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Electric Field Domain Interface in Helical Systems

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Abstract

The electric field bifurcation in helical plasmas under the condition of continuous fluxes is investigated. The stationary solution of the transport equation, together with charge neutrality condition, is investigated. It is shown that the anomalous flux plays an important role in determining multiple electric field solutions. The transition to the branch with a strong positive electric field occurs when the heat flux exceeds a critical value. Condition for the presence of transition is obtained. The radial structure of the electric field domain interface is obtained. The condition that the suppression of turbulence is expected to occur is discussed. Comparison with experimental observation is briefly mentioned.

Keywords:

helical systems, electric field bifurcation, domain interface, transition, suppression of turbulence

1. Introduction

Study of internal transport barrier has been progressed in this decade [1]. If the electric field domain interface could be established, the transport reduction at the interface takes place in helical plasmas. In some helical plasmas, a radial region with one polarity of radial electric field E_r touches a region with different polarity of E_r. Such regions are called electric field domains and the interface between them is called an electric field domain interface. The electric field domain interface has been identified in CHS plasma, and the internal transport barrier has been found in the CHS plasma [2]. In order to clarify the existence of the transport barrier in helical plasmas, the study of the electric field bifurcation [3] is necessary.

In this article, we study the electric field bifurcation in helical plasmas under the condition of continuous fluxes. The stationary solution of the transport equation, together with charge neutrality condition, is investigated for a given heat flux. The particle and wenergy fluxes are given by superpositions of the neoclassical flux and anomalous flux. It is shown that the anomalous flux plays an important role in determining the multiple electric field solutions. The transition to the branch with a strong positive electric field occurs when the heat flux exceeds a critical value. Condition for the presence of hard transition is analyzed. The spatial structure of the electric domain interface, across which regions of different electric polarity touch, is investigated. Comparison with experimental observation is discussed.

2. Existence of Domain Interface

Motivated by the experimental observation on ECH plasma in CHS, we consider a simplified case in which the ion energy balance and the ion temperature gradient are neglected, and the electron energy balance and particle balance are studied. We employ a simplified formula for the neoclassical ion particle flux after [4] in order to have an analytic insight as

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$$\Gamma_i^{NC} = \frac{nD_i}{1 + C_i\left(\frac{eaE_r}{T_e}\right)^2}\left(-\frac{n'}{n} + \frac{eE_r}{T_i}\right) \qquad (1)$$

with $D_j = 24\varepsilon^2\varepsilon_h^{1/2}\nu_j^{-1}v_D^2$ and $C_j = 36(\varepsilon/\varepsilon_h)^{1/2}(T_e/\nu_j rBea)^2$ and suffix j denotes species (e, i). Notation is as follows: $\varepsilon = r/R$ is the inverse aspect ratio, ε_h is the helical ripple, $v_D = T/eRB$ is the toroidal drift velocity, n is the density, T is the temperature, ν is the pitch-angle collision frequency, r is the minor radius, R is the major radius, and the prime' denotes the radial derivative d/dr. The coefficient C_j is related to the ratio between ∇B-drift rotation frequency and the collision frequency. The electron particle flux and energy flux are expressed in the regime of ripple diffusion as

$$\Gamma_e^{NC} = -\frac{n}{a}D_e\left(\frac{an'}{n} + \frac{aeE_r}{T_e} - \eta_{12}\frac{aT_e'}{T_e}\right) \qquad (2)$$

$$q_e^{NC} = -\frac{n}{a}D_e\gamma_e T_e\left(\frac{an'}{n} + \frac{aeE_r}{T_e} + \eta_{22}\frac{aT_e'}{T_e}\right), \qquad (3)$$

where coefficients $\gamma_e \simeq 5$, $\eta_{22} \simeq 4.5$ and $\eta_{12} \simeq 3.5$ are numerical constants. The total flux is assumed to be a sum of neoclassical ripple flux and anomalous flux as $\Gamma_{\text{tot}} = \Gamma^{NC} - D_{\text{anom}}n'$ and

$$q_{e,\text{tot}} = q_e^{NC} - \chi_{\text{anom}}\, nT_e'. \qquad (4)$$

The heat flux equation (4), with the constraint of the charge neutral condition $\Gamma_e = \Gamma_i$, is solved for given heat flux $q_{e,\text{tot}}$ [5]. In the strong temperature-gradient limit $|T_e'/T_e| >> |n_e'/n_e|$, one has

