

# MHD Stability Analysis on Helical System with Bootstrap Current Including Electric Field Effects

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## Abstract

The effect of the electrostatic potential  $\Phi$  on the bootstrap current has been calculated and its impact on the global ideal MHD stability of a LHD configuration has been investigated. The bootstrap current can destabilise a  $m/n = 2/3$  external kink when it causes the edge rotational transform to approach 1.5. At  $\Phi = 0$ , the unstable  $m/n = 2/3$  structure is strongly coupled with a  $m/n = 9/13$  term. Negative  $\Phi$  reduces the edge transform below 1.4 and the  $m/n = 2/3$  mode couples with a  $m/n = 5/7$  component instead. Positive  $\Phi$  displaces the edge transform closer to the critical value 1.5 weakening the contribution of toroidal sidebands like  $m/n = 9/13$ , but deteriorating the stability conditions slightly further. Global  $n = 1$ ,  $n = 2$  and  $n = 4$  modes are basically internal and have smaller growth rates than the  $n = 3$  external kink. These structures evolve from interchange-like for  $n = 1$  to ballooning-like for  $n = 4$ .

## Keywords:

electrostatic potential, bootstrap current, LHD, ideal MHD stability, external kink, mode coupling

## 1. Introduction

The bootstrap current in heliotrons depends sensitively on the plasma temperature, on the potential and on  $\beta$ . We have investigated a sequence of configurations that model the LHD device at fixed  $\beta = 3.5\%$  and temperature  $T = 2.5\text{keV}$  with electrostatic potential  $-2.5\text{keV} < \Phi < +2.5\text{keV}$  with vertical fields activated to centre the plasma at a major radius  $R = 3.6\text{m}$ . The magnetohydrodynamic (MHD) equilibria are computed with the VMEC code [1] and the global ideal MHD stability properties are studied with the TERPSICHORE code [2]. The bootstrap current is computed selfconsistently with the plasma temperature and electrostatic potential assumed using the formulas derived by Watanabe *et al.* for all collisionalities [3]. To explore external kink modes, we prescribe a dog-bone shaped conducting shell

that approximates the real conducting wall of the LHD device. We concentrate specifically on unstable structures with low order toroidal mode numbers, but retain higher order components that can couple strongly with the main resonant or near-resonant low order terms.

## 2. Effects of the Plasma Potential on External Kinks

The rotational transform profiles for the LHD configuration ( $\beta = 3.5\%$ ,  $T = 2.5\text{keV}$ ,  $R = 3.6\text{m}$ ) are shown in Fig. 1(a) for three different values of the electrostatic potential  $\Phi$ . For  $\Phi = 0$ , the rotational transform at the edge  $t_e \sim 1.4$ . A negative  $\Phi = -2.5\text{keV}$  damps the bootstrap current that reduces  $t_e$ , while positive  $\Phi = +2.5\text{keV}$  enhances this current and correspondingly  $t_e$ .

