma \) slightly deviates from \( 0^\circ \) (\( \beta = 6^\circ \)) under the influence of the residual helical ripples \( B_{12} \) and \( B_{24} \). On the other hand, pitch angle to minimize \( \gamma \) is \( \beta = 37^\circ \) for CHS. This angle corresponds to the pitch angle of the helical windings of CHS. As for the poloidal direction, \( \gamma \) in CHS-qa is between 0.3–0.4 m\(^{-1}\) which is slightly lower than that in the equivalent tokamak in spite of the same elongation averaged over the toroidal direction (\( Z_{10}/R_{10} = 1.54 \)). Though the reason for this has not been clarified yet, it may be attributed to distinctive three dimensional plasma shaping and magnetic axis of CHS-qa in the real space, as shown in the drawings found in Refs. [3,4]. This nature of low poloidal viscosity should be favorable for appearance of improved confinement modes associated with poloidal rotation.

Figure 2(b) summarises dependence of toroidal and poloidal \( \gamma \) on the normalized minor radius in CHS-qa and CHS. As is shown, the toroidal \( \gamma \) in CHS-qa is less than 0.1 m\(^{-1}\) which is drastically reduced by a factor of 5–20 than that in CHS over the entire plasma area. In the previous CHS experiment [9], an effective viscosity coefficient in the toroidal direction is much larger than that predicted from the neoclassical theory in inward-shifted configurations where toroidal \( \gamma \) near the center is about 0.1 m\(^{-1}\). This could be attributed to the shear viscosity (viscosity due to sheared flow between adjacent flux surfaces) which cannot be explained by the neoclassical theory. According to these experimental observations, \( \gamma \) below 0.1 m\(^{-1}\) in CHS-qa indicates that viscous damping is low enough to make the shear viscosity determine the toroidal rotation velocity. On the other hand, the poloidal \( \gamma \) is also reduced by a factor of 2–5, and is almost unchanged against the minor radius. This can be easily understood from the fact that the \( B_{10} \) component is dominant in CHS-qa as is shown in Fig. 1(a) because \( \langle \partial B/\partial \theta (\partial \theta/\partial s) \rangle \) for \( B_{10} \) component does not depend on the minor radius.
5. Effect of Bumpy Ripple Due to Discrete Modular Coils

In an actual device, higher-order bumpy ripple arising from discrete modular coils [10] may influence toroidal viscosity. The effect of this ripple on toroidal viscosity is discussed in this section. Configuration of the modular coils to reproduce the desired plasma shaping is calculated by the NESCOIL [11] code (developed by Merkel et al.). Once the coil configurations are given, Fourier coefficients of \( B_{\text{ax}}, R_{\text{ax}}, Z_{\text{ax}}, \) and \( \Phi_{\text{ax}} \) in Boozer coordinates can be calculated from the field line tracing code MAGN (developed at NIFS).

The Boozer spectrum of the magnetic field for a coil configuration designed for CHS-qa (2b32) clearly shows that the \( n = 20 \) bumpy ripple due to 20 modular coils is only 1% at the edge, and the other components are slightly smaller than those in Fig. 1(a). By taking changes in \( R_{\text{ax}}, Z_{\text{ax}}, \) and \( \Phi_{\text{ax}} \) into consideration, \( \gamma \) was calculated in the same manner described in Sec. 2. Consequently, distinct increase in the toroidal \( \gamma \) was clearly observed though the poloidal \( \gamma \) was not affected by the modular coils. The result for the toroidal \( \gamma \) is summarized in Fig. 3 to be compared with that for the original configuration. In spite of smaller components other than the \( n = 20 \) bumpy ripple, \( \gamma \) apparently increases over the entire plasma area. Therefore this increase can be attributed to the addition of small bumpy ripple by modular coils. However, \( \gamma \) still remains below 0.1 m\(^{-1}\) except for the outermost flux surface. This indicates that toroidal viscosity is low enough to be determined by shear viscosity even with bumpy ripples added by modular coils.

6. Conclusion

The modulation of magnetic field strength \( \gamma \) as a measure of viscous damping, is estimated in a QA stellarator CHS-qa configuration based on Fourier components of \( B, R, Z, \) and \( \Phi \) in Boozer angle variables. As a result, the toroidal \( \gamma \) is below 0.1 m\(^{-1}\) over the entire plasma area, in which case the shear viscosity cannot be negligible. In comparison with a conventional stellarator CHS, \( \gamma \) is drastically reduced in both toroidal and poloidal directions. Even with the bumpy ripple due to modular coils, the viscous damping in CHS-qa is low enough to allow fast plasma rotation in spite of three dimensional field structure. Because of low poloidal and toroidal viscosities, improved confinement modes due to flow shear are expected in a quasi-axisymmetric stellarator CHS-qa.

References