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Revisit to the Helicity and the Extended Generalized Self-organization Theory

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Abstract

It is clarified that the "helicity conservation law" is never "the conservation equation of the helicity K itself", but is merely "the time change rate equation of K". It is shown that since the total helicity K can never be conserved in the real experimental systems, the conjecture of the total helicity invariance is not physically available to real magnetized plasmas. With the use of auto-correlations for physical quantities, a novel extended generalized self-organization theory is presented, that neither based on the variational principle nor the energy principle. The self-organized states of every quantities may be realized during their own phases, and the dynamical system may evolve repeatedly out of phases self-organizations among quantities, depending on boundary conditions and input powers.

Keywords:

magnetic helicity, equation of change rate, conservation law, variational calculus, self-organization, auto-correlation, minimum change rate auto-correlation

1. Introduction

Since Dr. J.B. Taylor published his famous theory [1] to explain the appearance of the reversed field pinch (RFP) configuration [2], the magnetic helicity K has been believed to have important role as a global invariant in the self-organization process or the relaxation one of magnetized plasmas [3,4]. On the other hand, one of the authors (Y.K.) has been proposed the partially relaxed state model (PRSM) of the RFP in order to explain experimental data [5]. Without using the concept of the magnetic helicity, an energy integral was derived to deduce the PRSM and the mode transition point of the self-organized state in order to explain experimental data on the RFP [8.9].

In this paper, we study again the meaning of the magnetic helicity itself from the thought analysis [6], because of many evidences showing no invariance of the total helicity in simulations [7] and various experiments [9,10].

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2. Theoretical Thought Analysis

With the use of the thought analysis [6], we show here that although the energy conservation law is always physically correct, the "helicity conservation law" is never "the conservation equation of the helicity K itself".

Both of the energy conservation law and the helicity conservation one are derived from the following axiom set of physical laws of Maxwell's equations written in the MKSA unit used in the usual text books:

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}, \tag{1}$$

$$\frac{\partial \boldsymbol{D}}{\partial t} = -\boldsymbol{j} + \nabla \times \boldsymbol{H}, \tag{2}$$

$$\nabla \cdot \boldsymbol{D} = \rho, \tag{3}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{4}$$

We can get Poynting's energy conservation law, using Eqs.(1)–(4), as follows

$$W_{\mathrm{f}} = \int_V \left(\frac{\boldsymbol{E}\cdot\boldsymbol{D}}{2} + \frac{\boldsymbol{H}\cdot\boldsymbol{B}}{2} \right) \mathrm{d}V, \tag{5}$$

$$\frac{\partial W_{\mathrm{f}}}{\partial t} = -\int_V \boldsymbol{j}\cdot\boldsymbol{E}\,\mathrm{d}V - \frac{1}{\mu_0}\oint_S (\boldsymbol{E}\times\boldsymbol{B})\cdot \mathrm{d}\boldsymbol{S}. \tag{6}$$

The two physical laws of Eqs.(1) and (4) are rewritten equivalently by the following two equations with the use of the scalar and the vector potentials;

$$\frac{\partial \boldsymbol{A}}{\partial t} = -\nabla\phi - \boldsymbol{E}, \tag{7}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}. \tag{8}$$

The magnetic helicity K is basically defined by the following equation [1]:

$$K \equiv \frac{1}{\mu_0}\int_V \boldsymbol{A}\cdot\boldsymbol{B}\,\mathrm{d}V. \tag{9}$$

Here, we emphasize that even if we include "the external helicity", taking account of the gauge invariance [1], the following argument is still essentially correct and applicable. After the partial derivative of the definition Eq.(9) with respect to t, and using only two physical laws of Eqs.(1) and (4) with their equivalent Eqs.(7) and (8), we obtain the following equation for "the time change rate of K",

$$\frac{\partial K}{\partial t} = -\frac{2}{\mu_0}\int_V \boldsymbol{B}\cdot\boldsymbol{E}\,\mathrm{d}V + \frac{1}{\mu_0}\oint_S (\boldsymbol{E}\times\boldsymbol{A} - \phi\boldsymbol{B})\cdot \mathrm{d}\boldsymbol{S}. \tag{10}$$