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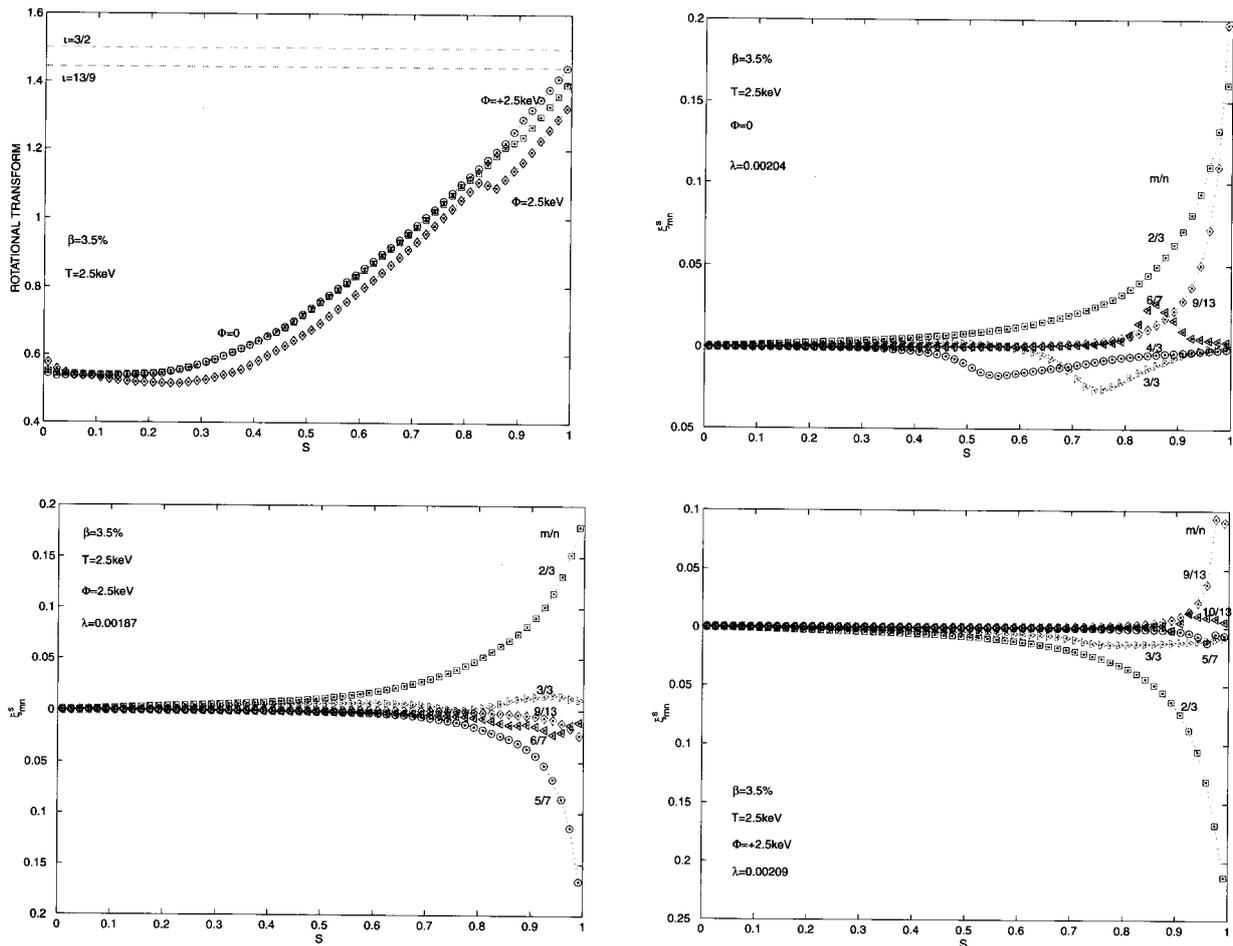


Fig. 1 (a) (top left) The rotational transform profiles for a shifted in LHD heliotron configuration at  $\beta = 3.5\%$  and  $T = 2.5\text{keV}$  for different values of the electrostatic potential  $\Phi$ . The locations of the critical resonant surfaces  $t_c = 3/2$  and  $t_c = 13/9$  are identified by the dashed lines. The leading Fourier amplitudes of the radial component of the perturbed displacement vector as a function of the radial variable of the  $n = 3$  of modes for (b) (top right)  $\Phi = 0$ , (c) (bottom left)  $\Phi = -2.5\text{keV}$  and (d) (bottom right)  $\Phi = +2.5\text{keV}$ .

The behaviour of the  $t$  profile as a function of  $\Phi$  constitutes a critical factor for the determination of MHD stability. In a 10 field period device such as LHD, mode components with toroidal mode number  $n = 3$  can couple through the background equilibrium with  $n = 13$  components. For vanishing  $\Phi$ , the resonant surfaces  $t = 13/9$  and  $t = 3/2$  are just outside the plasma. The close proximity of the  $t = 3/2$  and  $13/9$  favours a significant coupling between  $m/n = 2/3$  and  $m/n = 9/13$  components which is verified in Fig. 1(b). The leading Fourier amplitudes of the radial component of the displacement vector shows these two terms of the spectrum to be dominant and their structure reveals external kink features with an eigenvalue  $\lambda = -0.00204$ . For  $\Phi = -2.5\text{keV}$ ,  $t_e < 1.4$  and the eigenvalue reduces to  $\lambda =$

$-0.00187$  because the two critical resonances move away from the plasma edge. This is illustrated in Fig. 1(c) where the contribution of the  $m/n = 9/13$  term is weakened and replaced by a  $m/n = 7/5$  component because the  $t = 1.4$  surface moves into the vacuum domain. For  $\Phi = +2.5\text{keV}$ ,  $t_e$  gets closer to 1.5 which increases the unstable eigenvalue to  $\lambda = -0.00209$  even though the critical  $t = 13/9$  surface moves into the plasma in a region of large global magnetic shear as shown in Fig. 1(d).

