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Monte Carlo Simulations Study of Neoclassical Transport in Inward Shifted LHD Configurations

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Abstract

Neoclassical transport coefficients are computed in LHD configurations with two different values of magnetic axis shift in the major radius direction; the "standard" configuration (R_{ax} = 3.75 m) and the "inward shifted" configuration (R_{ax} = 3.6 m). We apply a newly developed Monte Carlo simulation code, DCOM, to evaluate local neoclassical diffusion coefficients. The obtained results are compared with the results by DKES code for benchmarking and we obtain good agreements between the results of two codes. It is found that the diffusion coefficients of the inward shifted configuration are about three to ten times smaller than those of the standard configuration. This difference can be seen not only in the $1/\nu$ regime but also in the plateau regime due to the larger amplitude of (1,10) magnetic field component. The diffusion coefficients are also evaluated in the presence of radial electric field and we found that the difference between diffusion coefficients in two configurations becomes smaller with stronger radial electric field in the $1/\nu$ regime.

Keywords:

LHD, neoclassical transport, radial electric field, Monte Carlo simulation

1. Introduction

In heliotron, radial drift motions of helically trapped particles enhance the radial transport of energy and particles. Thus, the neoclassical transport is an important issue to sustain a high temperature and high-density plasma in heliotrons. Especially, in the long-mean-free-path (LMFP) regime, the neoclassical transport coefficient increases as collision frequency decreases ($1/\nu$ regime) and the neoclassical transport would plays an important role as well as anomalous transport by turbulence. Also the radial electric field alters the orbit of helically trapped particle drastically and plays a significant role in the neoclassical transport in heliotrons.

Many studies have been done to evaluate the neoclassical transport coefficient analytically and numerically in helical systems. Among them the DKES (Drift Kinetic Equation Solver) code [1,2] has been commonly used for the experimental data analyses [3,4] and for theoretical predictions [5,6]. However, in the LMFP regime, a large number of Fourier modes must be used for distribution function and a convergence problem can be seen. On the other hand, the neoclassical transport coefficient has also been evaluated using Monte Carlo technique directly following the particle

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orbits [7-10]. Using this method we need a larger CPU time compared with DKES but there is no convergence problem.

In this paper we study the neoclassical transport numerically using Monte Carlo technique in the LHD configurations. We have developed a Monte Carlo simulation code, DCOM (Diffusion COefficient calculator by Monte carlo method), in which the neoclassical transport coefficients are estimated by the radial diffusion of test particles. We evaluate the neoclassical transport coefficients using DCOM code in the LHD configurations with two different values of magnetic axis shift in the major radius direction; the "standard" configuration (R_{ax} = 3.75 m) and the "inward shifted" configuration (R_{ax} = 3.6 m). The effect of magnetic axis shift is studied by comparing diffusion coefficients in two configurations.

2. Simulation Model

We evaluate a mono-energetic local diffusion coefficient, D, using Monte Carlo technique. In the simulation the mono-energetic N particles are released from the initial minor radius position, r_0, where the particles are randomly distributed in the poloidal and toroidal coordinates, and in the pitch angle space.

The test particle orbits are followed solving the equations of motion in the Boozer coordinates using 50 Fourier modes of magnetic field. The Boozer coordinates are constructed based on the MHD equilibrium obtained by VMEC code [11,12]. The pitch angle scatterings are taken into account applying the Monte Carlo collision operator based on the binominal distribution [13]. The pitch angle scattering after the time interval Δt is given in terms of $\lambda(= v_{\|}/v)$ by

$$\lambda_{n+1} = \lambda_n(1 - \nu\Delta t) + \sigma\left[\left(1 - \lambda_n^2\right)\nu\Delta t\right]^{1/2}, \qquad (1)$$

where σ takes +1 or −1 with equal probabilities. ν is the deflection collision frequency.