Assuming $(\boldsymbol{E}\cdot\boldsymbol{D})/2 \ll (\boldsymbol{H}\cdot\boldsymbol{B})/2$ in the plasma confinement experiments, we obtain the following equation for the field energy W_{f},

$$W_{\mathrm{f}} = \int_V \left(\frac{\boldsymbol{E}\cdot\boldsymbol{D}}{2} + \frac{\boldsymbol{H}\cdot\boldsymbol{B}}{2} \right) \mathrm{d}V \cong W_{\mathrm{m}}, \tag{11}$$

where W_{m} is the magnetic field energy component of W_{f}. From Eqs.(11) and (6), we obtain

$$\frac{\partial W_{\mathrm{f}}}{\partial t} \cong -\int_V \boldsymbol{j}\cdot\boldsymbol{E}\,\mathrm{d}V - \frac{1}{\mu_0}\oint_S (\boldsymbol{E}\times\boldsymbol{B})\cdot \mathrm{d}\boldsymbol{S}. \tag{12}$$

Using the simplified Ohm's law Eq.(13), we obtain the conservation laws of W_{f} and K as follows,

$$\text{Ohm's law:}\quad \boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} = \eta\boldsymbol{j}. \tag{13}$$

$$\frac{\partial W_{\mathrm{f}}}{\partial t} \cong -\int_V \{\eta\boldsymbol{j}\cdot\boldsymbol{j} + (\boldsymbol{j}\times\boldsymbol{B})\cdot\boldsymbol{v}\}\,\mathrm{d}V - \frac{1}{\mu_0}\oint_S (\boldsymbol{E}\times\boldsymbol{B})\cdot \mathrm{d}\boldsymbol{S}. \tag{14}$$

$$\frac{\partial K}{\partial t} = -\frac{2}{\mu_0}\int_V \eta\boldsymbol{j}\cdot\boldsymbol{B}\,\mathrm{d}V + \frac{1}{\mu_0}\oint_S (\boldsymbol{E}\times\boldsymbol{A} - \phi\boldsymbol{B})\cdot \mathrm{d}\boldsymbol{S}. \tag{15}$$

The misunderstanding on the "helicity conservation law" has been established from the following argument, using "the time change rate equation of K written by Eq.(15)" [1]. (A) At first, we consider "the ideal case" where the whole region of plasmas inside the boundary is filled with the ideally conducting plasma and the boundary surface is the ideally conducting wall, i.e., η = 0 and $\boldsymbol{E}$ = 0 and $\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{S}$ = 0 at the ideally conducting wall. We then get from Eq.(15) in this "ideal case" that "the time change rate of K" becomes as $\partial K/\partial t = 0$. (B) From this result, we may conclude as follows: Since the value of K is constant along the time variable t, the total helicity K is conserved and therefore it must be "the time invariant in the dynamical system in the case of ideal plasmas".

However, the part of (A) declares only that the value of K defined by Eq.(9) does not change along the time variable t in "the trivial case" of η = 0 plasmas filling fully within the ideally conducting wall.

From the following simple thought experiment, we can easily find that the total helicity K is never "the time invariant inside the ideally conducting wall". We consider a case, where some vacuum field regions with $\eta = \infty$ separate slightly the η = 0 plasmas from the ideally conducting wall. We then have to come back to Eq.(10), and we can put $\boldsymbol{E}$ = 0 in the plasma but have to leave $\boldsymbol{E}$ in the vacuum field region. In this simple case, the value of $\partial K/\partial t$ is passively and resultantly determined by the volume integral of $\boldsymbol{B}\cdot\boldsymbol{E}$ in Eq.(10). The total helicity K can never be conserved in the dynamical system in this simple case. The simple thought experiment shown above may lead us to a conclusion that the helicity conservation during the relaxation of magnetized plasmas does stay "Taylor's conjecture" forever.

On the other hand, the energy conservation law of Eq.(14) declares that even if η = 0 or η = 0 and $\boldsymbol{E}$ = 0 at the ideally conducting wall, the left-hand side $\partial W_{\mathrm{f}}/\partial t$ always balances with the volume integral term of $(\boldsymbol{j}\times\boldsymbol{B})\cdot\boldsymbol{v}$, which is called the dynamo term, in the right-hand side. The value of the helicity K has never been conserved in the computer simulations by R. Horiuchi and T. Sato [7], and also in all experiments on the reversed field pinch (RFP) by many authors [2,9], on the toroidal Z-pinch by K. Sugisaki [10] and on merging two Spheromaks into one field reversed configuration (FRC) or one Spheromak by Y. Ono, Katsurai et al. [11]. Especially, in the case of the toroidal Z-pinch experiments, the total helicity K increases to finite values from zero initial value within a few tens of μs [10]. These experimental results have been demonstrated that the conjecture of the total helicity invariance by Dr. J.B. Taylor is not physically available to real magnetized plasmas.