$$X^3 - \frac{b\,Q_t}{1+\chi_a-b}X^2 + \left(1 + \frac{(1+\chi_a)}{(1+\chi_a-b)}\frac{T_e}{T_i}\frac{D_i}{D_e}\right)X - \frac{b\,Q_t}{1+\chi_a-b} = 0, \qquad (5)$$

where $X = \sqrt{C_i}\,T_e^{-1}eaE_r$ is the normalized radial electric field, $Q_t = \sqrt{C_i}\,(\frac{n}{a}D_e\gamma_e T_e)^{-1}q_{e,\text{tot}}$ is the the normalized radial electron heat flux, $\chi_a = \chi_{\text{anom}}/\eta_{22}D_e\gamma_e$ and $b = \eta_{12}/\eta_{22} \simeq 7/9$ is a numerical coefficient.

Figure 1 shows the normalized radial electric field X and temperature gradient $Y = -\sqrt{C_i}\eta_{22}T_e^{-1}aT_e'$ as a function of the heat flux Q_t. Bifurcation exists if the

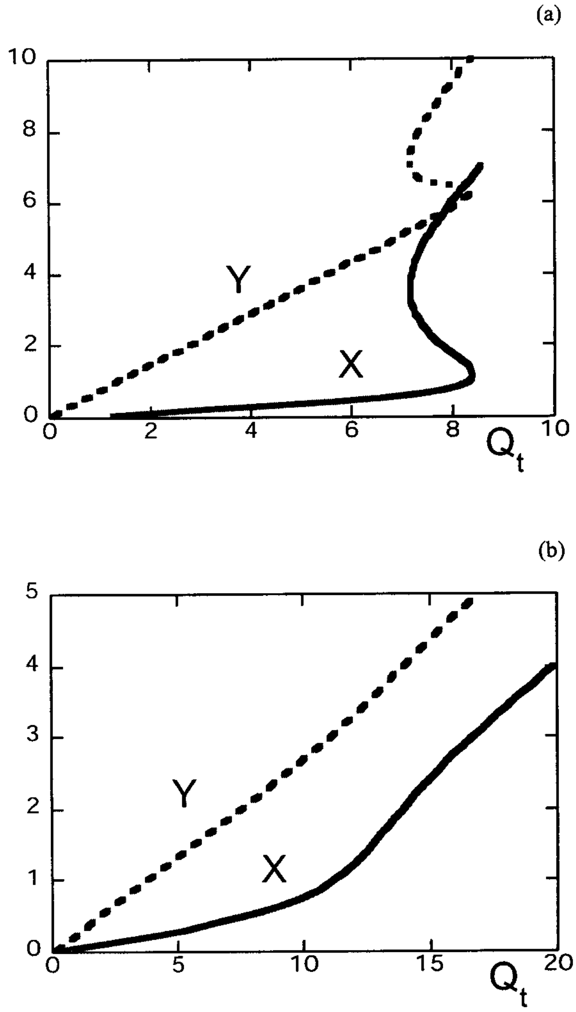

Fig.1 Normalized radial electric field X (solid line) and normalized temperature gradient Y (dashed line) vs electron heat flux for ECH plasmas. The case of hard transition (top) and that for soft transition (bottom) are shown. Parameters are: $T_e/T_i = 2$, $D_e/D_i = 0.26$, $b = 7/9$, $\chi_a = 0.5$ for case (a), and $T_e/T_i = 3$, $D_e/D_i = 1.09$, $b = 7/9$, $\chi_a = 3$ for case (b).

condition

$$b\left(1+\chi_a\right)^{-1} > 1 - T_e D_i / 8\,T_i D_e \qquad (6)$$

is satisfied. When Q_t exceeds a criterion, the transition to the other branch occurs. If the anomalous energy transport is strong and the ratio of the electron temperature to ion temperature is high, $b(1+\chi_a)^{-1} < 1 - T_eD_i/8T_iD_e$, a soft transition appears.