After several characteristic collisional time, τ, the diffusion coefficient can be evaluated by taking the mean square displacement of N particles as

$$D = \frac{1}{2Nt}\sum_{i=1}^{N}\left(r_i - \langle r\rangle\right)^2, \qquad (2)$$

where r_i is the radial position of i-th particle and $\langle r\rangle = (1/N)\sum_{i=1}^{N} r_i$. However, when some particles are lost from simulation region this expression does not give an accurate diffusion coefficient but smaller one. So we here employ the formula including the effects of these escaping particles from simulation region [14] as

$$D = \frac{-4L^2}{\pi^2 t}\ln\left\{\frac{1}{N}\sum_{i=1}^{N}\cos\left(\frac{\pi(r_i - r_0)}{2L}\right)\right\}, \qquad (3)$$

where L (= r_c−r_0) is the distance from the initial position, r_0, to the cutoff radii, r_c. This expression of D converges to eq. (2) when the distance L becomes infinity.

3. Magnetic Configuration of LHD

We, here, consider two LHD configurations with different values of the magnetic axis shift in the major radius direction. The first one is a configuration where the magnetic axis is shifted by 15 cm inwardly from the center of two helical coils in vacuum (R_{ax} = 3.75 m). This configuration is called "standard" configuration satisfying the requirements for highly balanced plasma performance (i.e. a high plasma beta, relatively good particle confinement, and creating a divertor configuration). The second one is a configuration where the magnetic axis is shifted by 30 cm inwardly from center of two helical coils in vacuum (R_{ax} = 3.6 m). This configuration is called "inward shifted'' configuration where the confinement of ripple trapped particles are improved drastically and the good confinement of energetic ions would be expected. Recent LHD experimental results have shown the better energy confinement in the inward shifted configuration R_{ax} = 3.6 m than that in the standard configuration R_{ax} = 3.75 m [15]. In the following we assume a vacuum magnetic configuration to study the essential difference of these configurations as a first step.

Figure 1 shows the change of magnetic field strength following the filed line at the radial position r/a = 0.5; (a) R_{ax} = 3.75 m and (b) R_{ax} = 3.6 m. We can see long and short period modulations due to toroidicity and helicity of magnetic field in the case of R_{ax} =3.75m. We can also see the two kind of modulations in the case of

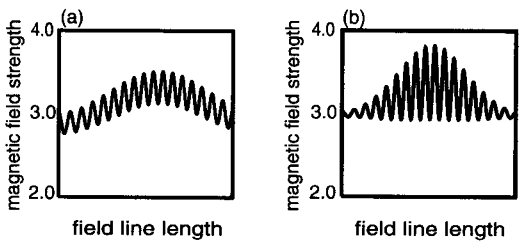

Fig. 1 The variations of the magnetic field strength B (T) along a magnetic field line are plotted for the r/a = 0.5 flux surface (a) in the standard configuration and (b) in the inward-shifted configuration.

R_{ax} = 3.6 m, while the minimum of magnetic strength is almost uniform. The Fourier spectrums of magnetic field in the Boozer coordinates (Fig. 2) show that this is due to the increase of (1,10) and (3,10) magnetic field components by shifting the magnetic axis inwardly, where (m, n) represents the Fourier component of magnetic field with the poloidal number, m, and the torodial number, n. This is close to the state so called sigma-optimized state [16]. The radial drift of helical trapped particles is reduced and thus the confinement of trapped particles is improved in this configuration [13]. A significant reduction of neoclassical transport also can be expected in the case of R_{ax} = 3.6 m.

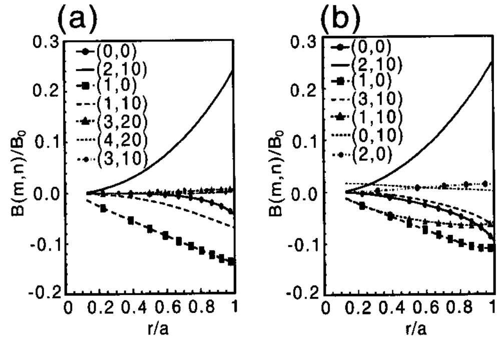

Fig. 2 The magnetic filed strength $B_{m,n}$ as a function of the normalized minor radius r/a (a) in the standard configuration and (b) in the inward-shifted configuration.