As is well known, the variational principle and the related or resultant dynamic equations are physically equivalent, i.e., we can start with either the variational principle or the related set of dynamic equations in order to analyze the systems. This fundamental physical thought is also the same for the energy principle and the related dynamic equations. Without using the concept of the helicity K, we can derive the Taylor state $\nabla \times \boldsymbol{B} = \lambda_T \boldsymbol{B}$ from the equation of the relaxed state of MHD plasmas written as $\nabla \times (\eta \nabla \times \boldsymbol{B}) = \lambda \boldsymbol{B}$, which includes the Taylor state of $\nabla \times \boldsymbol{B} = \lambda_T \boldsymbol{B}$ as a special case where η is spatially uniform [12].

3. Generalized Self-organization Theory

We develop here a novel extended generalized self-organization theory that is an extension of the last report in [12]. It should be emphasized here that the generalized self-organization theory with the use of auto-correlations for physical quantities is not fundamentally based on neither the variational principle nor the energy principle, and the auto-correlations is never time invariants.

Quantities with n elements in general dynamic systems of interest shall be expressed as $\boldsymbol{q}(t, \boldsymbol{x}) = \{q_1(t, \boldsymbol{x}), q_2(t, \boldsymbol{x}), \cdots, q_n(t, \boldsymbol{x})\}$. Here, t is time, $\boldsymbol{x}$ denotes m-dimensional space variables, and $\boldsymbol{q}$ represents a set of physical quantities having n elements. We consider a dissipative nonlinear dynamic system which may be generally described by

$$\frac{\partial q_i}{\partial t} = G_i[\boldsymbol{q}], \qquad (16)$$

where $G_i[\boldsymbol{q}]$ denotes linear or nonlinear dynamic operators, which may include no dissipative and/or dissipative terms. After taking the product of $q_i(t, \boldsymbol{x})$ and both sides of Eq.(16), and integrating both sides of the resultant equation over the volume V, we obtain the conservation equations for $q_i(t, \boldsymbol{x})$ as follows,

$$\int_V \{\frac{\partial}{\partial t}\frac{1}{2} q_i(t,\boldsymbol{x}) q_i(t,\boldsymbol{x})]\}\, dV = \int_V \{q_i(t,\boldsymbol{x})\, G_i[\boldsymbol{q}]\}\, dV. \qquad (17)$$

From the standpoint of observation on over all time evolution of the dynamic system, we can identify or define "the self-organized state" as "the state of the most unchangeable structure". The definition may be mathematically expressed by using auto-correlations, $q_i(t, \boldsymbol{x}) q_i(t + \Delta t, \boldsymbol{x})$, between the time, t, and slightly transferred time, $t + \Delta t$, with a small Δt in the following way, i.e., self-organized state is defined by

$$\min \left| \frac{\int q_i(t,\boldsymbol{x}) q_i(t+\Delta t,\boldsymbol{x})\, dV}{\int q_i(t,\boldsymbol{x}) q_i(t,\boldsymbol{x})\, dV} - 1 \right| \text{state}. \qquad (18)$$

Substituting the Taylor expansion of $q_i(t + \Delta t, \boldsymbol{x}) = q_i(t, \boldsymbol{x}) + [\partial q_i(t, \boldsymbol{x})/\partial t]\Delta t + \cdots$ into the definition Eq.(18), and taking account of the arbitrary smallness of Δt, we obtain the following equivalent definition for of the self-organized state from the first order of Δt in the definition Eq.(18):

$$\min \left| \frac{\int q_i(t,\boldsymbol{x}) \left[\partial q_i(t,\boldsymbol{x})/\partial t\right] dV}{\int q_i(t,\boldsymbol{x}) q_i(t,\boldsymbol{x})\, dV} \right| \text{state}. \qquad (19)$$

Substituting the original dynamic equation, Eq.(16), into Eq.(19), we obtain the following final condition for the self-organized state

$$\min \left| \frac{\int q_i(t,\boldsymbol{x})\, G_i[\boldsymbol{q}]\, dV}{\int q_i(t,\boldsymbol{x}) q_i(t,\boldsymbol{x})\, dV} \right| \text{state}. \qquad (20)$$

Since we have substituted the original dynamic equations into the definition of the self-organized state, we can recognize that "whole properties of the dynamic system is essentially embedded in the process of calculations to derive the self-organized state from the final condition of Eq.(20)".