### 3. The Perturbed Radial Magnetic Field Structure

The distribution of  $\sqrt{g}\delta B^r$  on a toroidal magnetic flux surface close to the edge of the plasma is displayed

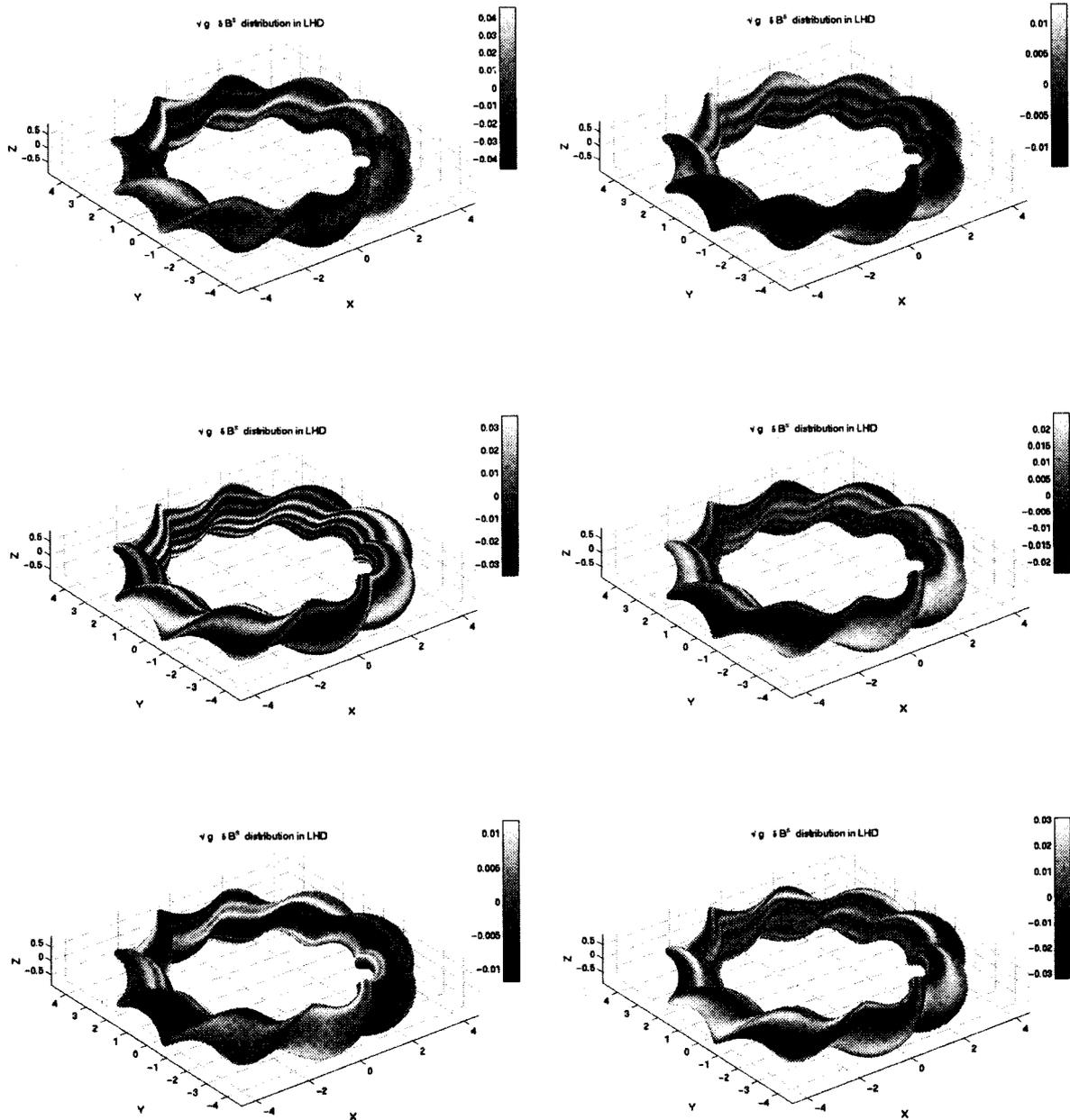


Fig. 2 The distribution at  $\sqrt{g}\delta B^s$  on a toroidal flux surface near the edge of the plasma in the LHD heliotron device at  $\beta = 3.5\%$  and  $T = 2.5\text{keV}$ . The external kink structure for which the  $n = 3$  family of modes dominate is shown on the column on the left hand side for electrostatic potential values of (a)  $\Phi = -2.5\text{keV}$  (top left), (c)  $\Phi = 0$  (middle left) and (e)  $\Phi = +2.5\text{keV}$  (bottom left). The right hand side column displays the  $\sqrt{g}\delta B^s$  structure at  $\Phi = 0$  for which the leading components at the boundary are dominated by (b) the  $m/n = 1/1, 6/9$  terms (top right), (d) the  $m/n = 2/2, 8/12$  terms (middle right) and (e) the  $m/n = 4/4$  term (bottom right).

in Fig. 2, where  $\sqrt{g}$  is the Jacobian and  $\delta B^s$  is the radial component of the perturbed magnetic field. The  $\sqrt{g}\delta B^s$  distribution of instability structures in which the  $m/n = 2/3$  term plays a leading role is shown on the left hand side column of Fig. 2. The cases of  $\Phi = -2.5\text{keV}$ , 0 and

$+2.5\text{keV}$  correspond to the top, middle and bottom figures, respectively. The mode structure appears to concentrate on the inside edge of the torus, but this is not an antiballooning feature because the maximum amplitudes on the inside are comparable to those on the outside.

The large number of oscillations in  $\sqrt{g\delta B^s}$  for  $\Phi = 0$  reflects the impact of the  $m/n = 13/9$  contribution to the instability structure.