4. Simulation Results

4.1 Diffusion Coefficient without E_r

We first study the neoclassical transport without radial electric field in LHD configurations. In order to obtain mono-energetic diffusion coefficients we assume an electron as a test particle (N = 800) and the energy to be 3 keV. The magnetic field is set to 3 T at the magnetic axis. The test particles are followed for several collisional time until the evaluated diffusion coefficient is converged.

Figure 3 shows the normalized diffusion coefficient D^* at r/a = 0.5 calculated by DCOM code (solid line) as a function of normalized collision frequency ν^* for the two configurations; (a) R_{ax} = 3.75 m and (b) R_{ax} = 3.6 m. We, here, normalize the collision frequency by $\nu\iota/R$, and the diffusion coefficient by the tokamak plateau value of mono-energetic case, D_p, as $D_p = (\pi/16)(v^3/\iota R\omega_c^2)$, where R, v, ι, and ω_c are the major radius, the velocity of test particles, the rotational transform, and the cyclotron frequency of test particle, respectively.

The dashed lines indicate the DKES results for the two configurations. We can see good agreements with DKES results from P-S regime through $1/\nu$ regime for both configuration cases. The small discrepancies can be seen in the low collision frequency region. This would be related to the treatment of ∇B drift on the magnetic flux surface. In DKES code, the relation $v_{E\times B} \gg v_{\nabla B}$ is assumed and the effect of ∇B drift on the magnetic flux surface is ignored. Therefore we can not obtain ν and $\sqrt{\nu}$ regimes without radial electric field in DKES code.

The dotted lines in Fig. 3 show the analytical results of diffusion coefficient D (= $D_a + D_{1/\nu}$) which is the sum of the non-axisymmetric contribution $D_{1/\nu}$ given by a multi-helicity model [13] and the axisymmetric contribution D_a given by $D_a = (D_{bp}^{3/2} + D_{PS}^{3/2})^{2/3}$, where $D_{bp} = D_b D_p/(D_b + D_p)$, $D_b = (B_{1,0}^{1/2}/\iota^2)(v_d R/c^2)$, $D_p = (2B_{1,0}^2/5\iota^2)(v_d v/c)$, and $D_{PS} = (7B_{1,0}^2/5\iota^2)(v_d R/c)$. In these expressions, $v_d = v^2/RB^2$ is the drift velocity of the test particle. We found that the diffusion coefficient obtained by DCOM is larger compared with the analytical one in the R_{ax} = 3.75 m case. While the coefficient given by DCOM is close to the analytical one in the plateau and PS regimes, and smaller in the $1/\nu$ regime in the case of R_{ax} = 3.6 m.

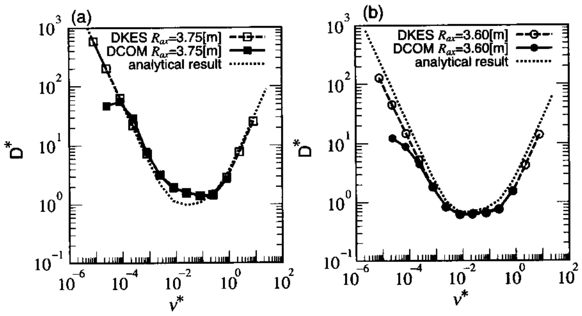

Fig. 3 The normalized mono-energetic diffusion coefficient as a function of normalized collision frequency at r/a = 0.5 (a) in the standard configuration and (b) in the inward-shifted configuration.

Comparing the diffusion coefficients for R_{ax} = 3.75 m and R_{ax} = 3.6 m cases, we can see the large difference between two configurations. In order to see the radial dependency of this difference in the $1/\nu$ regime, we evaluate the effective helical ripple, ε_{eff} [10], by which the diffusion coefficient in the $1/\nu$ regime, $D_{1/\nu}$, is given as