3. Structure of Domain Interface

The condition for the occurence of transition from one branch to another is given by the dynamical equation. A dynamical equation for the radial electric field is given by

$$\frac{\partial}{\partial t} E_r = \nabla \cdot \mu_i \nabla E_r - \frac{1}{\varepsilon_0 \varepsilon_\perp} J_r, \tag{7}$$

where μ_i is the ion shear viscosity [1]. When the electric domain interface exists in a stationary state, the location of the interface $r = r_*$ is dictated by the Maxwell's construction

$$\Delta\Phi\left(r_*; Q_t, \cdots\right) \equiv \int_{E_{r1}}^{E_{r2}} \left\{ \Gamma_i^{NC}\left(E_r, \cdots\right) - \Gamma_e^{NC}\left(E_r, \cdots\right) \right\} \mathrm{d}E_r = 0, \tag{8}$$

where E_1 and E_2 are two stable solutions which satisfy the local charge neutrality condition. Critical value of Q_t for transition is given by eq.(8). Analytic estimate is approximately given by the inflection point of Fig.1(a), namely,

$$q_{\mathrm{tot}} = q_{\mathrm{tot},c} \simeq \sqrt{3}\left(\frac{1+\chi_a}{b}\left(1 + \frac{D_i T_e}{4 D_e T_i}\right) - 1 \right) \left(\frac{\varepsilon_h}{\varepsilon}\right)^{1/4} \left(\frac{V_i reBa}{T_e}\right)\left(D_a \gamma_e \frac{nT_e}{a}\right). \tag{9}$$

The radial electric field in the vicinity of the domain interface is analyzed with the approximation of constant ion viscosity. The current-electric field relation $J_r^{NC}[E_r]$ is characterized by two parameters, i.e., the maximum and zeros of $J_r^{NC}[E_r]$, as

$$J_r^{NC} = \hat{J} E\left(E^2 - 1\right) \tag{10}$$

with $E = (2E_r - (E_{r1} + E_{r2}))/(E_{r2} - E_{r1})$. With the help of this simplification, one has the solution

$$E_r = \frac{\left(E_{r1} + E_{r2}\right)}{2} + \frac{\left(E_{r2} - E_{r1}\right)}{2} \cosh\left(\frac{r - r_*}{\Delta}\right) \tag{11}$$

and

$$\Delta = 2\sqrt{\varepsilon_0 \varepsilon_\perp \left| E_{r2} - E_{r1} \right| \hat{J}^{-1} \mu_i}, \tag{12}$$

where $\varepsilon_\perp = (1 + 2q^2)c^2 V_A^{-2}$. The influence of the electric field gradient on the shear viscosity and the solution are discussed in ref. [6].

The maximum gradient of the radial electric field is given by

$$E_r' = \frac{\left(E_{r2} - E_{r1}\right)}{\Delta}. \tag{13}$$

The criterion of the electric field gradient for suppression of turbulence is derived in ref. [7] for nonlinear current-diffusive interchange mode which could be excited in a system with magnetic hill (some operation mode of heliotron/torsatron configurations). Substitution of eq.(13) into the formula, $B^{-1}\,|\mathrm{d}E_r/\mathrm{d}r| > \sqrt{r/R}\, s c_s a^{-1}$, gives a condition

$$\sqrt{\left| E_{r2} - E_{r1} \right| \hat{J}^{-1}} > 2\sqrt{\left(r/R\right) \varepsilon_0 \varepsilon_\perp \mu_i}\; s B c_s a^{-1}, \tag{14}$$

where s is the magnetic shear. This result shows that the condition for the turbulence suppression in Heliotron plasma is satisfied, if the temperature is increased and the work done by the neoclassical current (being characterized by $|E_{r2} - E_{r1}|\,\hat{J}$) becomes high enough.

In the limiting case where the turbulence-driven cross-field momentum transport is suppressed, other mechanisms regulate the radial extent of the electric field domain interface. When the scale length of inhomogeneity of the radial electric field approaches to the ion banana width, a nonlocal property of the response $J(E_r)$ is not negligible. The ion response that contributes to $J(r)$ at r is determined by an average $\bar{E}_r$ as

$$J(r) = J\left[\bar{E}_r(r)\right], \tag{15}$$

where $\bar{E}_r$ is an average of E_r over a region of $r - \delta < r < r + \delta$ and δ is of the order of ion banana width ρ_b. Expanding $\bar{E}_r$ in a series of δ as $\bar{E}_r(r) = E_r(r) + E_r''(r)\delta^2/4 + \cdots$, one has an expression as

$$J(r) = J\left[E_r(r) + E_r''(r)\delta^2/4 + \cdots\right], = J\left[E_r(r)\right] + \frac{\delta^2}{4}\frac{\partial J}{\partial E_r} E_r''(r) + \cdots. \tag{16}$$