We have also explored the stability of the system with respect to other families of low order modes for  $\Phi = 0$ ,  $T = 2.5\text{keV}$  and  $\beta = 3.5\%$ . Though unstable, they typically have much smaller growth rates than the  $m/n = 2/3$  external kink. Furthermore, their structure corresponds basically to internal modes because the leading components peak inside the plasma in the vicinity of the appropriate rational surfaces. The  $\sqrt{g\delta B^s}$  distributions near the edge of the plasma for the  $n = 1$ ,  $n = 2$  and  $n = 4$  mode families are displayed in the right hand side column of Fig. 2, from top to bottom, respectively. The  $n = 1$  structure at the edge combines  $m/n = 1/1$  and  $m/n = 6/9$  features. The  $n = 2$  structure couples the  $m/n = 2/2$  component with  $n = 12$  terms, mainly  $m = 8$  at the edge. The  $n = 4$  family couples  $m/n = 4/4$  with various  $n = 14$  terms. The most noticeable difference between these edge structures is the variation of interchange-like character for  $n = 1$  to more ballooning character for  $n = 4$ .

#### 4. Conclusions

We have investigated the effect of the electrostatic potential  $\Phi$  on the global low order ideal MHD stability of a LHD configuration with axis shifted in to  $R = 3.6\text{m}$ , at  $\beta = 3.5\%$  and temperature  $T = 2.5\text{keV}$  with bootstrap current calculated selfconsistently. The potential can alter the rotational transform through its impact on the

bootstrap current. Positive  $\Phi$  increases the transform while negative  $\Phi$  decreases it. The bootstrap current can trigger external  $m/n = 2/3$  kink modes when it causes the edge- $t$  to approach 1.5. For  $\Phi = 0$ , the  $m/n = 2/3$  structure is strongly coupled with a  $m/n = 9/13$  component. For  $\Phi = -2.5\text{keV}$ , we have  $t_e < 1.4$ , and the unstable  $m/n = 2/3$  structure couples with a  $m/n = 5/7$  term. For  $\Phi = +2.5\text{keV}$ , the  $t_e$  approaches 1.5 and the coupling to toroidal sidebands diminishes. Internal ideal MHD structures have been computed for  $n = 1$ ,  $n = 2$  and  $n = 4$  mode families at  $\Phi = 0$ . The growth rates are smaller than that of the  $n = 3$  external kink. The mode structure is interchange-like for  $n = 1$  and becomes ballooning in character for  $n = 4$ .

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