$$D_{1/\nu} = \frac{8\sqrt{2}}{9\pi}\frac{v_d}{\iota}\varepsilon_{eff}^{3/2}.$$

We plot the evaluated ε_{eff} by DCOM results and that by

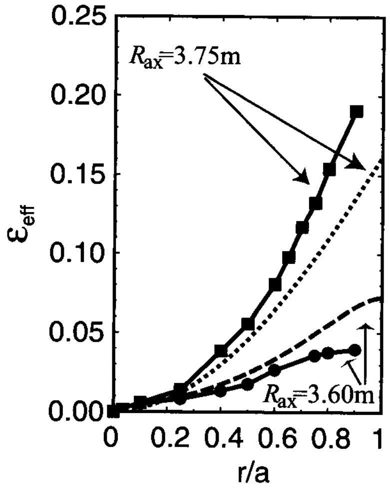

Fig. 4 The radial dependence of the effective helical ripple for ν^{-1} transport as a function of the normalized minor radius r/a. The full curves are obtained from the results of DCOM and the dotted (broken) curve is obtained from the analytic results.

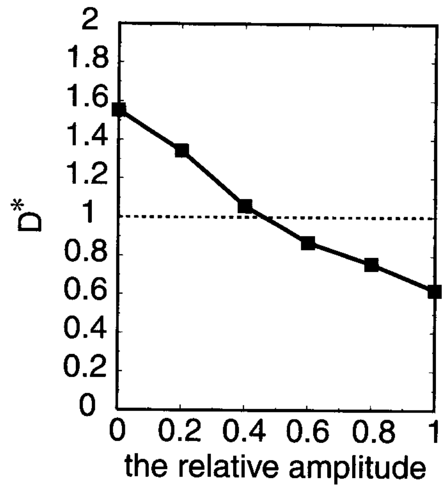

Fig. 5 The change of the plateau value given by DCOM at r/a = 0.5 in the inward-shifted configuration as a function of the relative amplitude of $B_{1,10}$ component from original value.

the multi-helicity model as a function of the normalized minor radius in Fig. 4. It is found that the difference of ε_{eff} between two configurations increases as the minor radius increases. In the outer region ($r/a \cong 0.9$) the diffusion coefficient of R_{ax} = 3.75 m case is about ten times larger than that of R_{ax} = 3.6 m case. Also we can see the increase of difference between the DCOM and multi-helicity model results in the outer regions for both configurations.

It is interesting to note that we can see the improvement of diffusion coefficient by axis shift not only in the $1/\nu$ regime but also in the plateau regime. The difference in the $1/\nu$ regime can be explained by the reduction of radial drift. However, the improvement of diffusion coefficient in the plateau regime can not be simply explained. This would be due to the coupling of toroidal and helical contribution by (1,10) magnetic field component in the plateau regime [17]. To make clear this point we have changed the value of (1,10) mode artificially from 0 to 100 %. Figure 5 shows the change of the plateau value given by DCOM at r/a = 0.5 as a function of the relative amplitude of (1,10) mode from original value. We clearly see the reduction of the plateau vale by increasing the relative amplitude of fraction. This shows the important role of the (1,10) mode in the improvement in the plateau regime.

4.2 Diffusion Coefficient with E_r

Next we study the effect of radial electric field, $E_r(= -\nabla\phi)$ on the neoclassical diffusion coefficient. We take into account the radial electric field introducing an electrostatic potential, ϕ, as $\phi = \phi_0\{1-(r/a)\}$. In Fig. 6 the diffusion coefficients of two configurations at r/a = 0.5 are shown changing the amplitude of radial electric field; strong reductions of the diffusion coefficients are observed in the $1/\nu$ regime in both cases and we can see the changes of collision frequency dependence of D from.

We can see that the effect of radial electric field is stronger in the R_{ax} = 3.75 m case $D \propto 1/\nu$ to $D \propto \sqrt{\nu}$ and $D \propto \nu$ and the difference between diffusion coefficients in two configurations becomes smaller in the ν regime with strong radial electric field. It is also found that the plateau values are almost independent with the value of radial electric field. We compare these coefficients with that by DKES and obtain good agreements from P-S regime through $1/\nu$ regime for both configurations.