The mathematical expressions with the use of the variational calculus for the definition of Eq.(19) and further the final condition Eq.(20) are written as follows, defining a functional F with use of a Lagrange multiplier λ_i:

$$F \equiv \int_V \{q_i(t,\boldsymbol{x})\, G_i[\boldsymbol{q}] + \lambda_i\, q_i(t,\boldsymbol{x}) q_i(t,\boldsymbol{x})\}\, dV. \qquad (21)$$

$$\delta F = 0, \qquad (22)$$

$$\delta^2 F > 0\,, \tag{23}$$

where δF and $\delta^2 F$ are respectively the first and the second variations of F "with respect to the variation $\boldsymbol{\delta q}(\boldsymbol{x})$ only for the spatial variable $\boldsymbol{x}$". Comparing Eqs.(17) and (21), we can find that the conservation equations concerning with the quantities $q_i(t, \boldsymbol{x})$ for the dynamic system of interest are naturally included in the present formulation of the generalized self-organization theory. The implicit assumption in this theory is that the dynamical system evolves all possible area in state phases.

When we apply the extended generalized self-organization theory to fusion plasmas, we use here the following three more physical laws, i.e., the conservation laws of the mass, Eq.(24), and the momentum, Eq.(25), and the generalized Ohm's law, Eq.(26).

$$\frac{\partial \rho_{\rm m}}{\partial t} = -\nabla \cdot (\rho_{\rm m} \boldsymbol{v})\,. \tag{24}$$

$$\rho_{\rm m} \frac{\partial \boldsymbol{v}}{\partial t} = -\rho_{\rm m}(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + [\rho_{\rm e}\boldsymbol{E} + \boldsymbol{j}\times\boldsymbol{B} - \nabla(P_{\rm e}+P_{\rm i})]\,. \tag{25}$$

$$\frac{\partial \boldsymbol{j}}{\partial t} = \frac{e^2 n_{\rm e}}{m_{\rm e}}\{\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} - \eta_{\rm ei}\,\boldsymbol{j} - \frac{1}{en_{\rm e}}(\boldsymbol{j}\times\boldsymbol{B}) + \frac{1}{en_{\rm e}}[\nabla P_{\rm e} - (m_{\rm e}/m_{\rm i})Z_{\rm i}\nabla P_{\rm i}]\}\,. \tag{26}$$

These three equations come from the Boltzmann kinetic equations for electrons and ions. Therefore, we start with an axiom set of seven physical laws, and the charge conservation law is included in Maxwell's equations, Eqs.(1)–(4). Conservation laws of $W_{\rm f}$, the kinetic energy $W_{\rm k}$, and the current defined by $W_{\rm c} = \int_V (1/2)\boldsymbol{j}\cdot\boldsymbol{j}\,{\rm d}V$ are obtained as follows:

$$\frac{\partial W_{\rm f}}{\partial t} = -\int_V \boldsymbol{j}\cdot\boldsymbol{E}\,{\rm d}V - \frac{1}{\mu_0}\oint_S (\boldsymbol{E}\times\boldsymbol{B})\cdot{\rm d}\boldsymbol{S}\,. \tag{27}$$

$$\frac{\partial}{\partial t}W_{\rm k} = \int_V \{-\frac{1}{2}\boldsymbol{v}\cdot\boldsymbol{v}\nabla\cdot(\rho_{\rm m}\boldsymbol{v}) - \rho_{\rm m}\boldsymbol{v}\cdot[(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}] + [\rho_{\rm e}\boldsymbol{E}\cdot\boldsymbol{v} + (\boldsymbol{j}\times\boldsymbol{B})\cdot\boldsymbol{v} - \rho_{\rm m}\boldsymbol{v}\cdot\nabla(P_{\rm e}+P_{\rm i})]\}\,{\rm d}V\,. \tag{28}$$

$$\frac{\partial}{\partial t}W_{\rm c} = \int_V \frac{e^2 n_{\rm e}}{m_{\rm e}}\{\boldsymbol{j}\cdot\boldsymbol{E} - (\boldsymbol{j}\times\boldsymbol{B})\cdot\boldsymbol{v} - \eta_{\rm ei}\,\boldsymbol{j}\cdot\boldsymbol{j} + \frac{1}{en_{\rm e}}\boldsymbol{j}\cdot[\nabla P_{\rm e} - (m_{\rm e}/m_{\rm i})Z_{\rm i}\nabla P_{\rm i}]\}\,{\rm d}V\,. \tag{29}$$