In the presence of a strong inhomogeneity, the second term in the right hand side of eq.(16) is influential. Combining eqs.(7) and (16) for a stationary state, one has

$$\left(\mu_i - \frac{\delta^2}{4\varepsilon_0 \varepsilon_\perp}\frac{\partial J}{\partial E_r}\right)\nabla^2 E_r = \frac{1}{\varepsilon_0 \varepsilon_\perp} J_r. \tag{17}$$

An effective viscosity is enhanced as $\mu_i - (\delta^2/4\varepsilon_0\varepsilon_\perp)\partial J/\partial E_r$. Replacing μ_i by $\mu_i - (\delta^2/4\varepsilon_0\varepsilon_\perp)\partial J/\partial E_r$ in eq.(12), the layer width is given by the relation

$$\Delta = 2\sqrt{\frac{\varepsilon_0\varepsilon_\perp\left|E_{r2}-E_{r1}\right|}{\hat{J}}\left(\mu_i - \frac{\delta^2}{4\varepsilon_0\varepsilon_\perp}\frac{\partial J}{\partial E_r}\right)} \ . \tag{18}$$

In the vicinity of the transition layer, an estimate $\partial J/\partial E_r \sim -\hat{J}/|E_{r2} - E_{r1}|$ holds. Substituting this estimate of $\partial J/\partial E_r$ into eq.(18), one has

$$\Delta = 2\sqrt{\frac{\varepsilon_0\varepsilon_\perp\left|E_{r2}-E_{r1}\right|}{\hat{J}}\mu_i + \frac{\rho_b^2}{4}} \ , \tag{19}$$

where the relation $\delta \sim \rho_b$ is used. In eq.(19), ρ_b is calculated by taking into account of the inhomogeneous radial electric field $\rho_b \sim \rho_{b,0}(1 - \rho_p^2 e E_r' T_i^{-1})^{-1}$ where $\rho_{b,0}$ is the one in the absence of inhomogeneous electric field [1]. When the shear viscosity is suppressed strongly, the layer width is given by the ion banana width including the effects of inhomogeneous radial electric field, $\Delta \sim \rho_b$. One obtains an estimate

$$\Delta \sim \left(1 \pm \frac{R}{r}\frac{e\left|E_{r2}-E_{r1}\right|\rho_{b,0}}{T_i}\right)\rho_{b,0}. \tag{20}$$

The sign of $\pm$ follows that of E_r'.

4. Summary and Discussion

The bifurcation of the electric field in helical plasma is analyzed, and the structure of the electric field domain interface is discussed. Condition for the existence of the bifurcation and electric field domain interface is obtained. The structure of the domain interface is analyzed. The analysis in this article is performed in a slab configuration. Investigations in a cylindrical configuration are performed in ref. [8], and application to LHD plasmas is discussed in ref. [9]. The analysis here provides a basis for understanding the simulation in realistic configurations.

One can test the theory by comparing the result with the observation of the CHS plasmas [10]. In the analysis on the dynamical change of the radial electric field, it has been shown that the radial current (the second term in the right hand side of eq.(7)) was found to be of the order of the prediction of the neoclassical theory. For the case of the 'dome' structure, plasma parameters have been observed as $n \simeq 2 \times 10^{18}$ m^{-3}, $\hat{J} \simeq$ 2Am^{-2} and $|E_{r2} - E_{r1}| \simeq 10^4$Vm^{-1}. Substitution of these parameters into eq.(12) gives an estimate $\Delta \simeq 5 \times 10^{-3}\sqrt{\mu_i}$(m) where μ_i is measured in a unit of m^{2}s^{-1}. Observation has given $\Delta \simeq 10^{-2}$ m. This gives an estimate of

$$\mu_i \simeq 5 \text{ m}^2\text{s}^{-1} \ . \tag{21}$$

This value of the ion viscosity is in the range of anomalous transport coefficient of the energy of CHS plasmas. Comparison of μ_i between the L-mode plasma and ITB plasma would confirm the reduction of anomalous transport.

The analysis in this article is limited to the steady state solution. The dynamical behaviour such as the electric pulsation [10] deserves a further analysis.

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