5. Conclusion

We have investigated the neoclassical transport in the inward shifted LHD configuration (R_{ax} = 3.6 m) comparing with that in the standard configuration (R_{ax} = 3.75 m). We have applied a newly developed Monte Carlo simulation code, DCOM, to evaluate local neoclassical diffusion coefficients. We have compared our results with the results by DKES code with and without a radial electric field, and obtain good agreements between two code results. This suggests

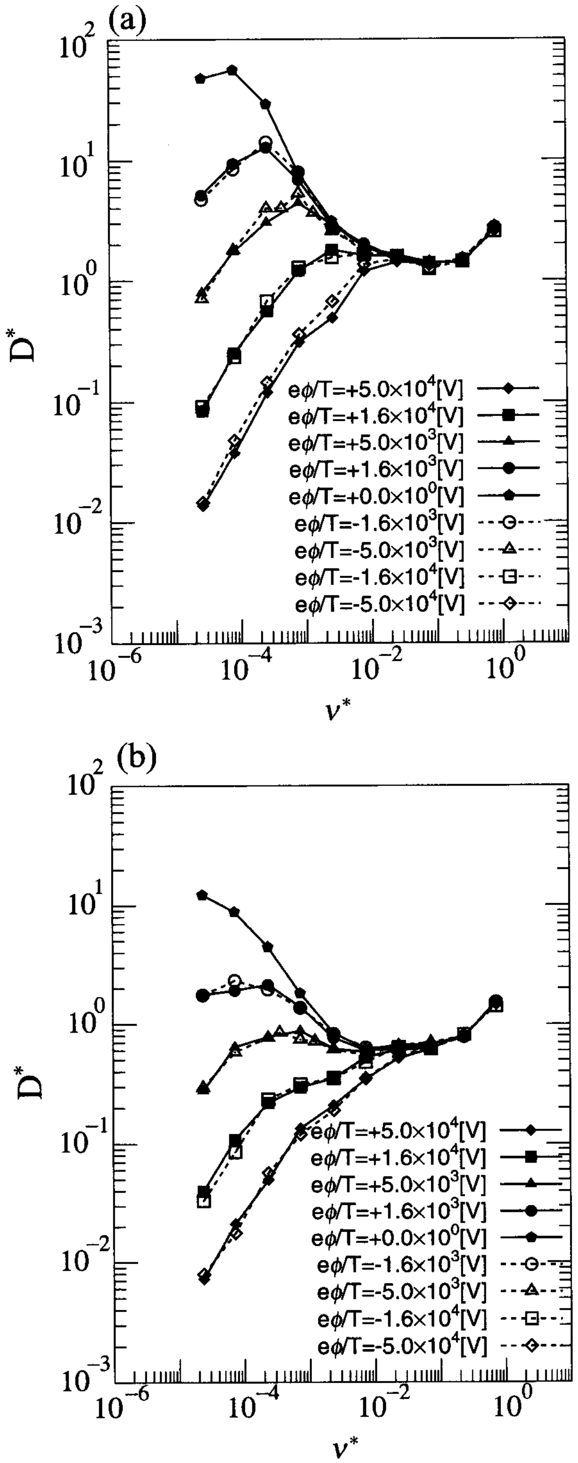

Fig. 6 The normalized mono-energetic diffusion coefficients as a function of the normalized collision frequency at r/a= 0.5 (a) in the standard configuration and (b) in the inward-shifted configuration. Here, the normalized electric potential $e\phi/T$ is used instead of radial electric fields E_r.

validity of our code for the analysis of neoclassical transport in LHD. Also a small discrepancy in the low collision frequency case without a radial electric field indicates the limit of the model assumed in the DKES.

It is found that the diffusion coefficients of R_{ax} = 3.6 m case is about three to ten times smaller than that of R_{ax} = 3.75 m case and this improvement can be seen not only in the $1/\nu$ regime but also in the plateau regime. We show that the (1,10) component of magnetic field plays important role in this improvement in the plateau regime. It is also found the analytic multi-helicity model gives smaller value for R_{ax} = 3.75 m case and larger value for R_{ax} = 3.6 m.

The effect of radial electric field on the neoclassical transport coefficient is also studied. We found that the difference of diffusion coefficient becomes smaller with strong radial electric field.

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