According to Eq.(21), we obtain the functional for the field energy $F_{\rm f}$, for the kinetic energy $F_{\rm k}$, and for the current $F_{\rm c}$, respectively, as follows,

$$F_{\rm f} = \int_V \{-\boldsymbol{j}\cdot\boldsymbol{E} + \lambda_{\rm f}(\frac{\varepsilon_0 \boldsymbol{E}\cdot\boldsymbol{E}}{2} + \frac{\boldsymbol{B}\cdot\boldsymbol{B}}{2\mu_0})\}\,{\rm d}V - \frac{1}{\mu_0}\oint_S (\boldsymbol{E}\times\boldsymbol{B})\cdot{\rm d}\boldsymbol{S}\,, \tag{30}$$

$$F_{\rm k} = \int_V \{-\frac{1}{2}\boldsymbol{v}\cdot\boldsymbol{v}\nabla\cdot(\rho_{\rm m}\boldsymbol{v}) - \rho_{\rm m}\boldsymbol{v}\cdot[(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}] + [\rho_{\rm e}\boldsymbol{E}\cdot\boldsymbol{v} + (\boldsymbol{j}\times\boldsymbol{B})\cdot\boldsymbol{v} - \rho_{\rm m}\boldsymbol{v}\cdot\nabla(P_{\rm e}+P_{\rm i})] + \lambda_{\rm v}\frac{\rho_{\rm m}}{2}\boldsymbol{v}\cdot\boldsymbol{v}\}\,{\rm d}V\,, \tag{31}$$

$$F_{\rm c} = \int_V \frac{e^2 n_{\rm e}}{m_{\rm e}}\{\boldsymbol{j}\cdot\boldsymbol{E} - (\boldsymbol{j}\times\boldsymbol{B})\cdot\boldsymbol{v} - \eta_{\rm ei}\,\boldsymbol{j}\cdot\boldsymbol{j}] + \frac{e}{m_{\rm e}}\boldsymbol{j}\cdot[\nabla P_{\rm e} - (m_{\rm e}/m_{\rm i})Z_{\rm i}\nabla P_{\rm i}] + \lambda_{\rm c}\,\boldsymbol{j}\cdot\boldsymbol{j}\}\,{\rm d}V\,. \tag{32}$$

In general, we take variations with respect to $\boldsymbol{\delta E}$, $\boldsymbol{\delta B}$, $\delta \boldsymbol{v}$, $\delta \boldsymbol{j}$, $\delta\rho_{\rm m}$, $\delta\rho_{\rm e}$, $\delta P_{\rm e}$, $\delta P_{\rm i}$, $\delta n_{\rm e}$, $\delta n_{\rm i}$, and $\delta\eta_{\rm ei}$. From the Euler-Lagrange equations for the solutions of Eq.(22), we will get new various equilibrium configurations of the self-organized states with the plasma flow, the shear flow, the space charge, the space potential, the deviation between the ion and the electron density profiles, the resistivity profile, and so on, depending on the boundary conditions and the external input sources such as the various energy injections and the particle beams. The resultant equilibrium configurations are far beyond the conventional MHD equilibrium ones by the Grad-Shafranov equation based on the equation of $\boldsymbol{j}\times\boldsymbol{B} = \nabla p$. The results of the present calculation will appear elsewhere.

Dividing Eq.(29) by $e^2 n_{\rm e}/m_{\rm e}$, we find the resultant equation becomes a power balance equation.

4. Concluding Remarks

Analyzing the logical and mathematical structures of the derivation process for the "helicity conservation law", and using the simple thought experiment for the case, where some vacuum field regions with $\eta = \infty$ separate slightly from the $\eta = 0$ plasma from the ideally conducting wall, we have clarified that "the so-called helicity conservation" is never "the conservation equations of the helicity K", but is merely the equations for "the time change rate of K". The total helicity K can never be conserved in the real experimental dynamic systems, as was observed in the toroidal Z-pinch experiments [10], the conjecture of the total helicity invariance by Dr. J.B. Taylor is not physically available to real magnetized plasmas.

In Section **3**, we have presented the extended generalized self-organization theory that is generalization of the theory in [6]. We have shown that "whole properties of the dynamic system is essentially embedded in the process of calculations to derive the self-organized state from the final condition of Eq.(20)". It should be emphasized here that the extended generalized self-organization theory can deduce the Taylor state without using the concept of the helicity, and further be applicable for any nonlinear dynamical systems [12,13,14,15]. It is important to point out that the self-organized states of every physical quantities of interest may be realized during their own peculiar phases, i.e., not at the same time but some of quantities reach to their self-organized states mutually at different times. The dynamical system may evolve repeatedly in a state phase diagram, having out of phase among quantities, i.e., "out of phases self-organizations for each quantity", due to boundary conditions and input beams and/or powers.

In order to realize the steady state of the confinement system of plasmas, we can extend conventional methods of plasma current drives with the use of the three conservation laws of Eqs.(27)–(29), using energy injections with use of various types of energies, such as magnetic energies, electromagnetic wave energies, internal energies of plasmoids by plasma guns, which induce the thermal plasma flow velocity, various particle beam energies, and so on. These additional injections give us more dynamic equations added to Eq.(16) for the system of interest.

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